An Efficient Example-based Method for CT Image Denoising based on Frequency Decomposition and Sparse Representation

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***Abstract*—In this paper we present an efficient example-based method for Gaussian denoising of CT images. In the proposed method, an image is considered as a sum of the three frequency bands: low-band, middle-band and high-band. We assume that the noise component is often mixed into the middle-band and the high-band in order to better preserve the high-frequency details in the image we perform denoising on these two bands. The method is based on a sparse representation model in which a set of standard images is used to construct the example dictionaries. The experimental results demonstrate that the proposed denoising method can preserve well the high-frequency details. The objective and subjective comparisons also show that the proposed our method outperforms other state-of-the-art denoising methods.**

***Index Terms*—Medical image denoising; Computed tomography (CT); Sparse representation; Example-based denoising.**

I. Introduction

Computed tomography (CT) imaging is the technique that creates cross-sectional images of the body by using X-rays beam. CT imaging is widely used in medical diagnosis and treatment. The quality of CT images highly depends on the radiation dose. It has been shown that low radiation imaging often deals with a number of quality-degrading artefacts, the most prominent of them being noise [1], [2]. Therefore, denoising plays an important role in improving image quality in CT imaging.

In fact, noise in a CT image relates to the number of X-ray photons that contribute to each small area of the image, and can be reduced by increasing the X-ray dose. However, such an increase may be harmful to patients and, hence, should not be prioritized. If, instead, we have a robust image processing algorithm for denoising the image, lower radiation scans become possible, thus bringing less damage to patients.

It is well-known that the noise in CT images often follows a Gaussian distribution [3]. For denoising this type of noise, various state-of-the-art methods have been proposed such as spatial filters [4]–[6], total variation-based methods [7], [8], sparse representation-based methods [9]–[12]. However, as shown in [13], noise in CT images, in fact, is more complex. The noise level in different image regions may be different. Therefore, methods which are based on the assumption of independent identically distributed additive noise might not be efficient enough. Regarding the specific nature of CT images, denoising while preserving as much as possible subtle details is not an easy task.

Among various directions explored in studying this problem of medical image denoising, the learning-based direction seems to be promising. Some recent example-based denoising methods that intend to preserve subtle details can be seen in [13]–[16]. In this paper, we are interested in the method proposed in [16], namely Markov Random Field Denoising (MRFD), which applied Markov Random Field to the design of an efficient denoising method for CT images. An interesting idea of this method is that an image is decomposed into three frequency bands namely low-band, middle-band, and high-band. The authors focus on estimating high-frequency band that is usually over-smoothed by traditional noise filters. Denoising is performed patch-wise with the help of a database of middle-high frequency patch pairs constructed from a given set of example images. The experimental results showed that MRFD is promising.

However, the MRFD method still has some drawbacks such as the high-frequency band in the output image is aggregated from the high-frequency patches selected directly from the database. This causes the performance of the method to depend heavily on the database constructed from the example images. Moreover, this method has high computational complexity. To overcome these drawbacks, we propose in this work a novel solution that outperforms MRFD. Unlike in MRFD, in the proposed method the middle- and high-frequency components in the desired image are estimated by finding the sparse linear combinations between the patches in the example-image database.

To evaluate the performance of the proposed method, we conduct many experiments on both images with simulated noises and real noisy images. The experimental results demonstrate that the proposed method is better than MRFD as well as some recently state-of-the-art denoising methods.

The paper is organized as follows. The proposed denoising method is described in Section II. The experimental results to evaluate the performance of the method are demonstrated in

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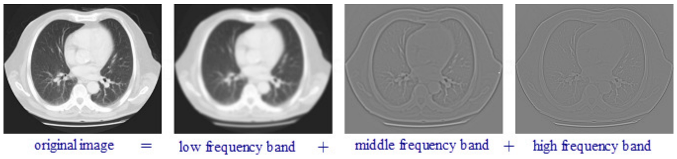


Fig. 1. Decomposing an image into three bands.

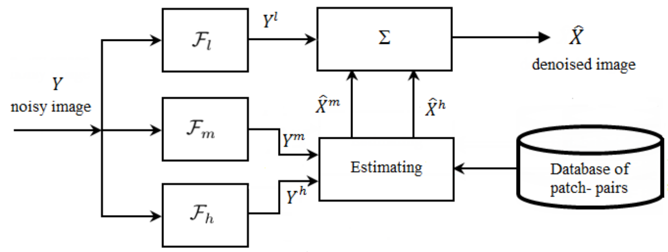


Fig. 2. Scheme diagram of the proposed denoising method.

Section III. Finally, conclusions and future works are presented in Section IV.

II. Proposed Method

Following [16], in the proposed method we consider a model of a CT image $\mathbf{X}$ that is composed of three components: low-frequency band $\mathbf{X}^{\ell}$, middle-frequency band $\mathbf{X}^{m}$, and high-frequency band $\mathbf{X}^{h}$ (see Fig. 1). It means that

$$\mathbf{X} = \mathbf{X}^{\ell} + \mathbf{X}^{m} + \mathbf{X}^{h}, \tag{1}$$

where the superscripts l, m, h denote low, middle and high, respectively.

Let $\mathbf{Y}$ be a noisy CT image that needs to be denoised. In this work, we use an assumption (as in [13]) that the noise component, denoted by η, in $\mathbf{Y}$ is additive. Accordingly, we can write

$$\mathbf{Y} = \mathbf{X} + \eta, \tag{2}$$

where $\mathbf{X}$ is a noise-free image that needs to be estimated. On each a small patch, the noise component can be approximated by a Gaussian distribution.

In order to remove η, MRFD in [16] proposed a reasonable assumption that if we decompose $\mathbf{Y}$ into three bands, as

$$\mathbf{Y} = \mathbf{Y}^{\ell} + \mathbf{Y}^{m} + \mathbf{Y}^{h}, \tag{3}$$

then the majority of the noise remains in $\mathbf{Y}^{h}$ and the rest in $\mathbf{Y}^{m}$. Thus, we can approximate $\mathbf{X}^{\ell}$ by $\mathbf{Y}^{\ell}$. Hence, denoising becomes finding the estimates $\hat{\mathbf{X}}^{m}$ of $\mathbf{X}^{m}$ and $\hat{\mathbf{X}}^{h}$ of $\mathbf{X}^{h}$ from $\mathbf{Y}^{m}$ and $\mathbf{Y}^{h}$. Finally,

$$\hat{\mathbf{X}} = \mathbf{Y}^{\ell} + \hat{\mathbf{X}}^{m} + \hat{\mathbf{X}}^{h} \tag{4}$$

will be the denoising result.

Let $\mathcal{F}_{\ell}(\cdot), \mathcal{F}_{m}(\cdot)$, and $\mathcal{F}_{h}(\cdot)$ be some low-pass, band-pass, and high-pass filters, respectively. These filters can be defined based on the classical filters such as the Gaussian filters. To denoise $\mathbf{Y}$, we first decompose it into the bands by

$$\mathbf{Y}^{\ell} = \mathcal{F}_{\ell}(\mathbf{Y}), \quad \mathbf{Y}^{m} = \mathcal{F}_{m}(\mathbf{Y}), \quad \mathbf{Y}^{h} = \mathcal{F}_{h}(\mathbf{Y}). \tag{5}$$

Then, we perform denoising on $\mathbf{Y}^{m}$ and $\mathbf{Y}^{h}$ by estimating $\mathbf{X}^{m}$ and $\mathbf{X}^{h}$ correspondingly. Here, $\mathbf{X}^{m}$ and $\mathbf{X}^{h}$ are estimated patch-wise with the help of a database of middle-high frequency patch pairs established from the given standard example images.

The proposed method mainly consists of two phases:

- **Database construction phase:** This phase generates a database of middle-high frequency patch pairs, denoted by (P_m, P_h), from a set of example standard images. Similarly to [13], [16], the example images are the noiseless ones, pre-processed, and are taken at nearly the same location as the noisy image.
- **Denoising phase:** Here, we estimate $\mathbf{X}^{m}$ and $\mathbf{X}^{h}$ patch-wise, based on a sparse representation model on the database (P_m, P_h).

An overview of the proposed method can be seen in Fig. 2 and more details of the two phases are described next.

A. Database Construction Phase

As mentioned above, in order to construct a database (P_m, P_h) for a given noisy image, a set of standard medical images denoted by $\{\mathbf{I}_t, t \in \Omega\}$ is used. These standard images are similar to the noisy image and considered as noiseless images. First each $\mathbf{I}_t$ is decomposed into three basic bands $(\mathbf{I}_t^{\ell}, \mathbf{I}_t^{m}, \mathbf{I}_t^{h})$ by the filters $\mathcal{F}_{\ell}$ and $\mathcal{F}_{m}$ following (5), that is

$$\mathbf{I}_t^{\ell} = \mathcal{F}_{\ell}(\mathbf{I}_t) \quad \text{and} \quad \mathbf{I}_t^{m} = \mathcal{F}_{m}(\mathbf{I}_t), \tag{6}$$

and $\mathbf{I}_t^{h}$ is then obtained by

$$\mathbf{I}_t^{h} = \mathbf{I}_t - \mathbf{I}_t^{\ell} - \mathbf{I}_t^{m}. \tag{7}$$

The database (P_m, P_h) stores the vectorized patch pairs $(\mathbf{u}_k^{m}, \mathbf{u}_k^{h})$ in which $\mathbf{u}_k^{m}$ and $\mathbf{u}_k^{h}$ correspond to the patches at the same position in $\mathbf{I}_t^{m}$ and $\mathbf{I}_t^{h}$, respectively.

B. Denoising Phase

In this phase, $\mathbf{X}^{m}$ and $\mathbf{X}^{h}$ are estimated patch-wise in four steps as follows:

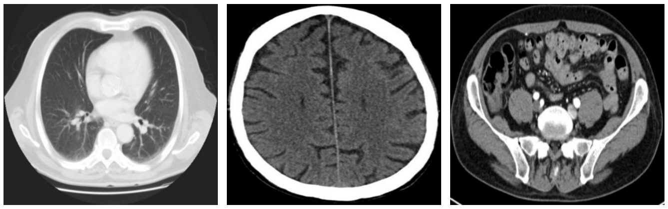
a) Step 1: The middle-frequency band $\mathbf{Y}^{m}$ is separated into a set of N vectorized overlapping patches $\{\mathbf{y}_i^{m}\}_{i=1}^{N}$. Our aim is to estimate the sets of corresponding patches $\{\mathbf{x}_i^{m}\}_{i=1}^{N}$, $\{\mathbf{x}_i^{h}\}_{i=1}^{N}$ of $\mathbf{X}^{m}$ and $\mathbf{X}^{h}$.

b) Step 2: For each $\mathbf{y}_i^{m}$, we find in the database (P_m, P_h) a sub-database $(\mathbf{D}_i^{m}, \mathbf{D}_i^{h}) \subset (P_m, P_h)$ where $\mathbf{D}_i^{m} = \{\mathbf{u}_k^{m}\}_{k=1}^{K}$ is the set of the K-nearest neighbors of $\mathbf{y}_i^{m}$ in P_m and $\mathbf{D}_i^{h} = \{\mathbf{u}_k^{h}\}_{k=1}^{K}$ such that $(\mathbf{u}_k^{m}, \mathbf{u}_k^{h})$ is a pair in (P_m, P_h).

c) Step 3: Estimate $\mathbf{x}_i^{m}$ and $\mathbf{x}_i^{h}$. We first find a sparse representation of $\mathbf{y}_i^{m}$ over $\mathbf{D}_i^{m}$ by solving the optimization problem

$$\hat{\alpha} = \arg\min_{\alpha} \frac{1}{2}\|\mathbf{D}_i^{m}\alpha - \mathbf{y}_i^{m}\|_2^2 \tag{8}$$

$$\text{subject to } \begin{cases} \|\alpha\|_0 \le L \\ \|\mathbf{D}_i^{h}\alpha - \mathbf{y}_i^{h}\|_2^2 \le \lambda\sigma_i^2 \end{cases} .$$



(a) Lung (b) Head (c) Abdomen

Fig. 3. CT images used for objective comparison of the denoising methods.

where σ_i is the standard deviation of noise in the noisy patch $\mathbf{y}_i$ of the noisy image $\mathbf{Y}$, and λ is a positive parameter. The estimates of $\mathbf{x}_i^m$ and $\mathbf{x}_i^h$ are now determined respectively by

$$\hat{\mathbf{x}}_i^m = \mathbf{D}_i^m \hat{\alpha}, \quad (9)$$

$$\hat{\mathbf{x}}_i^h = \mathbf{D}_i^h \hat{\alpha}. \quad (10)$$

Here, the problem (8) is solved by using the OMP algorithm [17].

d) Step 4: Finally, the estimates $\{\hat{\mathbf{x}}_i^m\}_{i=1}^N$ and $\{\hat{\mathbf{x}}_i^h\}_{i=1}^N$ are used to aggregate $\hat{\mathbf{X}}^m$ and $\hat{\mathbf{X}}^h$ according to (4).

III. Performance Evaluation

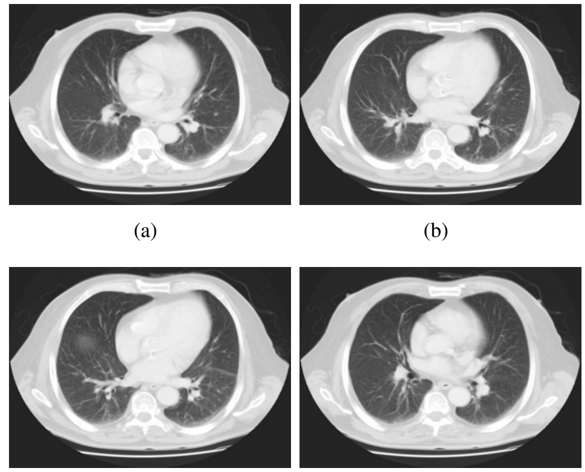
To evaluate the performance of the proposed method, we perform some experiments on images with both synthetic noise and real noise. The proposed method is compared to some state-of-the-art denoising methods, including Total Generalized Variation (TGV) [8], Non-Local Mean (NLM) [18] and MRFD [16]. For objective comparison, we use the Structural Similarity (SSIM) metric to evaluate the performance of the denoising methods on the images with synthetic noise.

A. On images with synthetic noise

We present here some experimental results on three CT images of lung, the abdomen and the head, as illustrated on Fig. 3. With each test image, we use four similar CT images which are considered as noise-free ones to build the corresponding databases. Fig. 4 shows the standard images selected to build database for denoising CT images of the lung. The testing images is then added with zero-mean Gaussian noise with standard deviations $\sigma = 10$, $\sigma = 20$, and $\sigma = 30$. The patch size is set to 9×9, 13×13, 17×17 corresponding to $\sigma = 10, 20$, and 30.

The objective results of the experiments are shown in Table I. Obviously, the SSIM values of the proposed method are higher than that of the others. It means that the proposed method outperforms the other methods.

For subjective comparison, we show in Fig. 5 the case of a CT image of the lung with noise level $\sigma = 20$. As can be seen in the image denoised by the proposed method (Fig. 5(e)), noise was effectively removed while small details were preserved better than the others (see the small rectangles).



(a) (b)

(c) (d)

Fig. 4. Noise-free CT images used to construct the database of lung images.

TABLE I
Structural-Similarity comparison on some CT scans

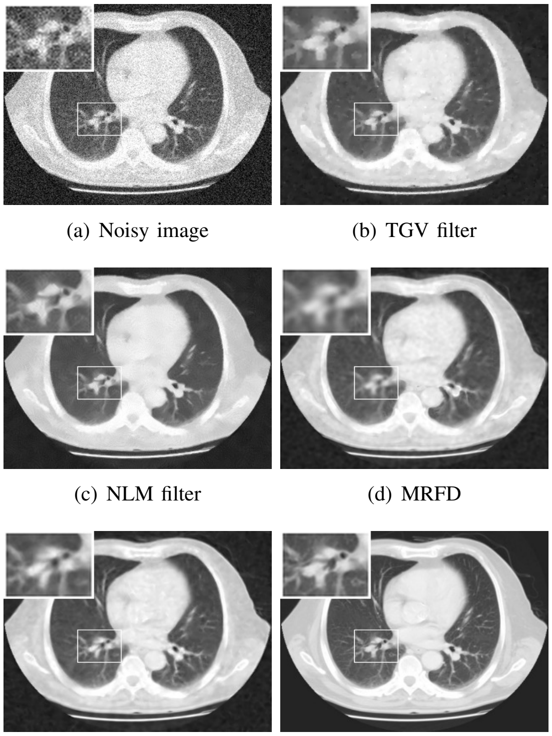
CT	σ	SSIM			
		TVG	NLM	MRFD	Proposed method
Lung	10	0.9035	0.9300	0.9229	**0.9397**
	20	0.8681	0.8458	0.8571	**0.8909**
	30	0.6145	0.7590	0.8205	**0.8541**
Head	10	0.7988	0.8855	0.8744	**0.9127**
	20	0.7908	0.7744	0.8038	**0.8156**
	30	0.6272	0.6924	0.7495	**0.7669**
Abdomen	10	0.8491	0.9106	0.8700	**0.9102**
	20	0.8251	0.8301	0.8068	**0.8392**
	30	0.6951	0.7510	0.7666	**0.7863**

B. On images with real noise

We present here an experiment on a real CT image of abdomen (Fig. 6(a)). It is heavily corrupted by tomographic noise. In the experiment, we use only a standard image (Fig. 6(b)) to build the corresponding database. The parameters are set as follows: patch size is equal to 13×13, $L = 3$, $K = 10$, and $\lambda = 169$. The results of different denoising methods are demonstrated in Fig. 6. Visually, the proposed method gives the best denoising result (Fig. 6(f)). As it can be seen, with the proposed method, noise is removed effectively while slightly enhancing contrast.

IV. Conclusions

In this paper we present an efficient denoising method for CT images. The proposed method can be seen as an improving version of the previous MRFD method in [16]. The proposed method is inspired by the idea of the MRFD that an image is decomposed into different frequency bands and denoising is performed on the middle- and high-frequency bands with the help of database of example middle-high frequency patch-pairs. By that way, we can process better the high-frequency

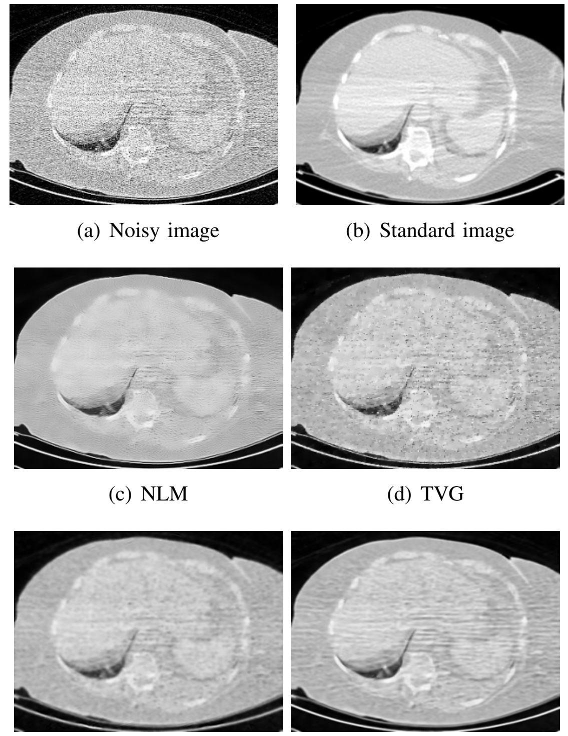


(a) Noisy image (b) TGV filter

(c) NLM filter (d) MRFD

(e) Our method (f) Original image

Fig. 5. Subjective comparison on the CT image of lung; $\sigma = 20$.



(a) Noisy image (b) Standard image

(c) NLM (d) TVG

(e) MRFD (f) Our method

Fig. 6. Subjective comparison on a noisy CT image of abdomen.

components in the image. The main contribution in this paper, as compared to [16], is that we have proposed a novel model based on sparse linear representation, leading to a more efficient denoising method. The experimental results show that the proposed method removes noise effectively while preserving subtle details better than some other state-of-the-art methods such as NLM, TGV, and MRFD. In the future works, we will find a solution for optimizing the example database as well as evaluating the effect of the parameters on the performance of the proposed method.

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