Integration of Compressed Sensing and Frequency Hopping Techniques for Ultrasound Tomography

Tran Quang-Huy[1,2], Tran Duc-Tan[2]
[1]Faculty of Physics, Ha Noi Pedagogical University No2, Hanoi, Vietnam
[2]Faculty of Electronics & Telecommunications, VNU University of Engineering & Technology, Hanoi, Vietnam
E-mail: tranquanghuy@hpu2.edu.vn, tantd@vnu.edu.vn

***Abstract*—The FH-DBIM (frequency hopping distorted Born iterative method) is a promising approach for imaging tumors at the level of biological tissues. This uses multiple frequencies for producing images, leading to large computational complexity and storage space (i.e. for each frequency, $N_tN_rN^2$ calculations are required). It is a main barrier that the FH-DBIM approach is not fully exploited in commercialized devices. Therefore, in this paper, we propose an alternative algorithm for significant reducing the computational complexity of the FH-DBIM by integrating compressed sensing and frequency hopping techniques for the DBIM. That is, the needed number of measurements for imaging is tremendously reduced in comparison with the conventional one. The proposed algorithm – CS-FH-DBIM– offers a new prospect for being able to image high-resolution images at the level of biological tissues in practice.**

Keywords—Ultrasound, tomography, inverse scattering, Distorted Born iterative method (DBIM), compressed sensing (CS), frequency hopping (FH).

I. Introduction

Conventional ultrasound imaging works based on a pulse echo method that cannot detect structures whose size is below the wavelength level [1]. Ultrasound tomography will offer a better resolution by placing the number of transmitters and receiver around the object, along with one of inverse scattering techniques [2]. Most of research on ultrasound tomography tries to estimate the sound contrast and the size of the structure (if exist) in the medium [3]. Due to the high computational cost of ultrasound tomography, commercialized devices are not popular.

In ultrasound tomography, Born approximation that assumes the scattering field is relatively small in comparison with the incident field is popularly used. The DBIM –Distorted Born iterative method– is popular in diffraction tomography [4]. In [1], edge detection during the iterative process was introduced in order to speed up the convergence and to enhance the quality of reconstruction, but the complexity and noise sensitivity issues remain. In [5], the multi-level fast multi-pole algorithm (MLFMA) was applied to the forward solver to further speed up the reconstruction process. However, MLFMA requires high set-up costs that make it difficult to implement in practice.

Recently, some hybrid imaging methods which combine the high resolution of ultrasound imaging method with the high contrast capabilities of another are developed for breast cancer detection. Some of those are PAT (photo-acoustic tomography) [6], TAT (thermo-acoustic tomography) [7], UMT (ultrasound modulated tomography), TE (Transient Elastography) [8] and MAT (magneto-acoustic tomography) [9] which provide potential breakthroughs in clinical applications of hybrid imaging method for early detection of cancer, functional imaging, and molecular imaging among others. However, these imaging methods require complicated configuration settings and multiple parameters needed for imaging.

Compressed sensing (CS), which is introduced by Candes and Tao [10] and Donoho [11] in 2006, could acquire and reconstruct sparse signals at a rate lower than that of Nyquist. Random measurement approach in the detection geometry configuration is proposed in [12, 13]. A set of measurements of the scattered field is performed using sets of receiver's random positions. This method can reduce the computational complexity and improve the quality of the reconstruction of the sound contrast. However, the image reconstruction process is quite long. In this paper, we proposed an approach to enhance the imaging quality by applying the compressed sensing technique along with the multi-frequency technique for DBIM.

II. The Distorted Born Iterative Method

A measurement configuration is set up for tranducers (i.e. transmitters and receivers), located in a circle around the object in order to obtain the scattered data (see Fig.1). Each tranducer can both transmit and receive. At an instance, only one transmitter and one receiver are active to for a corresponding measured data value. This data was processed using DBIM to reconstruct the sound contrast of scatters. In this way, any tissue can be detected in this medium.

Fig. 1. Delphinus SoftVue Ultrasound Imaging System [14]

It is assumed that there is an infinite space which contains a homogeneous medium (e.g. water) whose background wave number is k_0. An object (i.e. strange tumor) with constant density and a wave number k(r) is put inside this medium. The wave equation of the system can be shown as:

$$\nabla^2 p(\vec{r}) + k_0^2 p(\vec{r}) = -O(\vec{r})p(\vec{r}), \tag{1}$$

where

$$O(\vec{r}) = k_1^2 - k_0^2 - \rho(r)^{1/2}\nabla^2\rho(r)^{-1/2}, \tag{2}$$

$$k_1(r) = \frac{\omega}{c_1(r)} + i\alpha(r), \tag{3}$$

where $k_1(r)$ is the wave number, $c_1(r)$ is the sound speed, $\alpha(r)$ is the attennuation, $\rho(r)$ is the density, and ω is the angular frequency.

The incident wave is denoted as $p^{inc}(r)$, the scattered wave can then be obtained as follows:

$$p^{sc}(r) = \int_{\Omega} O(r')p(r')G_0(k_0, r - r')\, dr', \tag{4}$$

where $p(r) = p^{inc}(r) + p^{sc}(r)$ is the total pressure inside the inhomogenerous area Ω and $G_0(k_0, r - r')$ is the Green's function. When the background is homogenerous, G_0 is the 0-th Hankel function of the first kind:

$$G_0(k_0, r - r') = \frac{-i}{4} H_0^{(1)}(k_0|r - r'|) \tag{5}$$

The total pressure can be expressed as

$$p(r) = p^{inc}(r) + \int_{\Omega} O(r')p(r')G_0(k_0, r - r')\, dr'. \tag{6}$$

Eq. (6) can be effectively discreted by using the Method of Moment (MoM). The pressure in the grid points (see Fig.1) can be computed in vector form with size $N^2 \times 1$:

$$\bar{p} = \left(\bar{I} - \bar{C}.D(\bar{O})\right)p^{inc}. \tag{7}$$

The exterior points give scatter vector $N_t N_r \times 1$:

$$\bar{p}^{sc} = \bar{B}.D(\bar{O}).\bar{p}\,, \tag{8}$$

where $\bar{B}$ is the matrix with Green's coefficient $G_0(r,r')$ from each pixel to the receiver, $\bar{C}$ is the matrix with Green's coefficient $G_0(r,r')$ among all pixels, $\bar{I}$ is identity matrix, and D(.) is an operator that transform a vector into a diagonal matrix.

Two variables in equations (7) and (8), $\bar{p}$ and $\bar{O}$, are unknown. To solve this, the first Born approximation has been applied and the forward equation (7) and (8) can be rewritten [15]:

$$\Delta p^{sc} = \bar{B}.D(\bar{p}).\Delta\bar{O} = \bar{M}.\Delta\bar{O}, \tag{9}$$

where $\bar{M} = \bar{B}.D(\bar{p})$. For each transmitter and receiver, we will have amatrix $\bar{M}$ and a scalar value Δp^{sc}. Realize that unkown vector $\bar{O}$ has $N \times N$ variables which are equal to the number of pixels in RIO. Object function can be estimated by iterations:

$$\bar{O}^{n} = \bar{O}^{(n-1)} + \Delta\bar{O}^{(n-1)}, \tag{10}$$

where $\bar{O}^{n}$ and $\bar{O}^{(n-1)}$ are object functions at present and previous steps, respectively; $\Delta\bar{O}$ can be estimated by solving l_1 nonlinear regularization problem [16]:

$$\Delta\bar{O} = \arg\min_{\Delta\bar{O}} \left\|\Delta\bar{p}^{sc}{}_t - \overline{M_t}\Delta\bar{O}\right\|_2^2 + \epsilon\|\Delta\bar{O}\|_1\,, \tag{11}$$

where $\Delta\bar{p}^{sc}$ is the $(N_t N_r \times 1)$ vector, contains the difference between predicted and measured scattered ultrasound signals; $\bar{M}_t$ is system matrix $(N_t N_r \times N^2)$ formed by $N_t N_r$ different matrixes $\bar{M}_t$; and ϵ is the regularization parameter.

The CS-DBIM procedure is presented in Algorithm 1.

Algorithm 1. The Compressed Sensing Distorted Born Iterative Method (CS-DBIM)

Set up the measurement configuration for transducers
Choose initial values: $\bar{O}_{(n)} = \bar{O}_{(0)}$ and $\bar{p}_0 = \bar{p}^{inc}$ using (13)
For $n = 1\ to\ N_{sum}$, **do**
1. Calculate $\bar{B}$ and $\bar{C}$
2. Calculate p, $\bar{p}^{sc}$ corresponding to $\bar{O}_{(n)}$ using (7, 8)
3. Calculate $\Delta\bar{p}^{sc}$ using (9)
4. Calculate $\Delta\bar{O}_{(n)}$ using (11)
5. Calculate $\bar{O}_{(n+1)} = \bar{O}_{(n)} + \Delta\bar{O}_{(n)}$
End For

III. The CS-FH-DBIM Approach

It is noted that, an image reconstruction with a low frequency will offer a low resolution but a deeper penetration, and vice versa, an image reconstruction with a higher frequency will offer a higher resolution but a poor penetration. Born assumption can be made valid by decreasing the operating frequency or increasing the wavelength. This makes the heterogeneous medium relatively small in comparison with the wavelength, thus reducing the scattering strength and the difference between the total and scattered pressures (i.e., reduced error in Born assumption). The DBIM algorithm will converge at a low-enough operating frequency. However, the reconstructed image at this frequency has a poor spatial resolution. Therefore, we use the hopping frequency solution for enhancing spatial resolution. The proposed CS-FH-DBIM method consists of two stages. The first stage is a reconstruction process with a low frequency (f_1) using the CS-DBIM in N_{f1} iterations. We can easily obtain the convergence after N_{f1} iterations. Secondly, the obtained result at this stage is used as an initial estimate for the CS-DBIM with a high frequency (f_2) in N_{f2} iterations. The convergence can be achieved at high frequency because the result of the first stage does not has error and this current value of sound contrast is relatively close to the real one of sound contrast. This process can be repeated at multiple frequencies until achieving the desired spatial resolution. At each stage, the image reconstruction is completed by using the compressed sensing technique. That is, transducers will be randomly distributed on the measurement system and the image reconstruction is solved by using a sparse recovery algorithm (namely, l_1 least square problem). The advantage of this technique is that it allows reconstruct high-quality images with a small number of measurements ($N_t \times N_r$) and it is much smaller than the number of variables (N^2). This algorithm can extend the applicability of the DBIM in order to solve tumors at the level of biological tissues. The sampling ratio is defined as follows:

$$r = \frac{N_t N_r}{\mathrm{N} \times \mathrm{N}}. \tag{12}$$

Simulation parameters: Incident frequencies $f_1 = 0.5$ MHz, $f_2 = 1$ MHz; Number of transmitters $N_t = 17$; Number of receivers $N_r = 17$; Total number of iterations $N_{sum} = 4$; Number of pixels in the region of interest N = 20; $N_{f1} = 2$; $N_{f2} = 2$; Scattering area diameter = 7.3 mm; Sound contrast 3%; Gaussian noise 5%; Distances from transmitters and receivers to the center of the object = 60 mm. $N_t \times N_r = 17 \times 17 = 289$; $N \times N = 20 \times 20 = 400$; $r = 0.7225$.

The incident pressure for a Bessel beam of zero order in two-dimensional case is

$$\overline{\mathrm{p}}^{\mathrm{inc}} = \mathrm{J_0}(\mathrm{k_0}|\mathrm{r} - \mathrm{r_k}|), \tag{13}$$

where J_0 is the 0^{th} order Bessel function and $|r - r_k|$ is the distance between the transmitter and the k^{th} point in the ROI.

IV. Simulations And Results

Fig. 2 presents the error performance of the proposed method in comparison with the conventional one. With the same computational complexity (i.e. $N_t \times N_r \times N^2$), the normalized errors are 47.23%, 21.74%, 43.27%, 62.02% reduced after the first, second, third, fourth iterations, respectively. If the errors are the same in the conventional and proposed methods, the computational complexity is significantly reduced. Because the needed $N_t \times N_r$ for the proposed method is reduced, compared to the conventional method.

In the first stage of the proposed method (i.e. after the first two-iterations), the normalized error is almost stable, or not reduced. However, after applying the frequency hopping technique, the normalized error is plummeted. This means that the necessary number of iterations for applying the frequency hopping technique can be reduced to one iteration. Meanwhile, in the conventional method, the necessary number of iterations is $N_{sum}/2$) [17]. Therefore, in the proposed method, we can save a number of iterations that still maintain a good quality of the image reconstruction.

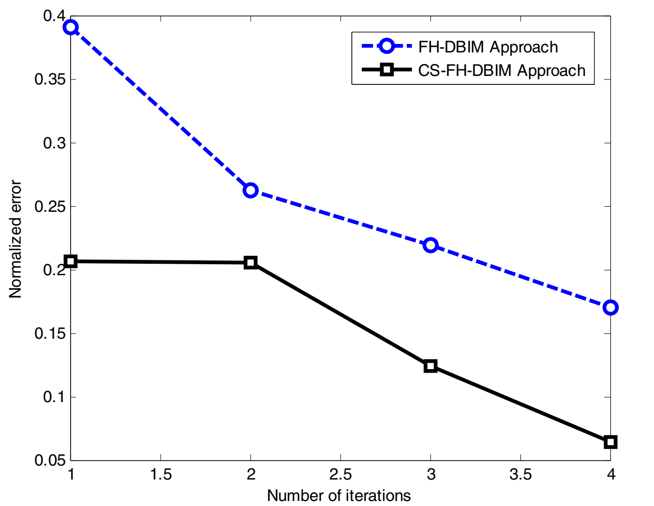
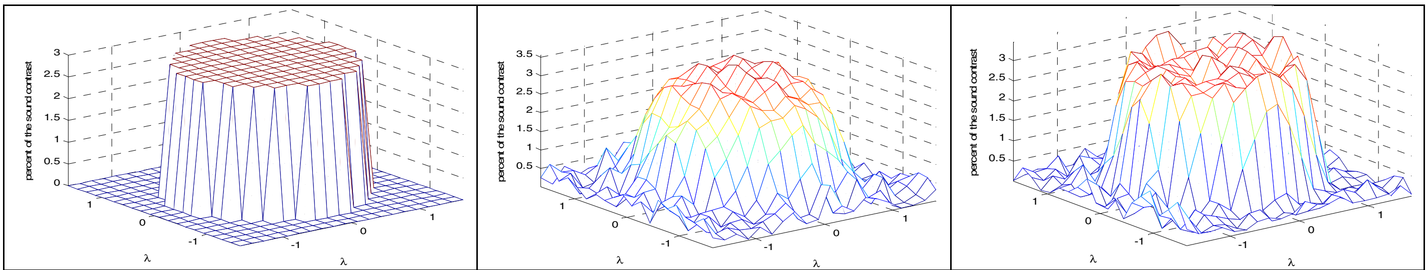


Fig. 2. Normalized error comparison of the FH-DBIM and CS-FH-DBIM approaches



(a)	(b)	(c)

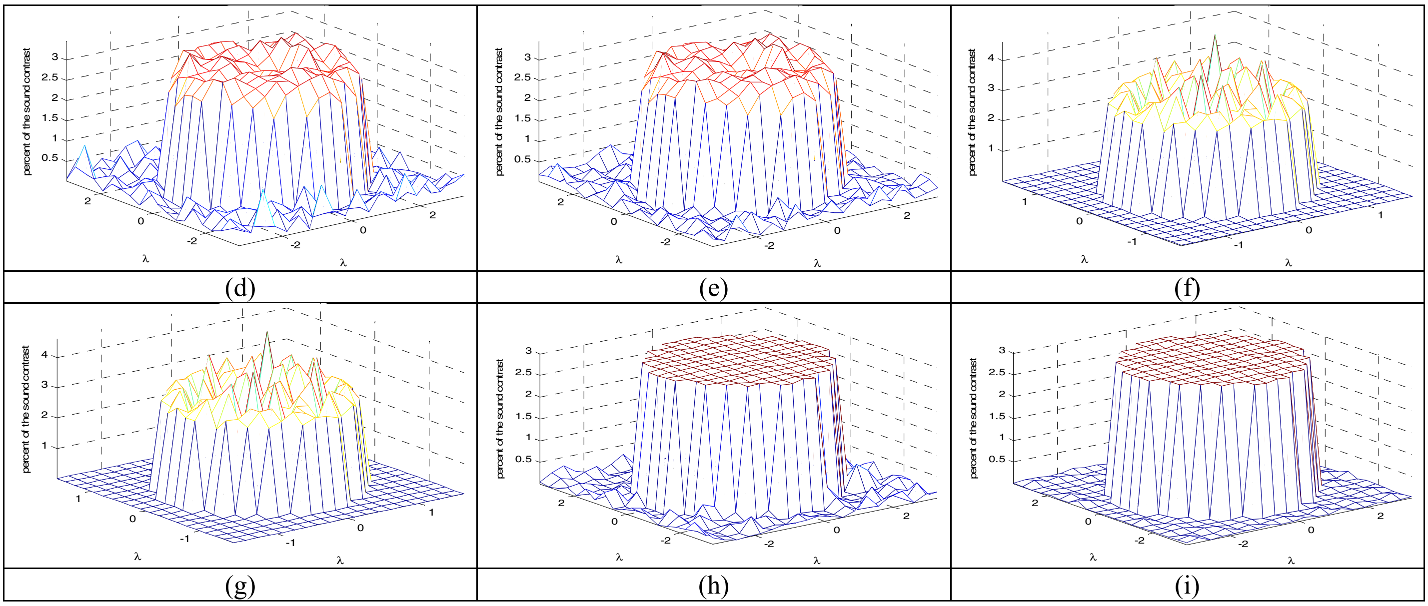


Fig. 3. (a). The ideal object function; (b), (c), (d), (e). The reconstructed results of the FH-DBIM approach after the first-to-fourth iterations; (f), (g), (h), (i). The reconstructed results of the CS-FH-DBIM approach after the first-to-fourth iterations.

Fig. 3a shows the ideal object function which is needed to reconstruct. Figs. 3b-to-3i show the reconstructed results using the conventional FH-DBIM method (in Figs. 3b, 3c, 3d, 3e) and the proposed CS-FH-DBIM one (in Figs. 3f, 3g, 3h, 3i). It can be seen that the background noise obtained by the proposed scheme is less than that in the conventional one. In total, the result of the proposed method is closer to the ideal object function than that using the conventional method.

V. CONCLUSIONS

This paper has successfully applied the compressed sensing technique and the hopping frequency technique in order to improve the quality of the image reconstruction and reduce the computational complexity of the FH-DBIM. The hopping frequency technique is used for achieving the desired spatial resolution and the compressed sensing technique is used for reducing the computational complexity. Numerical simulation scenarios of sound contrast reconstruction were implemented to prove the good performance of the CS-FH-DBIM method.

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