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On the Reuse of Shadowed CRs as AF Diversity Relays in Cooperative Spectrum Sensing in Correlated Suzuki Fading Channels

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SUMMARY Most recent work on cooperative spectrum sensing using cognitive radios has focused on issues involving the sensing channels and seemed to ignore those involving the reporting channels. Furthermore, no research has treated the effect of correlated composite Rayleigh-lognormal fading, also known as Suzuki fading, in cognitive radio. This paper proposes a technique for reuse of shadowed CRs, discarded during the sensing phase, as amplified-and-forward (AF) diversity relays for other surviving CRs to mitigate the effects of such fading in reporting channels. A thorough analysis of and a closed-form expression for the outage probability of the resulting cooperative AF diversity network in correlated composite Rayleigh-lognormal fading channels are presented in this paper. In particular, an efficient solution to the "PDF of sum-of-powers" of correlated Suzuki-distributed random variables using moment generating function (MGF) is proposed.
key words: *cognitive radio, cooperative spectrum sensing, amplify-and-forward diversity relaying, correlated composite fading, suzuki fading, MGF*

1. Introduction

Cooperative spectrum sensing using cognitive radio (CR) has proved to be a reliable technique for combating deep fading during the sensing a primary user [1], [2]. The sensing is carried out in two phases: in the sensing phase, the CRs independently measure and process the signal from the primary user, and in the reporting phase the CRs independently report the processed information to a fusion center (FC) which is usually a cognitive base station and which will make the final global sensing decision as to whether the primary user is present or absent. In many standard fusion rules, it is easy to see that the inclusion of deeply faded CRs, i.e. with low SNRs, in the decision fusion at the FC diminishes the reliability of the cooperative detection of the primary user. Thus by discarding the detection contribution from shadowed CR sensors, the detection probability of the cooperative sensing network can be improved. However, by doing so the CRs under shadowing are wasted. Signal-to-noise ratio (SNR) is a dominant metric of transmission quality affecting the detection performance of a CR sensor and can be used by the FC to decide if a CR should be rejected [4]. Various techniques can be used in wireless communications for a CR to efficiently estimate the SNR directly from its sensing energy samples without the knowledge of the transmitted signal power or the noise variance, e.g. [5], [11], [12]. In our paper, we choose the conventional method, energy detection [3], to evaluate the performance of detection in the cognitive radio network.

In reality, when the distances between CRs and the FC are of significant in suburban macrocells or under shadowing in urban microcells, loss and fading is a significant issue. Several researches have dealt with this issue [23]–[25]. As one of solutions, cooperative diversity relaying may be used to improve the performance of the reporting wireless channels. The cooperative diversity relay can process the information that it receives from the source and forward the information to the destination using either amplify-and-forward (AF) or decode-and-forward (DF) protocols.

In an earlier paper [6], we proposed an innovative reuse of those CRs under deep fading of the sensing channels by re-assigning them to act as diversity relays to their more healthy peers in the reporting channels, thus improving the global detection reliability of the fusion center. However, in [6], the fading mechanism in the reporting channels is assumed to be uncorrelated Rayleigh-lognormal distributed, also known as the Suzuki fading channel [7]. The proposed pairing algorithm which selects 'surviving-rejected' CR pairs to form cooperative diversity relaying networks, involves searching for pairs that produce lowest outage probabilities of the resulting networks. However, it is well known that the infinite integral in the probability density function (PDF) of the Suzuki fading distribution makes it difficult to derive a closed-form expression for channel outage probability, thus preventing any efficient and fast search.

As in [7], Suzuki fading is a composite fading consisting of two components: Rayleigh fading and log-normal fading. In a model of diversity branches with MRC receiver in a Suzuki fading environment, if Rayleigh components of the subchannels or log-normal components of those are correlated, we have correlated Suzuki fading channels. In this paper, we would like to consider a more complicated and more realistic context, cooperative relay over correlated

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Suzuki faded reporting channels. The mathematical complexity arises in the calculation of the PDF of the sum of two lognormal or correlated lognormal random variables is well known [8]–[10]. Relevant to our paper is the sum of two correlated composite Rayleigh-lognormal RVs at the FC destination of the resulting cooperative diversity AF relaying network - one directly from the surviving CR and the other forwarded from the relay. To the best of our knowledge, this problem has not been solved in the published literature to date. In the case of independently faded RVs, this PDF can be calculated using the moment generating function (MGF) technique [8], [10] followed by the inverse Laplace Transform (ILT) [10]. In this paper, we present a new idea to derive a closed-form expression for the sum of two correlated Suzuki distributed RVs, hence for the outage probability of the cooperative diversity AF relaying network. While the main motivation of our paper is clearly to re-use the shadowed sensing CRs which otherwise will be wasted, its main contribution is more towards the mathematical and computational advance for cooperative diversity relaying using AF protocol.

The rest of this paper is organized as follows: The principles of spectrum sensing using cognitive radios are introduced in Sect. 2. A brief description of the cooperative diversity relaying technique using amplify-and-forward (AF) relaying protocol is given in Sect. 3 together with the theory for calculating the PDF of the power sum of two correlated Suzuki random variables (RVs). Then the outage probability of cooperative relay cognitive network over correlated Suzuki fading channels will be presented. Section 3 offers the main contribution of our paper. Algorithm for discarding and reusing deeply faded CRs as diversity relays for other CRs are introduced in Sect. 4. Section 5 provides simulation results of the above issues. Some conclusions will be given in Sect. 6.

2. Spectrum Sensing Using Cognitive Radios

A simple diagram of a cooperative spectrum sensing network in Cognitive Radio is depicted in Fig. 1. As shown in the figure, there are two groups of channels: sensing channels and reporting channels in a decentralized Cognitive Radio Network (CRN). They correspond to two phases: sensing phase and reporting phase. In the sensing stage, each cognitive radio normally uses an energy-based detector [3] to independently sense the presence or absence of the signal transmitted from a primary user (PU) and makes a local decision.

In order to explain the operation of spectrum sensing of CRs, we consider the ith cognitive radio. The decision of local spectrum sensing of the ith CR can be formulated by the binary hypothesis [1]:

$$x_i(t) = \begin{cases} w_i(t) & H_0 \text{ (PU is absent)}, \\ h_i(t)s(t) + w_i(t) & H_1 \text{ (PU is present)}. \end{cases} \quad (1)$$

where $x_i(t)$ is the received signal at the ith CR and $s(t)$ is the

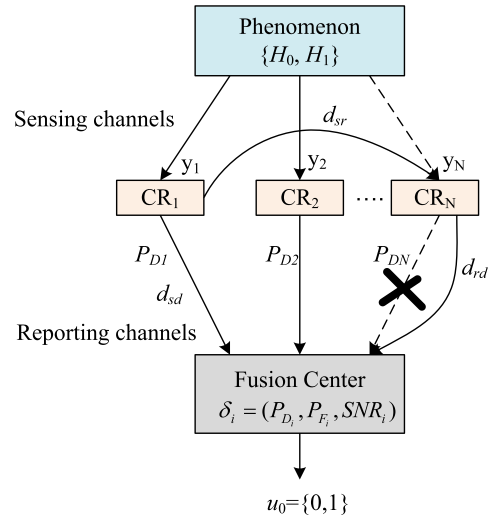

Fig. 1 A model of cooperative spectrum sensing in Cognitive Radio in a deep fading environment.

signal transmitted from the PU, $w_i(t)$ is the additive white Gaussian noise (AWGN) at the ith CR and $h_i(t)$ is the channel gain of the sensing channel between the PU and the ith CR.

To obtain a collective decision about the presence of the PU over the sensing channels, CRs send their local decisions to a Fusion Center (FC) over the Common Control Channels (CCC) [20], [21] which are so-called reporting channels. The control channel can be assigned as a dedicated channel in licensed or unlicensed bands, or an underlay ultra - wideband (UWB) channel [22]. The cooperating CRs use a Multiple Access Control (MAC) scheme to access the control channel. Based on the collected information from the CRs, the FC makes a global decision whether there is a spectrum hole over the channels ($u_0 = 1$) or not ($u_0 = 0$) as seen in Fig. 1.

There are 2 parameters used to evaluate the performance of spectrum sensing: the false-alarm probability and the detection probability. False-alarm probability (P_f) is the probability that a CR wrongly detects the presence of the PU, i.e. it decides that there is a signal of PU occupying the frequyency band of interst while there is no signal. In other words, in the case of using energy detection, the detected signal energy y is greater than a threshold λ, and that the SU is not allowed to access the channel, but in fact, the channel is empty. False-alarm probability is defined as [1]:

$$P_f = P(y > \lambda | H_0) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \quad (2)$$

where u denotes the time bandwidth product, i.e., $u = TW$, $\Gamma(.)$ and $\Gamma(.,.)$ are the gamma and the incomplete gamma function, respectively.

Detection probability (P_d) is the probability that a CR detects correctly the presence of signal from the PU over the sensing channel and that the secondary user (SU) is not allowed to use this channel. That means the received energy

y at the CR is larger than threshold λ. In deed, there is a signal of the PU occupying the frequency band of interest. Detection probability under fading condition is defined as [1]:

$$P_d = \int_\gamma Q_u(\sqrt{2ux}, \sqrt{\lambda}) f_\gamma(x) dx \tag{3}$$

where $f_\gamma(x)$ is the PDF of the SNR γ in the fading channel and $Q_u(.,.)$ is the generalized Marcum Q-function [15] defined as follows:

$$Q_u(a,b) = \int_b^\infty \frac{x^u}{a^u} e^{-\frac{x^2+a^2}{2}} I_{u-1}(ax) dx$$

where $I_{u-1}(.)$ is the $(u-1)$th order modified Bessel function of the first kind.

In a Rayleigh fading channel with average SNR $\overline{\gamma}$, the probability of detection of a local CR is given in [2] as

$$\begin{aligned} P_{D,Rayleigh}(\overline{\gamma}) =& \frac{\Gamma(u-1, \frac{\lambda}{2})}{\Gamma(u-1)} + \exp\left(-\frac{\lambda}{2(1+u\overline{\gamma})}\right) \\ & \times \left(1+\frac{1}{u\overline{\gamma}}\right)^{u-1} \left[1 - \frac{\Gamma\left(u-1, \frac{\lambda u \overline{\gamma}}{2(1+u\overline{\gamma})}\right)}{\Gamma(u-1)}\right] \end{aligned} \tag{4}$$

Now we consider a composite Rayleigh-lognormal fading channel. The PDF of its power gain is [7], [10], [17]:

$$\begin{aligned} f_{R-Ln}(p) =& \int_0^\infty \frac{1}{x} \exp\left(-\frac{p}{x}\right) \frac{\xi}{x\sigma_Z \sqrt{2\pi}} \\ & \times \exp\left(-\frac{(10\log_{10} x - \mu_Z)^2}{2\sigma_Z^2}\right) dx \end{aligned} \tag{5}$$

in which $\xi = 10/\ln 10$ is the log base conversion factor, and the lognormal component is e^Z where $Z \sim N(\mu_z, \sigma_z^2)$.

Thus by inserting (5) into (3), it can be shown that the probability of detection in the composite Rayleigh-lognormal fading channel can be approximated in terms of its corresponding probability of detection in a Rayleigh channel in (4), is [10]:

$$P_{D,Suzuki} \approx \frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_p} w_n P_{D,Rayleigh}\left(\overline{\gamma} = e^{(\mu_Z + a_n \sigma_Z \sqrt{2})}\right) \tag{6}$$

in which w_n and a_n are, respectively the weights and the abscissas of the Gauss-Hermite polynomial. The approximation becomes more and more accurate with increasing approximation order N_p, and high accuracy is achieved for $N_p > 12$ [8]–[10].

3. Outage Probability of Cooperative Diversity Relaying Networks over Correlated Suzuki Fading Channels

3.1 AF Cooperative Relaying Protocol Definition

In this paper, we consider the Amplify-and-Forward (AF) Relaying Diversity. In Fig. 2, a simple model of cooperative diversity relaying network with M-relay branches is presented. Note that x_s, P_s, y_{ij}, h_{ij} and n_{ij}, are, respectively the normalized transmit signal, i.e. $E(|x|^2) = 1$, the power from the source, the received signal, the channel gain (or loss), and the additive noise on the channel link from i to j. The additive noise has Gaussian distribution with zero mean and variance σ^2. In the case of reusing a shadowed CR as relay, we just consider $M = 1$. That means each healthy CR uses only one relay. With this assumption, corresponding to Fig. 1, the source is a healthy CR, the destination is the FC and the relay is a reused CR. In the AF protocol, the relay operates in two phases: the relay-receive phase and the relay-transmit phase. In the relay - receive phase, the source broadcasts its message to both the relay and the destination. In the relay - transmit phase, the source is off, the relay amplifies the received signal by a factor α_r and sends it to the destination. A thorough analysis of the cooperative AF relaying network is given in [18].

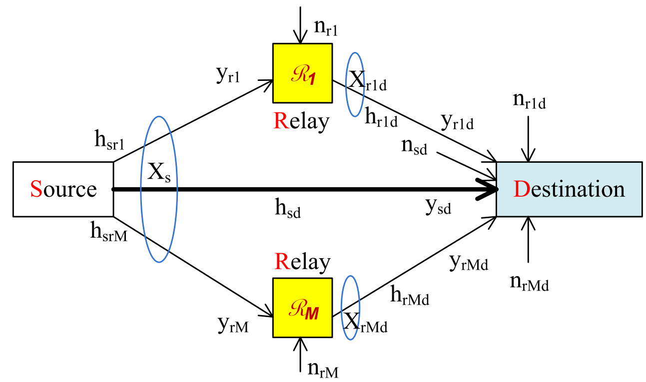

Fig. 2 Diagram of an M-relay cooperative diversity relaying network.

The SNR of the relayed signal at the destination is given as follows [18]:

$$\gamma_R = \frac{|\alpha_r h_{sr} h_{rd}|^2 P_s}{|\alpha_r h_{rd}|^2 \sigma_{sr}^2} = \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1} \tag{7}$$

where $\gamma_{sr} = \frac{|h_{sr}|^2 P_s}{\sigma_{sr}^2}$, $\gamma_{rd} = \frac{|h_{rd}|^2 P_r}{\sigma_{rd}^2}$.

Assuming that the additive noise is the same in all the channels, then we have unfaded SNR $\frac{P}{\sigma^2} = SNR_0$ being also the same in all the channels. As a result, the power gain of the relay channel is:

$$|h_R|^2 = \frac{|h_{sr}|^2 |h_{rd}|^2}{|h_{sr}|^2 + |h_{rd}|^2 + 1/SNR_0} \tag{8}$$

Finally, the *end-to-end* power gain of the cooperative diversity AF relaying wireless network using MRC reception at the destination is:

$$|h_{AF}|^2 = |h_{sd}|^2 + |h_R|^2 \tag{9}$$

3.2 Distribution of the End-to-End Output of the AF Relaying Network

The statistical distribution of $|h_R|^2$ has been a point of inter-

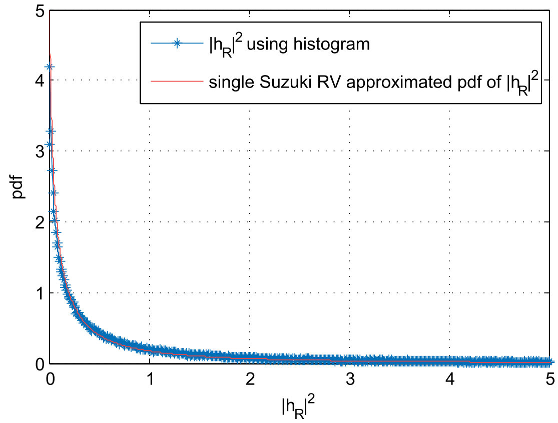

Fig. 3 Matching of the histogram of $|h_R|^2$ calculated from (8) to a single Suzuki PDF.

est of many authors and the lack of its analytical expression has been a bottleneck preventing any accurate analysis of error and outage performance of amplify-and-forward (AF) diversity relaying protocol. In the following, we justify that $|h_R|^2$ may indeed be assumed to be Suzuki distributed when both $|h_{sr}|^2$ and $|h_{rd}|^2$ are Suzuki distributed. In Fig. 3, we generated RV $|h_{sr}|^2$ with Gaussian parameters $\mu_{sr} = 0$ dB and $\sigma_{sr} = 8$ dB, and $|h_{rd}|^2$ with $\mu_{rd} = 9.0309$ dB and $\sigma_{rd} = 8$ dB, then calculated $|h_R|^2$ from (8) and its PDF ($|h_R|^2$) using histogram method. We then fitted a single Suzuki with PDF, $f_{R-Ln}(|h_R|^2)$, as in (5) with estimated Gaussian distribution parameters $(\hat{\mu}_R, \hat{\sigma}_R^2)$. The matching of the two PDF curves were done at two points $(|h_{R1}|^2, |h_{R2}|^2) = (0.1, 0.2)$ to give two simultaneous non-linear equations which were then solved using Matlab function *fsolve*. The solution was found to be $\hat{\mu}_R = -2.5159$ dB and $\hat{\sigma}_R = 6.7813$ dB. By visual inspection of Fig. 3, we conclude that the assumption above is acceptable.

Since the two diversity channels h_{sd} and h_R are in the proximity of each other, they are likely to be correlated and $|h_{AF}|^2$ in (9) can be considered as the power sum of two correlated Suzuki distributed RVs.

In this section, the main contribution of our paper is presented. The mathematical derivation for the PDF of the MRC receiver's output with L correlated Suzuki distributed diversity input branches, requires the following definitions: $p = \sum_{i=1}^{L} p_{R-Ln}(i) = \sum_{i=1}^{L} p(i)$. Note that p is the resultant power gain of the Rayleigh-lognormal RVs at the output of the MRC combiner. Function $f_{R-Ln}(p_1, p_2, \ldots, p_L)$ denotes the joint distribution of the correlated composite Rayleigh-lognormal input powers, while its marginal distribution of a single RV, as given in (5), is:

$$\begin{aligned} f_{R-Ln}(p) &= \int_0^\infty f_R(p|x) f_{Ln}(x) dx \\ &= \int_0^\infty \frac{1}{x} \exp\left(-\frac{p}{x}\right) \frac{\xi}{x\sigma_Z\sqrt{2\pi}} \\ &\quad \times \exp\left(-\frac{(10\log_{10} x - \mu_Z)^2}{2\sigma_Z^2}\right) \end{aligned} \tag{10}$$

in which $f_{Ln}(\mathbf{x} = x_1, x_2, \ldots, x_L)$ is the joint distribution of the log-normal shadowing component with its associated generating Gaussian vector $\mathbf{z} = (z_1, z_2, \ldots, z_L)$. The elements z_i are normally distributed with mean μ_{z_i} and variance $\sigma_{z_i}^2$ and are correlated with covariance matrix $\mathbf{C_z}$, i.e. in vector form

$$f_{\mathbf{Z}}(\mathbf{z}) = \frac{1}{(2\pi)^{L/2}|\mathbf{C_Z}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z}-\mu_{\mathbf{z}})^T \mathbf{C_Z}^{-1}(\mathbf{z}-\mu_{\mathbf{z}})\right) \tag{11}$$

Then the Moment Generating Function (MGF) of p is:

$$M_{p,MRC}(s) = \int_0^\infty \ldots \int_0^\infty G(s, p_1, \ldots, p_L) dp_1 \ldots dp_L \tag{12}$$

in which

$$G(s, p_1, \ldots, p_L) = \prod_{i=1}^{L} \exp(-sp_i) f_{R-Ln}(p_1, \ldots, p_L)$$

and s is the transform variable in the Laplace domain. By using (10) for p_1 conditioned on x_1 and re-arranging terms with p_1 together, then we have (13) as shown on top of next page. We can immediately recognize that the integral with respect to p_1 inside the square brackets gives the MGF of the exponentially-distributed RV [16], we have (14) shown on top of the next page. We proceed in the same way for $p_2, p_3, \ldots, p_L$ to obtain

$$\begin{aligned} M_{p,MRC}(s) = \int_{x_L}^\infty \ldots \int_{x_1}^\infty \prod_{i=1}^{L} M_{exp}(s, x_i) \\ \times f_{Ln}(x_1, \ldots, x_L) dx_1 \ldots dx_L \end{aligned} \tag{15}$$

in which x_i is the local power of the corresponding log-normal diversity branch. In the vector form for $\mathbf{x} = (x_1, x_2, \ldots, x_L)$, (15) can be rewritten as

$$M_{p,MRC}(s) = \int_0^\infty \prod_{i=1}^{L} M_{exp}(s, x_i) f_{Ln}(\mathbf{x}) d\mathbf{x} \tag{16}$$

By equating $f_{Ln}(\mathbf{x})d\mathbf{x} = f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z}$ in (16) and by using (11) we have:

$$\begin{aligned} M_{p,MRC}(s) = \int_{-\infty}^\infty \frac{1}{(2\pi)^{L/2}|\mathbf{C_z}|^{1/2}} \prod_{i=1}^{L} \left(\frac{1}{1 + s\exp(\frac{z_i}{\xi})}\right) \\ \times \exp\left(-\frac{1}{2}(\mathbf{z}-\mu_{\mathbf{z}})^T \mathbf{C_z}^{-1}(\mathbf{z}-\mu_{zv})\right) d(\mathbf{z}) \end{aligned} \tag{17}$$

To decorrelate (17), we make a change of variable $\mathbf{z} = \sqrt{2}\mathbf{C_z}^{1/2}\mathbf{u} + \mu_{\mathbf{z}}$, (17) becomes

$$\begin{aligned} M_{p,MRC}(s) = \int_{-\infty}^\infty \frac{1}{\pi^{L/2}} \left[1 + s\exp\left(\frac{\sqrt{2}}{\xi}\sum_{j=1}^{L} c_{ij}u_j + \frac{\mu_i}{\xi}\right)\right]^{-1} \\ \times \exp(\mathbf{u}^T\mathbf{u}) d\mathbf{u} \end{aligned} \tag{18}$$

$$M_{p,MRC}(s) = \int_{p_L}^{\infty} \cdots \int_{p_2}^{\infty} \left\{ \int_{x_1=0}^{\infty} \left[\int_0^{\infty} \left(\frac{e^{-(s+\frac{1}{x})p_1}}{x_1} \right) \right] f_{Ln}(x_1, p_1, \ldots, p_L) dx_1 \right\} dp_2 \ldots dp_L \tag{13}$$

$$M_{p,MRC}(s) = \int_{p_L}^{\infty} \cdots \int_{p_2}^{\infty} \left\{ \int_{x_1=0}^{\infty} \left(\frac{1}{1+sx_1} \right) f_{Ln}(x_1, p_1, \ldots, p_L) dx_1 \right\} dp_2 \ldots dp_L \tag{14}$$

where c_{ij} is the (i, j) element of $\mathbf{C}_\mathbf{z}^{1/2}$ which is obtained from $\mathbf{C}_\mathbf{z} = \mathbf{C}_\mathbf{z}^{1/2}(\mathbf{C}_\mathbf{z}^{1/2})^T$ using Cholesky decomposition. Since $\mathbf{C}_\mathbf{z}$ is symmetric and positive definite, $\mathbf{C}_\mathbf{z}^{1/2}$ is lower triangular. The integral in (18) can be accurately approximated using Gauss-Hermite expansion. The MGF of the sum of L correlated composite Rayleigh-lognormal power gains finally is:

$$M_{p,MRC}(s, \mu_\mathbf{z}, \mathbf{C}_\mathbf{z}) \approx \sum_{n_L=1}^{N_p} \cdots \sum_{n_1=1}^{N_p} \frac{w_{n_1} \ldots w_{n_L}}{\pi^{L/2}} \times \prod_{i=1}^{L} \left[1 + s \exp\left(\frac{\sqrt{2}}{\xi} \sum_{j=1}^{L} c_{ij} a_n + \frac{\mu_i}{\xi} \right) \right]^{-1} \tag{19}$$

As in [6] for the case of MRC diversity reception when only lognormal shadowing exists, in this paper we have found that (19) gives a very accurate result for (18) when $N_p = 12$.

Now we are ready to apply (19) to our AF diversity relaying network with $L = 2$ to find the PDF of $|h_{AF}|^2$ in (9). We deduce from (19) that

$$M_{|h_{AF}|^2}(s) = \frac{1}{\pi} \left\{ \sum_{n=1}^{N_p} \frac{w_n}{k_{n,sd}[s + k_{n,sd}^{-1}(.)]} \right\} \left\{ \sum_{n=1}^{N_p} \frac{w_m}{k_{m,R}[s + k_{m,R}^{-1}(.)]} \right\} \tag{20}$$

in which

$$k_{n,sd}(a_n, \mu_{sd}, \sigma_{sd}) = 10^{\sqrt{2}\sum_{i=1}^{2} c_{1i}a_n + \mu_{sd}} = \exp\left(\frac{\sqrt{2}}{\xi} \sum_{i=1}^{2} c_{1i} a_n + \frac{\mu_{sd}}{\xi} \right) \tag{21a}$$

$$k_{m,R}(a_n, \hat{\mu}_R, \hat{\sigma}_R) = 10^{\sqrt{2}\sum_{i=1}^{2} c_{1i}a_n + \hat{\mu}_R} = \exp\left(\frac{\sqrt{2}}{\xi} \sum_{i=1}^{2} c_{1i} a_n + \frac{\hat{\mu}_R}{\xi} \right) \tag{21b}$$

Since PDF is the inverse Laplace transform of MGF, we have from (20):

$$f_{|h_{AF}|^2}(p) = \text{ILT}\left(M_{|h_{AF}|^2} \right) = \frac{1}{\pi} \sum_{m=1}^{N_p} \sum_{n=1}^{N_p} w_m w_n \frac{\left(e^{-p/k_{n,sd}} - e^{-p/k_{n,sd}} \right)}{k_{n,sd}(.) - k_{m,R}(.)} \tag{22}$$

From (A·12) of the Appendix A, for a given correlation coefficient ρ_{LN} between two lognormal RVs, we have (23) as on the top of next page, which is required for the calculation of parameters k's in (21a) and (21b). Figure 4 illustrates the PDF of the power sum of two correlated Suzuki variables $|h_{sd}|^2$ and $|h_R|^2$ with corresponding Gaussian parameters $[\mu_{sd}, \sigma_{sd}] = [9.0309, 8]$ dB, $[\hat{\mu}_R, \hat{\sigma}_R] = [-2.5159, 6.7813]$ dB and correlation coefficient $\rho = 0.5$. We can see that the theoretical curve of the PDF of $|h_{AF}|^2$ based on (22) matches very well with the simulated one.

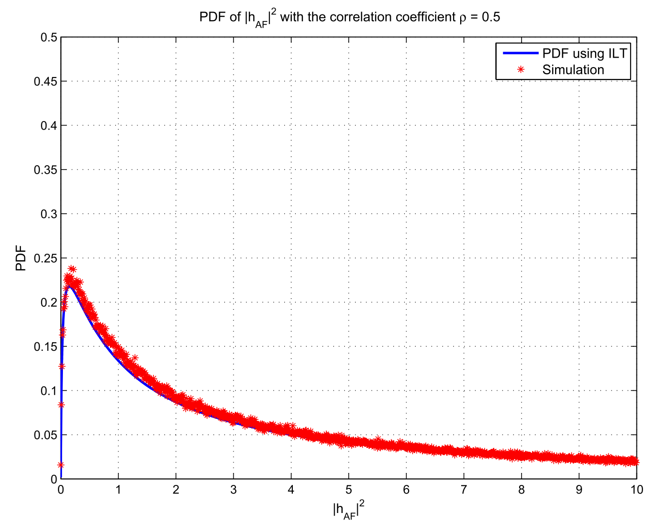

Fig. 4 PDF of the power sum of two correlated Suzuki channel.

3.3 Outage Probability of AF-relaying Network under Correlated Suzuki Fading Condition

The outage probability is a key parameter in wireless communications. It represents the probability that the transmitted signal can be lost due to the fluctuation in wireless channel response due to fading. With a given outage information rate threshold R_{th}, the outage probability between two end points i and j via M relays is defined by:

$$P_{|h_{ij}|^2}^{out}(SNR_0, R_{th}) = \Pr\left(|h_{ij}|^2 < \mu_{th} \right) = F_{|h_{ij}|^2} \tag{24}$$

where $F(.)$ denotes the cumulative density function (CDF) of the fading distribution and μ_{th} denotes the channel gain threshold given by:

$$\mu_{th} = \frac{2^{(M+1)R_{th}} - 1}{SNR_0} \tag{25}$$

Therefore in the AF cooperative diversity relaying network,

$$\mathbf{C_z} = \begin{bmatrix} \sigma_{sd}^2/\xi^2 & \ln(1+\rho_{LN}\sqrt{[\exp(\sigma_{sd}^2/\xi^2)][\exp(\hat{\sigma}_R^2/\xi^2)]}) \\ \ln(1+\rho_{LN}\sqrt{[\exp(\sigma_{sd}^2/\xi^2)][\exp(\hat{\sigma}_R^2/\xi^2)]}) & \sigma_{sd}^2/\xi^2 \end{bmatrix} \tag{23}$$

the outage probability is:

$$P^{out}_{|h_{AF}|^2}(SNR_0, R_{th}) = \Pr\left(|h_{AF}|^2 < \mu_{th}\right) = F_{|h_{AF}|^2}(\mu_{th}) \tag{26}$$

With the PDF of $|h_{AF}|^2$ given in (22), the outage probability in (26) of the cooperative AF relaying network, i.e. the CR reuse network (S_j, R_i, D) formed by the source CR S_j, the relay CR R_i and the destination fusion centre, is given as:

$$P^{out}_{|h_{AF}|^2}(S_j, R_i, \mu_{th}) = \frac{1}{\pi}\sum_{n=1}^{N_p}\sum_{m=1}^{N_p} w_n w_m \times\left[1 - \frac{k_{n,s_jd}(.)e^{-\frac{\mu_{th}}{k_{n,s_jd}(.)}} - k_{m,R_i}(.)e^{-\frac{\mu_{th}}{k_{m,R_i}(.)}}}{k_{n,s_jd}(.) - k_{m,R_i}(.)}\right] \tag{27}$$

For the case of a single Suzuki faded link h_{sd} between a CR and the FC, we can obtain the MGF of its power gain by setting $L = 1$ and $\rho_{LN} = 0$ in (19) or directly from [16]. Therefore,

$$M_{|h_{sd}|^2}(s, \mu_{sd}, \sigma_{sd}) \approx \sum_{n=1}^{N_p} \frac{w_n}{\sqrt{\pi}}\left[1 + e\exp\left(\frac{\sqrt{2}}{\xi}\sigma_{sd}a_n + \frac{\mu_{sd}}{\xi}\right)\right]^{-1} \tag{28}$$

By taking the Inverse Laplace Transform (ILT) of (28), we have:

$$f_{|h_{sd}|^2}(p) = \mathrm{ILT}\left(M_{|h_{sd}|^2}(s)\right) = \frac{1}{\sqrt{pi}}\sum_{n=1}^{N_p} w_n \frac{e^{-p/k_{n,sd}}}{k_{n,sd}} \tag{29}$$

where $k_{n,sd} = \exp\left(\frac{\sqrt{2}}{\xi}\sigma_{sd}a_n + \frac{\mu_{sd}}{\xi}\right)$. Therefore, the outage of this direct link is given as:

$$P^{out}_{|h_{sd}|^2} = F_{|h_{sd}|^2} = \int_0^{\mu_{th}} \frac{1}{\sqrt{\pi}}\sum_{n=1}^{N_p} w_n e^{-p/k_{n,sd}}dp = \frac{1}{\sqrt{\pi}}\sum_{n=1}^{N_p} w_n\left(1 - e^{-\mu_{th}/k_{n,sd}}\right) \tag{30}$$

4. Algorithm of Re-Use Deep Faded CRs as Relays

In this section, we will consider two algorithms: first, to identify CRs which are in deep fading; second, to assign eliminated CRs as relay to improve the reliability of transmission between surviving CRs and the FC.

4.1 Algorithm to Eliminate Deeply Faded CRs.

The precision of the final decision by the FC about the presence or absence of the primary user's signal obviously depends on the reliability of the information sent from individual CRs. Therefore it is essential that the FC is able to discriminate against information from untrustworthy CRs which are corrupted by deep fading. It is obvious that the reliability of the sensing information which finally reaches the FC depends on the fading condition of both the sensing and reporting channels. To keep the spectrum sensing overhead down, a realistic CR eliminating algorithm will only make this decision once at the start of the sensing session. This is equivalent to assuming that the channel quality, hence its SNR, due to fading remains invariable for the entire spectrum sensing session. However, this issue is not a topic of interest in this paper and we simply assume that if the SNR associated with the sensing information from a CR is below a given threshold, then the sensing information from that CR will be discarded from the decision process by the FC. These deeply faded CRs will no longer be used for sensing but will be reassigned by the FC to assist the healthier CRs by acting as cooperative diversity relays to the latters in the reporting network.

4.2 Algorithm to Assign Eliminated CRs as Relays

The pairing algorithm is based on the minimum probability of outage of the entire reporting network, i.e.

$$\left(S_j, R_i\right) = \arg_{(j,i)}\min\left\{P^{out}_{|h_{AF}|^2}(S_j, R_i, \mu_{th})\right\} \tag{31}$$

of the cooperative diversity relay network resulted from each pair (S_j, R_i) is calculated from (27). The proposed algorithm is simply to select pairs with the lowest outage probability $P^{out}_{|h_{AF}|^2}(S_j, R_i, \mu_{th})$.

5. Scenario and Numerical Results

5.1 Scenario

In this paper, we consider a model of the cognitive radio reporting network as shown in Fig. 5. In this model, CRs are evenly distributed over a circle. In details, the SNRs in the raw signals measured at the 8 CRs are SNR_i = [2.8782, 8.3683, 9.3683, 5.1447, 2.0362, −3.458, 0.2434, −6.8987] dB, and the threshold set by the FC is $\mathrm{SNR}_{\mathrm{th}}$ = 0.5 dB. Here, we also assume that propagation paths between sensors and PU are under Suzuki fadings. Based on

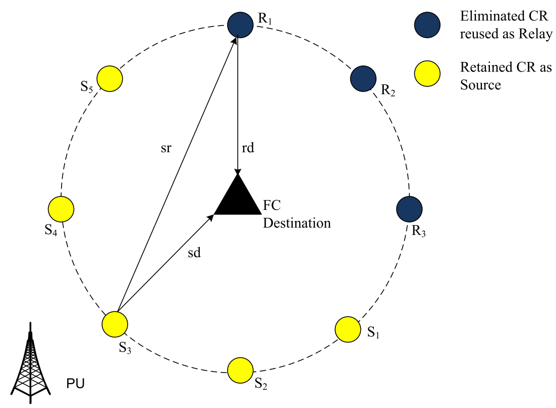

Fig. 5 Simulation model of reporting network to illustrate the proposed pairing algorithm.

SNR information, FC eliminates 3 CRs (S_6, S_7, S_8) which have average SNR lower than SNR_{th} from participating in the fusion process. Then, the detection probability of each surviving CR_i under Suzuki fading and without cooperating relay is calculated as in (6).

In the reporting phase, the FC reassigns discarded CRs to act as relays for surviving CRs to improve the transmission reliability of the CR-to-FC reporting channels, thus forming 3-terminal relay networks with the surviving CRs as sources (S), the eliminated CRs as relays (R), and the FC as the destination (D). In this case, each relay forwards information from only one CR.

Furthermore, we assume that Suzuki fading channels in reporting phase all have $\sigma = 8$ dB and that μ is proportional to a negative exponent of the propagation distance, $d^{-\alpha}$. At the distance of $2d$ in which d denotes the radius of the circle, we normalize $\mu_{\text{normalized}} = 0$ dB. Therefore, the mean μ of a link with the length of d_μ is defined as below:

$$\mu = 10 \log 10 \left[\left(\frac{d_\mu}{2d} \right)^{-\alpha} \right] + \mu_{\text{normalized}} \tag{32}$$

5.2 Results

Table 1 shows the means of each link between (S_j, R_i). Assuming an urban environment, we choose attenuation index $\alpha = 3$. Note that for the simulation model in Fig. 5, $\mu_{rd} = \mu_{sd} = 9.0309$ dB which are required for computation in Sect. 3.2. From these simulation data, the Gaussian parameters of h_R are found for each cooperative diversity relay network (S_j, R_i, D) as shown in Table 2. The outage probability of all the networks (S_j, R_i, D) as calculated from (27) is shown in Table 3 for rate threshold $\mu_{th} = 0.1$. Based on Table 3, we choose the 3 best pairs (S_j, R_i) with the lowest outage probability. They are (S_1, R_3), (S_2, R_2) and (S_5, R_1) corresponding to the outage probabilities of 0.0058, 0.0151 and 0.0058 with $\mu_{th} = 0.1$. The result on Table 3 is rather expected, i.e. the pairing rule is simply between the two closest CRs.

Because the reporting channels between the CRs and the FC are subjected to fading and hence link outages, the reported parameters received at the FC are usually deteriorated. The effective probability of detection received from the jth CR by the FC is:

$$P_{De}(j) = P_{D_j} \left\{ 1 - P^{out}(j) \right\} \tag{33}$$

where P_{D_j} is the original probability of detection of the PU during the sensing phase calculated from (6), i.e. the reported probability of detection if the reporting channel is lossless, and $P^{out}(j)$ is the outage probability of the channel between the jth CR and the FC either with (using (27)) or without diversity relaying (using (30)). It can be easily observed that the outage probability in (27) is lower with the use of the relay compared to that without the relay, hence improving the local effective probability of detection in (33) received by the fusion center. In this paper, we simply assume that the FC use AND rule to make the global decision [19].

Figure 6 shows the results of the receiver operating characteristic (ROC) curves of the cooperative sensing network in three operating environments: lossless reporting channels, correlated Suzuki fading reporting channels without re-use of eliminated CRs as diversity relays, and correlated Suzuki fading reporting channels with re-use of eliminated CRs as diversity relays. All ROC curves are for the simulation model in Fig. 5. As shown in the figure, the black curve which represents the ROC with 3 relays gives a better performance than the red one which is the ROC without any relay. Thus, we can declare that the detection performance of the system is improved when using relays to support survival CRs sending sensing data to the FC over the correlated Suzuki environment.

Furthermore, we investigate the effect of the number

Table 1 Values of μ(in dB) of (S_j, R_i) channel with $\mu_{\text{normalized}} = 0$ dB at the distance $2d$, $\alpha = 3$, calculated from (32).

CR	S_1	S_2	S_3	S_4	S_5
R_1	1.0313	0	1.0313	4.5154	12.5142
R_2	4.5154	1.0313	0	1.0313	4.5154
R_3	12.5142	4.5154	1.0313	0	1.0313

Table 2 Estimated $\hat{\mu}_R$ and $\hat{\sigma}_R$ of h_R of the Cooperative Relay Network (S_j, R_i, D) in dB using *fsolve* function matching at $p_1 = 0.1$ and $p_2 = 0.2$ (See Fig. 3).

CR	S_1	S_2	S_3	S_4	S_5
R_1	−1.7922 6.6235	−2.5159 6.7813	−1.7922 6.6235	0.3847 6.4352	4.6806 6.6576
R_2	0.3847 6.4352	−1.7922 6.6235	−2.5159 6.7813	−1.7922 6.6235	0.3847 6.4352
R_3	4.6806 6.6576	0.3847 6.4352	−1.7922 6.6235	−2.5159 6.7813	−1.7922 6.6235

Table 3 Matrix of Probability of Outage of Cooperative Relay Network (S_j, R_i, D) with $\mu_{th} = 0.1$.

CR	S_1	S_2	S_3	S_4	S_5
R_1	0.0151	0.0168	0.0151	0.0109	0.0058
R_2	0.0109	0.0151	0.0168	0.0151	0.0109
R_3	0.0058	0.0109	0.0151	0.0168	0.0151

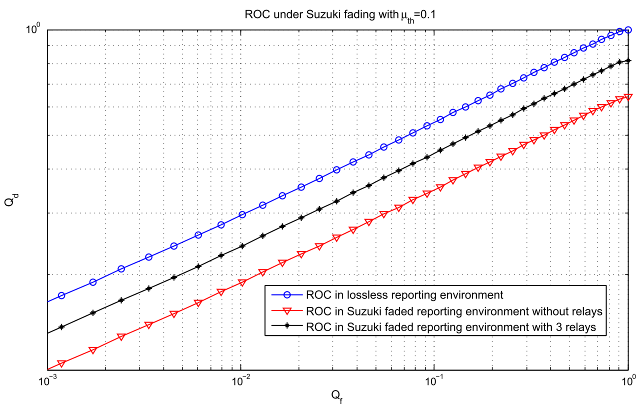

Fig. 6 Sensing performance of the cooperative spectrum sensing network under fading with and without re-use of poor CRs as diversity relays, outage threshold $\mu_{th} = 0.1$.

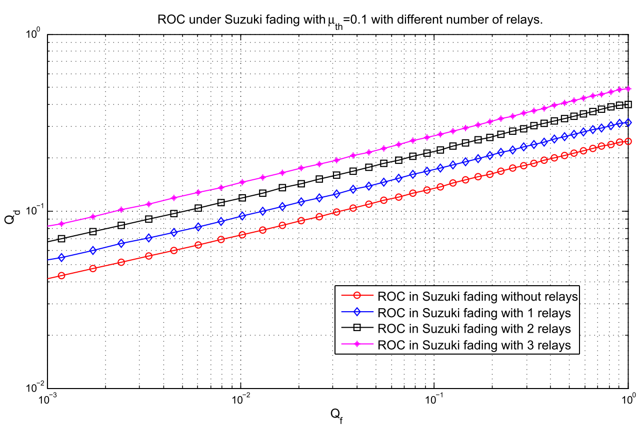

Fig. 7 Sensing performance of the cooperative spectrum sensing network under fading with and without re-use of poor CRs as diversity relays, outage threshold $\mu_{th} = 0.1$.

of relays on detection performance of cooperative spectrum sensing as depicted in Fig. 7. Using the obtained results, we have already the set of 3 relays which have the best probabilities of outage as shown in Table 3. We gradually reduce the number of relays in descending order by excluding the one with the highest probabilty. In particular, in the case of using 2 relays to forward sensing data to the FC, we choose the best pairs of (S_1, R_3) and (S_2, R_2). For the rest case of 1 relay, we choose the best pair of (S_1, R_3). As shown in Fig. 7, the more relays are used, the better detection performance of cooperative spectrum sensing we obtain.

6. Conclusion

The paper has successfully presented an accurate analysis of our proposed strategy of re-using shadowed sensing CRs, which would otherwise be discarded, as diversity AF relays to improve the performance of surviving peers against correlated composite Rayleigh-lognormal fading in the reporting network. The most significant contribution of this paper is the derivation of a closed-form and accurate expression for the probability of outage probability P_{out} of the resulting three-terminal cooperative diversity AF relaying network under such correlated fading. In particular, an efficient solution is proposed to the fundamental problem of "PDF of sum-of-powers" of correlated Suzuki-distributed random variables using Gauss-Hermite polynomial approximation to their moment generating function (MGF). This expression allows us to calculate the effective probability of detection P_D and to greatly speed up the execution of the proposed re-use algorithm, giving us the incentive to research a more sophisticated and efficient algorithm. The effectiveness of the strategy was judged on the basis of resulting global ROC curves, i.e. global probability of detection, Q_D, versus global probability of false alarm, Q_F. The benefit of the proposed re-use of shadowed sensing CRs as diversity relays is also proved.

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Appendix: Calculate $\mathbf{C_z}$ from $\mathbf{C_{Ln}}$

For simplicity, we consider the case $L = 2$ for two adjacent channels h_1 and h_2, but the analysis below can be generalized to the case of two non-adjacent channels h_i and h_j.

Given h_1 and h_2 being both log-normal random variables and their associated normal $Z_1 \sim N(\mu_{Z_1}), \sigma_{Z_1})^2)$ and $Z_2 \sim N(\mu_{Z_2}), \sigma_{Z_2})^2)$, i.e.

$$h_1 = \exp(\mu_{Z_1}) exp(X_1) \text{where} X_1 \sim N(\mu_{Z_1}, \sigma^2_{Z_1}) \quad \text{(A· 1a)}$$

$$h_2 = \exp(\mu_{Z_2}) exp(X_2) \text{where} X_2 \sim N(\mu_{Z_2}, \sigma^2_{Z_2}) \quad \text{(A· 1b)}$$

The correlation coefficient between X_1 and X_2 is ρ_Z,we have:

$$E[h_1] = \exp(\mu_{Z_1}) E(X_1) = \exp\left(\mu_{Z_1} + \frac{1}{2}\sigma^2_{Z_1}\right) \quad \text{(A· 2a)}$$

$$E[h_2] = \exp(\mu_{Z_2}) E(X_2) = \exp\left(\mu_{Z_2} + \frac{1}{2}\sigma^2_{Z_2}\right) \quad \text{(A· 2b)}$$

and

$$\begin{aligned} \text{var}[h_1] &= \exp\left[2\mu_{Z_1} + \sigma^2_{Z_1}\right]\left[\exp(\sigma^2_{Z_1}) - 1\right] \\ &= E[h_1]^2 \left[\exp(\sigma^2_{Z_1}) - 1\right] \quad \text{(A· 3a)} \end{aligned}$$

$$\begin{aligned} \text{var}[h_2] &= \exp\left[2\mu_{Z_2} + \sigma^2_{Z_2}\right]\left[\exp(\sigma^2_{Z_2}) - 1\right] \\ &= E[h_2]^2 \left[\exp(\sigma^2_{Z_2}) - 1\right] \quad \text{(A· 3b)} \end{aligned}$$

also

$$\begin{aligned} E[h_1 h_2] &= \exp(\mu_{Z_1} + \mu_{Z_2}) E[\exp(X_1 + X_2)] \\ &= \exp(\mu_{Z_1} + \mu_{Z_2}) \exp\left(\frac{1}{2}\sigma^2_{X_1+X_2}\right). \quad \text{(A· 4)} \end{aligned}$$

Since

$$\sigma^2_{X_1+X_2} = \text{var}(X_1 + X_2) = \sigma^2_{Z_1} + 2\rho_Z \sigma_{Z_1}\sigma_{Z_2} + \sigma^2_{Z_2} \quad \text{(A· 5)}$$

Then, we have:

$$\begin{aligned} &E[h_1 h_2] \\ &= \exp(\mu_{Z_1} + \mu_{Z_2}) \exp\left[\frac{1}{2}\left(\sigma^2_{Z_1} + 2\rho_Z \sigma_{Z_1}\sigma_{Z_2} + \sigma^2_{Z_1}\right)\right] \quad \text{(A· 6)} \end{aligned}$$

(A· 6) can be re-arranged to give:

$$\begin{aligned} E[h_1 h_2] &= \exp\left(\mu_{Z_1} + \frac{1}{2}\sigma^2_{Z_1}\right) \exp\left(\mu_{Z_2} + \frac{1}{2}\sigma^2_{Z_2}\right) \exp(\rho_Z \sigma_{Z_1}\sigma_{Z_2}) \\ &= E(h_1)E(h_2) \exp(\rho_Z \sigma_{Z_1}\sigma_{Z_2}). \end{aligned} \quad \text{(A· 7)}$$

Therefore:

$$\begin{aligned} \text{cov}(h_1 h_2) &= E[h_1 h_2] - E[h_1]E[h_2] \\ &= E[h_1]E[h_2] \{\exp(\rho_Z \sigma_{Z_1}\sigma_{Z_2}) - 1\} \quad \text{(A· 8)} \end{aligned}$$

And:

$$\rho_{LN} = \frac{\text{cov}(h_1 h_2)}{\sqrt{\text{var}(h_1)\text{var}(h_2)}} = \frac{\{\exp(\rho_Z \sigma_{Z_1}\sigma_{Z_2}) - 1\}}{\sqrt{\left[\exp\left(\sigma^2_{Z_1} - 1\right)\right]\left[\exp\left(\sigma^2_{Z_2} - 1\right)\right]}} \quad \text{(A· 9)}$$

Giving

$$\rho_Z \sigma_{Z_1}\sigma_{Z_2} = \ln\left(1 + \rho_{LN} \sqrt{\left[\exp\left(\sigma^2_{Z_1} - 1\right)\right]\left[\exp\left(\sigma^2_{Z_2} - 1\right)\right]}\right) \quad \text{(A· 10)}$$

The covariance matrix of two correlated log-normal variables h_1 and h_2, by definition is:

$$\mathbf{C_{Ln}} = \begin{bmatrix} \text{var}(h_1) & \rho_{LN}\sqrt{\text{var}(h_1)\text{var}(h_1)} \\ \rho_{LN}\sqrt{\text{var}(h_1)\text{var}(h_1)} & \text{var}(h_2) \end{bmatrix}$$

Using (A· 9) and (A· 3) we have (A· 11) as shown on the top of next page while the covariance matrix of two correlated variables Z_1 and Z_2, by definition is:

$$\mathbf{C_z} = \begin{bmatrix} \sigma^2_{Z_1} & \rho_Z \sigma_{Z_1}\sigma_{Z_2} \\ \rho_Z \sigma_{Z_1}\sigma_{Z_2} & \sigma^2_{Z_1} \end{bmatrix}$$

Using (A· 10) we have (A· 12) as on the top of next page.We can generalize to the case of non-adjacent channels, $L > 2$ as

$$c_z(i, j) = \text{cov}\left(Z_i, Z_j\right) = \ln\left(1 + \rho_{LN}^{|i-j|} \frac{\sqrt{\text{var}(h_i)\text{var}(h_j)}}{E(h_i)E(h_j)}\right) \quad \text{(A· 13)}$$

$$\mathbf{C_{Ln}} = \begin{bmatrix} E[h_1]^2\left[\exp\left(\sigma_{Z_1}^2 - 1\right)\right] & \left\{\exp\left(\rho_Z \sigma_{Z_1}\sigma_{Z_2}\right) - 1\right\} E[h_1]E[h_2] \\ \left\{\exp\left(\rho_Z \sigma_{Z_1}\sigma_{Z_2}\right) - 1\right\} E[h_1]E[h_2] & E[h_2]^2\left[\exp\left(\sigma_{Z_2}^2 - 1\right)\right] \end{bmatrix} \tag{A$\cdot$ 11}$$

$$\mathbf{C_z} = \begin{bmatrix} \sigma_{Z_1}^2 & \ln\left(1 + \rho_{LN}\sqrt{\left[\exp\left(\sigma_{Z_1}^2\right) - 1\right]\left[\exp\left(\sigma_{Z_1}^2\right) - 1\right]}\right) \\ \ln\left(1 + \rho_{LN}\sqrt{\left[\exp\left(\sigma_{Z_1}^2\right) - 1\right]\left[\exp\left(\sigma_{Z_1}^2\right) - 1\right]}\right) & \sigma_{Z_2}^2 \end{bmatrix} \tag{A$\cdot$ 12}$$

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