A closed-form of Cooperative Detection Probability using EGC-Based Soft Decision under Suzuki Fading

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***Abstract*—In cooperative spectrum sensing based on energy detection, several researchers have concluded that Soft Decision has better detection performance than Hard Decision. In this paper, we focus on Equal Gain Combining (EGC)-based soft decision under Suzuki fading which is a composite Rayleigh-lognormal fading. We use Moment-Generating function (MGF) to approximate Probability Density Function (PDF) of power sum of received signals at Fusion Center. Then we propose a novel method to evaluate cooperative detection performance under the effect of i.i.d Suzuki fading by using Gauss-Hermite approximation and MGF matching. Finally, we compare the results of EGC-based Soft Decision with those of Hard Decision.**

***Index Terms*—cooperative spectrum sensing, soft decision, EGC, Suzuki fading, Gauss-Hermite integration, MGF matching.**

I. Introduction

The scarcity of radio spectrum in wireless communication has become a serious problem. The radio spectrum, however, is not utilized efficiently while the demand for frequency spectrum for wireless communication services has increased significantly in last few years. Cognitive Radio (CR) proposed by Mitola [1] has been becoming an emerging technology to utilize spectrum more efficiently and alleviate spectrum scarcity. A CR network allows Secondary Users (SU), i.e. unlicensed users, to access a spectrum when it is not used by Primary Users (PU), i.e the licensed users, and without causing harmful interference to PUs [2].

To detect spectrum holes which is not occupied by PU and provide spectrum access for SUs, spectrum sensing techniques have an important and essential role. In particular, spectrum usage and PU presence is detected by SUs due to spectrum sensing techniques.

Various sensing techniques have been addressed in [3]. The authors pointed out that each method has pros and cons, in which energy detector (ED) is an appropriate candidate to enable CR functionalities because ED has low computational complexity and does not require any the transmitted signal's prior information.

A number of studies have focused on analyzing the performance of ED over several fading channels [4-7]. With aims to enhance the detection performance of spectrum sensing under effect of fadings, the CRs do a collaboration by sending their sensing information to a Fusion Center (FC) [4], [8]. Two methods of cooperative spectrum sensing, Hard Decision and EGC-Based Soft Decision, has already been discussed in research papers [4], [5]. When utilizing EGC-Based Soft Decision, the performance of cooperative spectrum sensing gets a higher detection probability when compared with Hard Decision [5]. Motivated from this observation, EGC-Based Soft Decision under fading environment, such as Rayleigh or lognormal fading, is obtained by numerical methods [4], [9]. Up to now, based on our studies there is no published research about the EGC-based soft decision under Suzuki fading. Therefore, in this paper, we study this issue and evaluate the detection performance of cooperative spectrum sensing of the CR network under the effect of Suzuki fading. We propose a closed-form of cooperative detection probability by utilizing MGF matching technique and Gauss-Hermite integration. Based on mathematical analysis, we investigate the detection performance of cooperative spectrum sensing under Suzuki fading.
The rest of paper is organized as follows: Section II introduces a system model of cooperative spectrum sensing. We study the detection performance of cooperative spectrum sensing under Suzuki fading in Section III. We discuss some obtained results in Section IV. Finally, in Section V, we conclude the paper.

II. Model of Cooperative Spectrum Sensing System

A. Local Spectrum Sensing

A cognitive radio network which comprises n CRs and a Fusion Centre (FC) is shown in Fig. 1. In the case of local

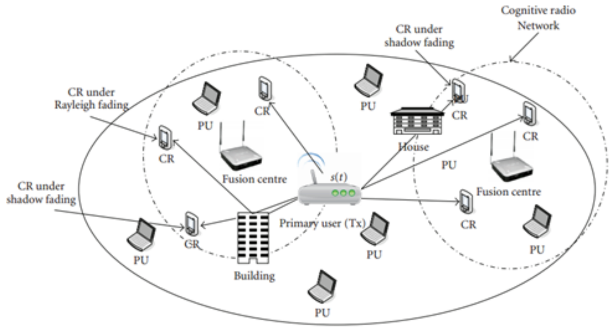

Fig. 1. System Model of Cognitive Radio Network [10]

spectrum sensing, each CR performs spectrum sensing individually. Each CR makes a decision based on the received PU's signal. Hence, the spectrum sensing problem can be regarded as a binary hypothesis testing problem defined mathematically as follows,

$$x(t) = \begin{cases} n(t), H_0 \\ hs(t) + n(t), H_1 \end{cases}$$

where $x(t)$, $s(t)$, h and $n(t)$ denote respectively the signal received by the CR, the transmitted PU signal, the amplitude gain of the channel and the Additive White Gaussian Noise (AWGN). The signal-to-noise ratio, SNR, is determined as $\frac{P}{N_0 W}$ whereas P is the transmited power of PU, N_0 is the AWGN power spectral density and W denotes the channel bandwidth.

According to the previous work of Urkowitz [11], the average received energy, Y, is compared with a threshold to make a decision (H_0 or H_1). To assess the performance of local spectrum sensing, we take into account the following parameters: false-alarm probability P_f, detection probability P_d, and missed detection probability P_m. In AWGN channel, h is deterministic. The false-alarm and detection probabilities are given as follows [4],

$$P_f = P\{Y > \lambda | H_0\} = \frac{\Gamma(m, \lambda/2)}{\Gamma(m)} = G_m(\lambda) \tag{1}$$

$$P_d = P\{Y > \lambda | H_1\} = Q_m(\sqrt{2m\gamma}, \sqrt{\lambda}) \tag{2}$$

Whereas, $\Gamma(a, b)$ and $Q_m(.,.)$ denote respectively the incomplete gamma function [11] and the generalized Marcum Q-function [12]. From Equations (2) and (1), P_d and P_f are related to each other via parameter λ . Combining these two equations, we obtain the following relationship,

$$P_d = Q_m\left(\sqrt{2m\gamma}, \sqrt{G_m^{-1}(P_f)}\right) \tag{3}$$

With the effect of fading and shadowing, h is varying and P_d becomes a conditional probability dependent on the instantaneous SNR γ . As a results, the average detection probability may be obtained by averaging P_d in (3) over statistics of fading,

$$P_{d,fading} = \int\limits_{\gamma} Q_m\left(\sqrt{2mx}, \sqrt{G_m^{-1}(P_f)}\right) f_\gamma(x)dx \tag{4}$$

whereas, $f_\gamma(x)$ is the PDF of SNR γ under fading.

B. Local Spectrum Sensing under Suzuki fading channel

The Suzuki distribution was recommended by Hirofumi Suzuki in 1977 [13]. The PDF of Suzuki fading can be derived by equating the local average power of the Rayleigh faded signal to the instantaneous power of the arriving lognormal signal [14]. In Suzuki fading, there is no significant loss of power in the local multipath channel, i.e. the average power gain, $E\left[|h_R|^2\right]$, is 1. Then, the distribution of the power gain p under Suzuki fading channel can be modeled as PDF of the product of power gain of Rayleigh channel $|h_R|^2$ and power gain of lognormal channel $|h_{Ln}|^2$,

$$p = |h_{R-Ln}|^2 = |h_R|^2 |h_{Ln}|^2 \tag{5}$$

Using the Jacobian transformation technique, we can obtain the PDF of the composite fading channel as follows,

$$\begin{aligned} f_{R-Ln}(p) &= \int_0^\infty \frac{1}{x} f_R(\frac{p}{x}) f_{Ln}(x)dx \\ &= \int_0^\infty \frac{1}{x}\left(\frac{1}{\overline{p}_R}\exp\left(-\frac{p}{\overline{p}_R x}\right)\right) \\ &\times \frac{1}{x\sigma_z\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \mu_z)^2}{2\sigma_z^2}\right)dx \end{aligned} \tag{6}$$

where (μ_z, σ_z^2) are the parameters of the lognormal fading. Since, $\overline{p}_R = E|h_R|^2 = 1$, (6) may be rewritten as,

$$f_{R-Ln}(p) = \int\limits_0^\infty \frac{1}{x^2}\exp\left(-\frac{p}{x}\right)\frac{1}{\sigma_z\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \mu_z)^2}{2\sigma_z^2}\right)dx \tag{7}$$

The detection probability under Suzuki fading can be obtained by substituting from (7) into (4),

$$\begin{aligned} P_{d,Suzuki} = &\int\limits_0^\infty \int\limits_0^\infty \frac{1}{x}\exp\left(-\frac{p}{x}\right)\frac{1}{x\sigma_z\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \mu_z)^2}{2\sigma_z^2}\right) \\ &\times Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right)dxdp \end{aligned} \tag{8}$$

A closed-form of the detection probability can be approximated as in our previous work [16],

$$P_{d,Suzuki} = \frac{1}{\sqrt{\pi}}\Sigma_{i=1}^{N_p} w_i P_{d,Rayleigh}(\overline{\gamma} = e^{\sqrt{2}\sigma_z a_i + \mu_z}) \tag{9}$$

where a_i, w_i and N_p are respectively ascissas, weight factors number of samples of the Gauss-Hermite integration. a_i and w_i for different values of N_p are available in [11, Table (25.10)]. Higher value of N_p give more accurate approximation. High accuracy is attained when $N_p \geq 6$.

C. Cooperative Spectrum Sensing

1) Hard Decision Combing: In Hard Decision method, each CR performs a local spectrum sensing and sends its individual sensing information ($u_i = 0, 1$) to a FC. If $u_i = 1$, the hypothesis H_1 will be chosen, otherwise, hypothesis H_0 will be chosen. The FC then colligates the incoming information

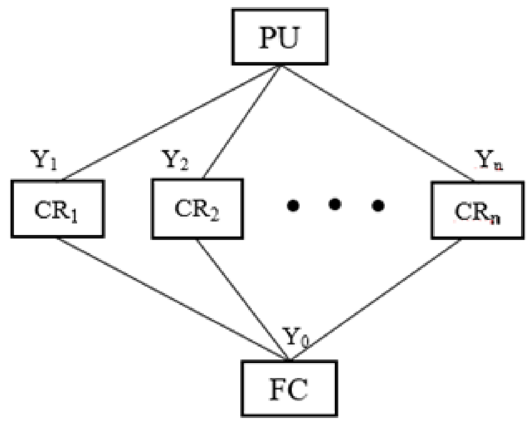

Fig. 2. Model of cooperative spectrum sensing using EGC-based Soft Decision

to make a decision about the presence of a PU signal. For simplicity, we assume that:

- The sensing channels, i.e, links between PU and CRs are affected by i.i.d fading.
- The reporting channels, i.e, links between CRs and FC, are ideal which means information from CRs to FC is not lost or changed.
- The FC applies the Hard Decision according to (k-out-of-n) rule.

For special cases, $k = 1$ denotes OR rule, $k = n$ denotes AND rule and $k = [n/2]$ denotes Majority rule. Hence, the total probability of detection Q_d and the total probability of false-alarm Q_f when n CRs join the cooperative spectrum sensing [4] are

$$Q_d = \Sigma_{i=k}^{n} C_n^i P_d^i (1 - P_d)^{n-i} \tag{10}$$

$$Q_f = \Sigma_{i=k}^{n} C_n^i P_f^i (1 - P_f)^{n-i} \tag{11}$$

whereas P_f and P_d were defined in (2) and (4), respectively. The total probability of missed detection is

$$Q_m = 1 - Q_d \tag{12}$$

2) EGC-based Soft Decision: In this section, collaborating spectrum sensing using EGC-based Soft Decision is investigated. Fig. 2 shows a model of cooperative spectrum sensing using EGC-based Soft Decision. In this method, each CR senses energy from PU and sends it to FC. At the FC, power gains Y_i are combined to obtain the sum of power gains Y_0. Then, FC compares Y_0 to a given threshold to select the appropriate hypothesis (H_0 or H_1.) We have the sum of energies:

$$Y_0 = \Sigma_{i=1}^{n} Y_i$$

Therefore, the PDF of combiner output given by [4]:

$$f_{Y_0|H_0}(y) = \frac{y^{nm-1} e^{-y/2}}{\Gamma(nm) 2^{nm}} \tag{13}$$

$$f_{Y_o|H_0} = \frac{y^{m-1} e^{(} y + 2m\Sigma_{i=1}^{n}\gamma_i)/2}{\Gamma(nm) 2^{nm}} \times {}_0F_1\left(nm, \frac{my\Sigma_{i=1}^{n}\gamma_i}{2}\right) \tag{14}$$

whereas, γ denotes instant Signal to Noise Ratio (SNR) and ${}_0F_1(.,.)$ is the confluent hyper-geometric limit function [12]. So, we can calculate the probabilities of detection false-alarm as:

$$\begin{aligned} Q_d &= P\{Y_0 > \lambda | H_1, \gamma_1 = l_1, \ldots, \gamma_n = l_n\} \\ &= Q_{nm}\left(\sqrt{2m\Sigma_{i=1}^{n} l_i}, \sqrt{\lambda}\right) \end{aligned} \tag{15}$$

$$Q_f = P\{Y_0 > \lambda | H_0\} = \frac{\Gamma(nm, \lambda/2)}{\Gamma(nm)} \tag{16}$$

However, the conditional probability Q_d is only a function of $\gamma_0 = \Sigma_{i=1}^{n}\gamma_i$. The detection probability under fading channels can be defined as follows [4]:

$$Q_d = \int_{\gamma_0} Q_{nm}\left(\sqrt{2mx}, \sqrt{\lambda}\right) f_{\gamma_0}(x) dx \tag{17}$$

whereas $f_{\gamma_0}(x)$ denotes the PDF of SNR γ_0.

III. Performance of Cooperative Spectrum Sensing using EGC-Based soft decision under Suzuki fading

As mentioned before, detection performance of Soft Decision under different fadings, such as, lognormal or Rayleigh fading, are investigated. In this section, we will deal with Suzuki fading and propose a closed-form to calculate the cooperative detection performance Q_d . As shown in (17), Q_d can be calculated if the PDF of γ_0 is determined. With the assumption that all received SNRs, γ_i ($i = 1, \ldots, n$), have i.i.d Suzuki distribution, then γ_0 is the sum of n i.i.d Suzuki random variables. We use MGF matching [17] to approximate γ_0 as a Suzuki RV. MGF of a Suzuki RV $\gamma_i = e^{X_i}$ with $X_i \sim N(\mu_{x_i}, \sigma_{x_i}^2)$ can be written as [18]

$$\Psi_{\gamma_i}(s) = \frac{1}{\sqrt{\pi}} \Sigma_{n=1}^{N_p} \frac{w_n}{1 + s\exp(\sqrt{2}\sigma_{x_i} + \mu_{x_i})} \tag{18}$$

MGF of sum of n Suzuki RVs $\gamma_0 = \Sigma_{i=1}^{n}\gamma_i$ is given as follows

$$\Psi_{\gamma_0}(s) = \Pi_{i=1}^{n} \Psi_{\gamma_i(s)} \tag{19}$$

where N_p, a_n, and w_n denote respectively order of Gauss - Hermite integration, abscissas, weight. Terms w_n and a_n can use results in Table I when we choose $N_p = 8$ [19]. We match the MGF of $\hat{\gamma}$ with the MGF of γ_0 in (19) at two different points, real and positive values of s, namely, s_1 and s_2 using *fslove* function in Matlab. Then we obtain the estimated parameters $(\hat{\mu}, \hat{\sigma}^2)$ of Suzuki RV $\hat{\gamma}$.
Therefore, we can rewrite PDF of γ_0, $f_{\gamma_0}(p)$, as follows:

$$f_{\gamma_0}(p) = \int_0^{\infty} \frac{1}{x^2} \exp\left(\frac{p}{x}\right) \frac{1}{\hat{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \hat{\mu})^2}{2\hat{\sigma}^2}\right) dx \tag{20}$$

Fig. 3 illustrates an example of MGF matching of two

TABLE I
WEIGHT AND ABSCISSAS WITH $N_p = 8$

a_n	w_n
0.38118	0.66144
1.15719	0.20780
1.98165	0.017077
2.93063	0.0001996
-2.93063	0.0001996
-1.98165	0.017077
-1.15719	0.20780
-0.38118	0.66144

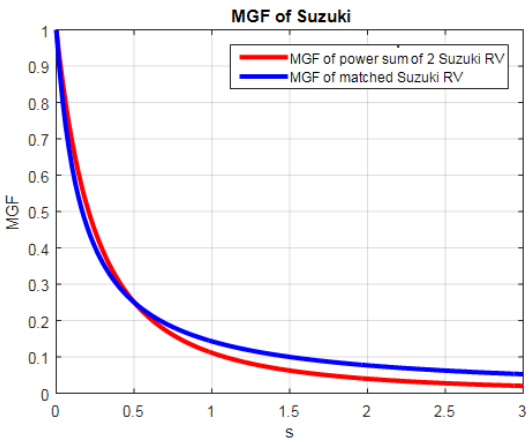

Fig. 3. MGF matching of two i.i.d Suzuki RVs with a Suzuki RV,

i.i.d Suzuki RVs with $\mu = 3$dB, $\sigma = 0$ dB at two points $(s_1, s_2) = (0.4, 1)$. The estimated parameters are $\hat{\sigma} = 0.011$ dB and $\hat{\mu} = 7.7718$ dB.

Replacing $f_{\gamma_0}(p)$ in Equation (20) into Equation (17), we get the cooperative detection probability $Q_{d,Su}$ under Suzuki fading as follows:

$$\begin{aligned} Q_{d,Su} &= \int_0^\infty \int_0^\infty \frac{1}{x} \exp(-\frac{p}{x}) \frac{1}{x\hat{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \hat{\mu})^2}{2\hat{\sigma}^2}\right) \\ &\times Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right) dxdp \\ &= \int_0^\infty \left[\int_0^\infty \frac{1}{x} \exp(-\frac{p}{x}) Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right) dp\right] \\ &\times \frac{1}{x\hat{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \hat{\mu})^2}{2\hat{\sigma}^2}\right) dx \end{aligned} \tag{21}$$

The function inside the square bracket is the cooperative probability of detection under Rayleigh channel. As presented in [6], the closed-form of Q_d under Rayleigh fading channel can be written as follows:

$$\begin{aligned} Q_{d,Ray}(\overline{\gamma}) &= \int_0^\infty \frac{1}{\overline{\gamma}} \exp(-\frac{p}{\overline{\gamma}}) Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right) dp \\ &= e^{-\frac{\lambda}{2\sigma^2}} \Sigma_{i=0}^{mn-2} \frac{\left(\frac{\lambda}{2\sigma^2}\right)^i}{i!} + \left(\frac{2\sigma^2 + a\overline{\gamma}n}{a\overline{\gamma}n}\right)^{mn-1} \\ &\times \left[e^{-\frac{\lambda}{2\sigma^2 + a\overline{\gamma}n}} - e^{\frac{\lambda}{2\sigma^2}} \Sigma_{i=0}^{mn-2} \frac{\left(\frac{\lambda a\overline{\gamma}n}{2\sigma^2(2\sigma^2 + a\overline{\gamma}n)}\right)^i}{i!}\right] \end{aligned} \tag{22}$$

whereas, $\overline{\gamma}$ denotes the average SNR under Rayleigh fading and $a = 2$.

Substituting Equation (22) into (21), we obtain the following equation:

$$Q_{d,Su} = \int_0^\infty Q_{d,Ray}(\overline{\gamma} = x) \frac{1}{x\hat{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \hat{\mu})^2}{2\hat{\sigma}^2}\right) dx \tag{23}$$

Let

$$u = \frac{\ln x - \hat{\mu}}{\sqrt{2}\hat{\sigma}} \rightarrow du = \frac{1}{\sqrt{2}\hat{\sigma}x} dx$$

The probability of detection can be rewritten as follows:

$$Q_{d,Su} = \frac{1}{\sqrt{\pi}} \int_0^\infty Q_{d,Ray}\left(\overline{\gamma} = e^{\sqrt{2}\hat{\sigma}u + \hat{\mu}}\right) \exp\left(-u^2\right) du \tag{24}$$

Using Gauss - Hermite integration, we can approximate:

$$Q_{d,Su} = \frac{1}{\sqrt{\pi}} \Sigma_{i=1}^{N_p} w_i Q_{d,Ray}\left(\overline{\gamma} = e^{\sqrt{2}\hat{\sigma}a_i + \hat{\mu}}\right) \tag{25}$$

Expression (25) can be considered as a closed-form of co-operative detection probability. This helps us improve the complexity of calculating and evaluating the performance of cooperative spectrum sensing using EGC-based Soft Decision under Suzuki fading.

IV. RESULTS AND DISCUSSION

Based on analysis in Section III, we used Matlab to evaluate the detection performance of cooperative spectrum sensing of a CR network using EGC-based Soft Decision as depicted in Fig. 2. In our study, we assumed that all sensing channels are under i.i.d Suzuki fading channels. Sensing information sent over reporting channels is assumed lossless.

Fig. 4 illustrates an example of detection performance of EGC-based Soft Decision under i.i.d Suzuki fading channels. In this case, we investigated the detection performance with varying number of cooperative CRs, n. We consider three different values of n equal to 4, 8 and 15. Parameters of lognormal components are set $\mu = 3$dB, $\sigma = 0$ dB. Since we consider i.i.d Suzuki channels, these parameters are the same for all channels. Black, blue and red lines are ROCs corresponding to $n = 4, 8, 15$. As shown in the figure, when

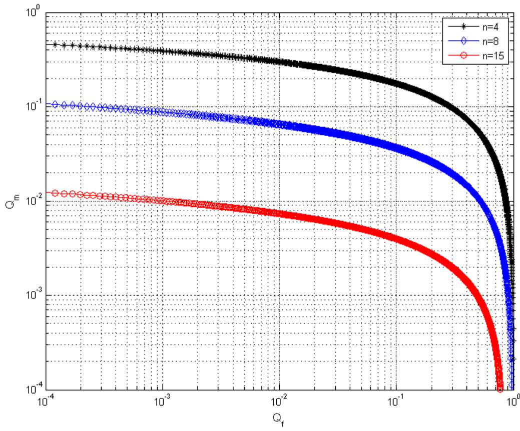

Fig. 4. Complementary receiver operating characteristics curves (ROCs) (Q_m vs Q_f) of cooperative spectrum sensing using EGC-based Soft Decision with various values of n.

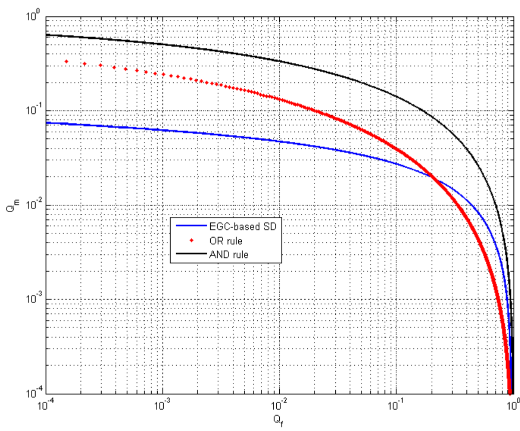

Fig. 5. Comparison between EGC-based Soft Decision and Hard Decision with $n = 6$.

n increases, the detection performance of CR network is improved significantly.

Furthermore, we performed a comparison between EGC-based Soft Decision and Hard Decision using k-out-of-n rule. In this evaluation, we choose $\mu = 1$ dB and $\sigma = 3$ dB and number of cooperative CRs $n = 6$. As shown in Fig. 5, blue, red and black lines are ROCs corresponding to EGC-based Soft Decision, OR rule and AND rule. Soft Decision give a much better performance than Hard Decision, in particular, for OR rule for small values of Q_f. When compared with AND rule, EGC-based Soft Decision is totally better.

V. Conclusion

In this paper, we have studied EGC-based Soft Decision for cooperative spectrum sensing over Suzuki fading channel in cognitive radio. We have presented the use of MGF matching to approximate power sum of n Suzuki RVs as a single Suzuki RV. Then, we proposed a closed-form of detection probability Q_d under Suzuki fading to evaluate detection performance of cooperative CR network. The results obtained have shown that using the Soft Decision method is better than using the Hard Decision method.

Acknowledgement

This work was supported by National Foundation for Science and Technology Development (NAFOSTED), Vietnam.

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