On the nonlinear stability of eccentrically stiffened functionally graded annular spherical segment

Vu Thi Thuy Anh, Nguyen Dinh Duc*

Vietnam National University, Hanoi – 144 Xuan Thuy – Cau Giay – Hanoi – Vietnam

Introduction

In recent years, many authors have focused on the static and dynamic of eccentrically stiffened shell structures because the shells as other composite structures, usually reinforced by stiffening members to provide the benefit of added load-carrying static and dynamic capability with a relatively small additional weight penalty. In additions, eccentrically stiffened shallow shell is a very important structure in engineering design of aircraft, missile and aerospace industries. As a result, there are many researches on the static and dynamic of eccentrically stiffened shell structures, especially structures made of composite material.

As well as know a functionally graded material (FGM) is a two-component composite characterized by a compositional gradient from one component to the other. In contrast, traditional composites are homogeneous mixtures, and they therefore involve a compromise between the desirable properties of the component materials. Since significant proportions of an FGM contain the pure form of each component, the need for compromise is eliminated. The properties of both components can be fully utilised. This is mainly due to the increasing use of FGM as components of structures in the advanced engineering.

In this paper, the nonlinear analysis of eccentrically stiffened FGM annular spherical segment shells is investigated. The segment-shells are reinforced by eccentrically longitudinal and transversal stiffeners made of full metal or full ceramic depending on situation of stiffeners at metal-rich side or ceramic-rich side of the shell respectively. The paper analyzed and discussed the effects of material and geometrical properties, elastic foundations and eccentrically stiffeners on the stability of the eccentrically stiffened FGM annular spherical segment.

Nomenclature

k - The volume fraction index (non-negative number)

w - The deflection of the annular spherical shell

k_1 - The Winkler foundation modulus

k_2 - The shear layer foundation stiffness of Pasternak.

$\varepsilon_r^0, \varepsilon_\theta^0$ - The normal strains

$\gamma_{r\theta}^0$ - The shear strain at the middle surface of the spherical shell

$\chi_r, \chi_\theta, \chi_{r\theta}$ - The changes of curvatures and twist

s_1, s_2 - The distance between eccentrically longitudinal and latitude stiffeners respectively.

A_1, A_2 - The cross-sectional area of eccentrically longitudinal and latitude stiffeners respectively.

d_1, d_2, h_1, h_2 - The width and height of eccentrically longitudinal and latitude stiffeners respectively.

n_1, n_2 - The numbers of eccentrically longitudinal and latitude stiffeners respectively.

E_0 - The Young's modulus of the stiffeners. if the stiffeners are reinforced at the surface of the ceramic-rich, if the stiffeners are reinforced at the surface of the metal-rich.

Model annular spherical shell

An FGM annular spherical segment or a FGM open annular spherical shell limited by two meridians and two parallels of a spherical shell resting on elastic foundations with radius of curvature R, base radius of lower and upper bases r_1, r_0 respectively, open angle of two meridional planes β and thickness h. The FGM annular spherical segment reinforced by eccentrically longitudinal and transverse stiffeners is subjected to external pressure uniformly distributed on the outer surface as shown in Fig.1.

Basic equations

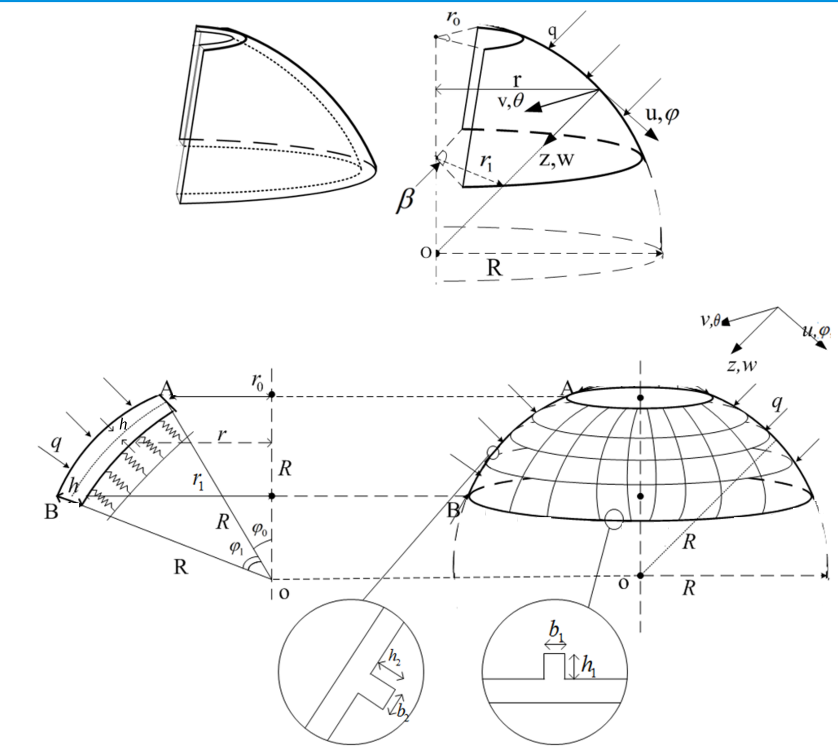

Fig. 1. Configuration of a FGM annular spherical segment shells and eccentrically stiffened FGM annular spherical shell.

The nonlinear equilibrium equations of a perfect shell based on the classical shell theory [1]

$$\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial \theta} + \frac{N_r}{r} - \frac{N_\theta}{r} = 0, \qquad (1)$$

$$\frac{\partial N_\theta}{r\partial \theta} + \frac{\partial N_{r\theta}}{\partial r} + \frac{2N_{r\theta}}{r} = 0, \qquad (2)$$

$$\frac{\partial^2 M_r}{\partial r^2} + \frac{2}{r}\frac{\partial M_r}{\partial r} + 2\left(\frac{\partial^2 M_{r\theta}}{r\partial r\partial \theta} + \frac{1}{r^2}\frac{\partial M_{r\theta}}{\partial \theta}\right) + \frac{1}{r^2}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{1}{r}\frac{\partial M_\theta}{\partial r} +$$
$$+\frac{1}{R}(N_r + N_\theta) + N_r\frac{\partial^2 w}{\partial r^2} - 2N_{r\theta}\left(\frac{1}{r^2}\frac{\partial w}{\partial \theta} - \frac{\partial^2 w}{r\partial r\partial \theta}\right) +$$
$$+N_\theta\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + q - k_1 w + k_2\Delta w = 0. \qquad (3)$$

The constitutive stress-strain equations by Hooke law for the shell material are omitted here for brevity.

$$N_r = \left(A_{11} + \frac{E_0 A_1}{s_1}\right)\varepsilon_r^0 + A_{12}\varepsilon_\theta^0 - (B_{11} + C_1)\chi_r - B_{12}\chi_\theta,$$

$$N_\theta = A_{12}\varepsilon_r^0 + \left(A_{22} + \frac{E_0 A_2}{s_2}\right)\varepsilon_\theta^0 - B_{12}\chi_r - (B_{22} + C_2)\chi_\theta,$$

$$N_{r\theta} = A_{66}\gamma_{r\theta}^0 - 2B_{66}\chi_{r\theta},$$

$$M_r = (B_{11} + C_1)\varepsilon_r^0 + B_{12}\varepsilon_\theta^0 - \left(D_{11} + \frac{E_0 I_1}{s_1}\right)\chi_r - D_{12}\chi_\theta, \qquad (4)$$

$$M_\theta = B_{12}\varepsilon_r^0 + (B_{22} + C_2)\varepsilon_\theta^0 - D_{12}\chi_r - \left(D_{22} + \frac{E_0 I_2}{s_2}\right)\chi_\theta,$$

$$M_{r\theta} = B_{66}\gamma_{r\theta}^0 - 2D_{66}\chi_{r\theta},$$

Depending on the in-plane behavior at the edge of boundary conditions will be considered in cases the edges are simply supported, immovable and movable.

Case A: The edges of the annular spherical segment are simply supported and movable.

Case B: The edges of the annular spherical segment are simply supported and immovable.

From each boundary conditions in case A and B, approximate solutions for the nonlinear equations of u, v, w, introduction of these solutions into obtained 3 nonlinear equations of u, v, w, we obtain the equations, which have form $R_i(u,v,w) = 0, i = 1,2,3.$

Applying Galerkin method for the resulting, that are

$$\int_{r_0}^{r_1}\int_0^{\beta} R_1 \cos\frac{m\pi(r - r_0)}{r_1 - r_0}\sin(n\theta)\, r dr d\theta = 0;$$

$$\int_{r_0}^{r_1}\int_0^{\beta} R_2 \sin\frac{m\pi(r - r_0)}{r_1 - r_0}\cos(n\theta)\, r dr d\theta = 0,$$

$$\int_{r_0}^{r_1}\int_0^{\beta} R_3 \sin\frac{m\pi(r - r_0)}{r_1 - r_0}\sin(n\theta)\, r dr d\theta = 0. \qquad (5)$$

we obtain the following equations form for 2 case

$$a_{11}U + a_{12}V + a_{13}W + a_{14}W^2 = 0,$$

$$a_{21}U + a_{22}V + a_{23}W + a_{24}W^2 = 0, \qquad (6)$$

$$a_{31}U + a_{32}V + a_{33}W + (a_{34}U + a_{35}V + k_1 a_{36} + k_2 a_{37})W +$$
$$+a_{38}W^2 + a_{39}W^3 + a_{310}q = 0,$$

Eq. (13) allows determine the deflection curve equation with form

$$q = c_{11}W^3 + c_{12}W^2 + c_{13}W + (c_{14}k_1 + c_{15}k_2)W. \qquad (7)$$

Numerical results

Eq. (7) is used for determining the nonlinear stability of eccentrically stiffened functionally graded annular spherical segment under uniform external pressure.To validate the proposed approach, the critical loads of eccentrically stiffened FGM annular spherical segment with elastic foundations are compared with the critical load of FGM annular spherical segment under uniform external pressure by Phuong in the same conditions and geometrical parameters, the results are presented in table 1.

k	Phuong [1]	Case A	Case B
0	1.3859	1.3613	1.4062
1	0.7485	0.7378	0.7503
5	0.4508	0.4317	0.4632

Table 2. Effects of the elastic foundations and mode on the critical loads of annular spherical segments under external pressure

(K_1,K_2)	(0,0)	(10,0)	(100,10)	(0,10)	(10,20)
(m,n)	$R/h=800, r_0/R=0.05, r_1/R=0.5, \beta=\pi/6, k=1.$				
(1,1)	0.2302 (A)	1.3279 e5 (A)	1.3272 e6 (A)	74.1165 (A)	1.3286 e5 (A)
	- 6.2012 (B)	3.8912 e6 (B)	4.2804 e7 (B)	3.8912 e6 (B)	1.1673 e7 (B)
(5,1)	0.6531(A)	2.2487 e5 (A)	2.2441 e6 (A)	516.0362 (A)	2.3467 e4 (A)
	1.0938 (B)	2.8529 e5 (B)	3.1382 e6 (B)	2.8529 e5 (B)	8.5587 e5 (B)
(9,1)	0.4313 (A)	0.6834 e5 (A)	0.6838 e6 (A)	448.7510 (A)	6.9242 e4 (A)
	0.5935 (B)	0.8949 e5 (B)	0.9844 e5 (B)	8.9499 e4 (B)	2.6849 e5 (B)
(1,3)	0.5813 (A)	1.0258 e5 (A)	1.0263 e6 (A)	434.1690 (A)	1.0345 e5 (A)
	10.2791 (B)	1.9897 e6 (B)	2.1887 e7 (B)	1.9897 e6 (B)	5.9692 e6 (B)
(1,5)	1.9205 (A)	0.7404 e5 (A)	0.7413 e6 (A)	858.3343 (A)	0.7576 e5 (A)
	5.6218 (B)	1.0608 e6 (B)	1.1669 e7 (B)	1.0608 e6 (B)	3.1825 e6 (B)

Fig.2. shows the effects of volume fraction index k (0,1.5) on the nonlinear stability of eccentrically stiffened functionally graded annular spherical segment subjected to external pressure (mode (m,n)=(3,1)). As can be seen, the load–deflection curves become lower when increases.

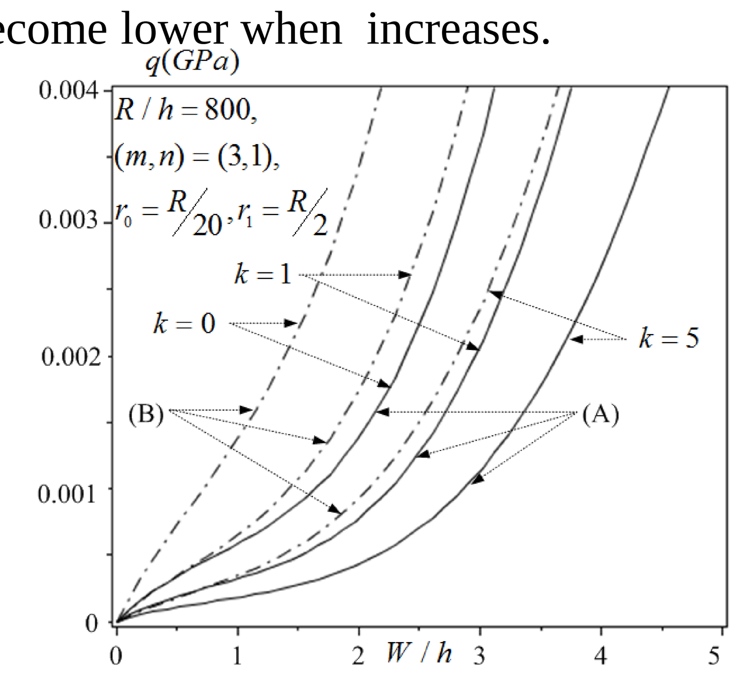

Fig. 2. Effects of k on the nonlinear stability of eccentrically stiffened FGM annular spherical segment

Conclusions

The present paper aims to propose a nonlinear analysis of eccentrically stiffened FGM annular spherical segment shells on elastic foundations under uniform external pressure. Approximate solutions are assumed to satisfy the simply supported boundary condition and Galerkin method is applied to obtain closed-form relations of bifurcation type of nonlinear stability. The effects of material, geometrical properties, elastic foundations, combination of external pressure and stiffener arrangement, stiffener number on the nonlinear stability of eccentrically stiffened FGM annular spherical segment are analyzed and discussed.

References

[1] Bich DH, Phuong NT. Buckling analysis of FGM annular spherical shells and segments subjected to mechanical loads.VNU J of Mathem-Phys 2013;29(3):14-31.
[2] Anh VTT, Bich DH, Duc ND. Nonlinear buckling analysis of thin FGM annular spherical shells on elastic foundations under external pressure and thermal loads. Eur J of Mechan – A/Solids 2015; 50: 28-38.
[3] Anh VTT, Duc ND. The nonlinear stability of axisymmetric FGM annular spherical shells under thermo-mechanical load. Mechan of Adv Mater and Struct, 2016, DOI: 10.1080/15376494.2015.1091528