

# Solving the staff rescheduling problem in Lai Chau hydropower station

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**Abstract**—This article studies the complex staff rescheduling problem arising in the context of Lai Chau hydropower station. Currently, rescheduling task is implemented manually by agreement of switching shifts among workers or by a scheduler. It is a complex and time-consuming operation but obtained schedules have been far below the expectation of the staff as they in general violate working rules and contract agreements. In this research, we propose an approach based on Mixed Integer Programming to support the scheduler solving the problem. Our model includes multiple objectives which minimize the number of changed shifts and the number of affected employees. Experimental results show the effectiveness of our method.

## I. INTRODUCTION

The Lai Chau hydropower plant, located in Muong Te District in Lai Chau Province, is the third largest hydropower plant in Viet Nam. The station has begun operations since 2016. According to the Electricity of Vietnam (EVN), the Lai Chau hydropower plant has three turbines with a combined capacity of 1,200 MW and total output of 4.7 billion kWh [8]. In addition to supplying electricity to the national grid, during the plant operation process, the plant has also supplied electric power and water for the Red River Delta and has contributed to promote socio-economic development in Lai Chau and neighbouring provinces. Due to its importance in many sectors, the operation of the station must be supervised strictly 24 hours per day and 7 days per week.

In a world where demand for round-the-clock services is higher and higher leading to an increase of shift works in a wide variety of industries, a good staff schedule plays an important role to the success of an organization. Differences among industries, their goals and restrictions, often mean that specific models and methods must be developed for each of them [3].

In the context of Lai Chau hydropower station, the staff is divided into several groups. Each group contains a subset of employees; each has a specific position (skill) required in supervising shifts. The main mission of employees is to supervise the whole operation of the station according to their skills but they also have to take in charge of other tasks. A feasible staff schedule in the station has to satisfy a number of hard constraints imposed by work rules and quota requirements. The objective of the problem is to provide schedule so that the number of workers in a group doing a

supervising work is maximized. The staff scheduling problem in the station is solved in the end of each month to provide a schedule for the next month (See [4] for further information).

Staff rostering is another complex scheduling task that occurs quite frequently each month in Lai Chau hydropower plant. It arises whenever there is an unplanned schedule disruption such as a sudden absence of a sick worker. In simple cases, the station lets workers bargain with their colleagues for replacement before asking to be off. In other situations where the worker cannot find any replacement, a scheduler has to arrange the situation by manual efforts. Due to limited computational capacity, he cannot reconstruct the schedule in a larger scale on all employees. They are stressful and time-consuming operations because of the problem complexity but the results in general have not completely met the scheduling constraints leading to a dissatisfaction of the staff. In this research, we propose a method based on Mixed Integer Programming to help the scheduler solving the problem. Our model aims at providing new schedule with minimum changes, subject to reported absences while satisfying all constraints as in original schedule.

This article is organized as follows. In Section 2, we present a brief overview of related literature. The problem definition is described in details in Section 3. Solution methods are proposed in Section 4. In Section 5, we report our implemented experiments. And finally, we make conclusions in Section 6.

## II. LITERATURE REVIEW

Scheduling (or rostering) problem is one of the most studied problems in the field of operations research. It is defined as the allocation of human resources to shifts to not only meet the requirements about quota, work rules, workplace agreements,... but also obtain equity among workers and workers' satisfaction. The problem has become increasingly important as business becomes more service oriented and cost conscious [3]. Readers are recommended to [1], [2], [3], [5] for more details. Unlike the staff scheduling which provides a complete schedule for all employees in a whole planning horizon, staff rostering is the process of changing a planned schedule of some employees. It occurs when there are incident absences as a worker asks for being off in some working assigned days. The issue that almost organizations must face

up to in these situations is related to workforce shortage. The rescheduling process also affects to employees since unexpected changes in typical schedules may create personal inconveniences that may lead to frustration among workers [14]. Therefore, the objective of the staff rescheduling problem is in general minimizing changes to the original schedule while minimizing extra costs. As in typical schedule, new schedule is still expected to meet management goals, staffing requirements, individual worker agreements, work rules and workers' preferences, subject to reported absences.

Although the staff rescheduling problem has widely appeared in the variety of industries, the number of articles on the problem is quite limited. Most of researches in the literature considered the nurse rescheduling problem. Moz and Pato in a series of papers proposed various techniques for solving the problem arising in the context of a hospital in Lisbon. The first one [12] studied the problem where at least one nurse informs that she will be unable to perform the shifts assigned on one or more future working days. The problem was formulated as an integer multi-commodity flow model, with the objective of minimizing deviations from the original roster. A similar study was carried out in 2004 [13], taking into account the issue of overstaffing. In 2007, they propose a hybrid method of the multi-commodity model with genetic algorithm and a constructive heuristic [14]. In 2008, they improved their work by adding another objective aimed at minimizing overtime [15].

Heuristic and metaheuristic have been considered to be efficient approaches to solve the rostering problem. In 2010, Kitada et al. [7] modeled the nurse rostering problem using a recursive heuristic method with tree search. In 2011, Maenhout and Vanhoucke [9] proposed an evolutionary metaheuristic approach that combines principles from various meta-heuristics. In 2013, Maenhout et al. [10], [11] presented an Artificial Immune System and evolutionary-based approaches for the nurse rostering problem.

Recently, in 2017, Shih-Min Wu et al [16] considered the Multi-objective Nurse Rostering Problem, subject to a given set of hard and soft constraints. They introduced a formal framework of Multi-Objective Nurse Rostering Problem (MO-NRP) that aimed to find trade-off solutions among "optimality" and "stability", then defined a novel solution criterion called an egalitarian solution for a MO-NRP.

To the best of our knowledge, the staff rescheduling problem has never been studied in the context of a power station. This research proposes a method based on mixed integer programming to tackle the problem of Lai Chau hydroelectric plant in Vietnam. Besides specific constraints related to the electric sector and objectives of the original schedule, our model incorporates additional objectives which minimize the number of shift changes and the number of affected employees.

### III. PROBLEM DEFINITION

#### A. Works and workforce

Lai Chau hydropower station [8] has the labor force of 74 employees. A manager, a vice - manager and a scheduler are

not mentioned in the scheduling process. We just consider 71 remaining employees that are divided into 6 groups: five groups have 12 employees each and one group has 11 employees. Each worker takes a role in a supervising shift, which requires a workforce of 11 people in 8 positions as follows: one shift supervisor, one group leader, one central control operator, two spillway area and intake gate supervisors, one auxiliary type-1-equipment supervisor, one auxiliary type-2-equipment supervisor, one gas-insulated switchgear supervisor, and three machine hall supervisors. Moreover, workers are gathered by positions for replacement in urgent situations. A worker can only replace another of the same position in a supervising shift. There are 3 shifts per day: a day shift, an evening shift and a night shift. In addition to supervise the station operation, workers participate in carrying out some other works ( $T^*$ ), which are also considered to be a working day as supervising work.

Table I. Description of works implemented in the station.

Work	Frequency	Working time	# of required workers per shift
Supervising	Everyday	Three shifts	11 workers of 8 positions
Cleaning 1	All days except Sundays	Day shift	4 workers
Cleaning 2	At least 15 days per month	Day shift	4-5 workers
Administrative work	Everyday	Day shift	At least 2 workers
Emergency response exercise	One day per month	Day shift	All workers of a pre-selected group

#### B. Work rules

In order to ensure workers' health, it is not allowed to work two consecutive shifts, i.e. in 16 hours continuously. Moreover, workers need break time to recover, two consecutive working shifts should be as far as possible to each other (at least 8 hours). Specially, the supervising task is more stressful than other tasks so the minimum break time between 2 consecutive supervising shifts must be at least 16 hours. In addition, to avoid being tired after a long working-day sequence, a worker should not work many days in a row. Finally, supervising tasks carried out in three shifts (K1, K2, K3) are different from each other. As a result, the number of consecutive similar supervising shifts is also restricted as the worker might get accustomed to work in only a shift and forget tasks of other shifts.

#### C. Day-off and preference

The number of days off in a month of a worker is equal to the number of weekends (Saturday, Sunday) and national (or international) holidays (if any) in that month. Fixed days off of a worker express his preference. If some preferences cause shortage, the manager will decide who is allowed to be off. Once the worker's preference is accepted, the scheduler must satisfy his desired absences. . Moreover, the station allows

workers to have 6 consecutive days-off each month. Remaining days-off will be arranged to cut down long working sequences.

There are also employees who have no preference. For these people, the scheduler will decide their consecutive days-off sequences. It is worthy noting that the final days-off sequence of the previous month could be taken into account in this situation. For example, if the last 3 days of the previous month are days off of a worker, the scheduler can assign him to be off in the first 3 days of the current month to complete six-consecutive-days-off sequence.

Depending on the destination of the long vacation, a worker would be assigned a suitable shift right before his vacation so that he has time to prepare for the trip. More precisely, if an employee travels to Son La city - a local area, he should be given a priority to work in either day shift or evening shift to be able to take the coach which passes through Lai Chau at night. If an employee travels to Hanoi, he should be assigned a day shift to be able to take the train in the evening. Furthermore, it takes time for him to get back to work when finishing the vacation, his next-day schedule should be an evening shift or a night shift. This makes workers more comfortable and reduces the possibility of being late at work.

#### D. Staff rescheduling

As mentioned above, a planned schedule rarely takes place as expected. It is quite often that workers ask for sudden absences due to personal issues. A worker is allowed to be off urgently no more than 6 days. Currently, to deal with such situations, the station lets the absent worker find a suitable replacement from one of their colleagues who are in the same position. The substitute will swap the schedule with the absent employee. By this way, the scheduler do not have to reconstruct the schedule. However, it is difficult for the worker to find a feasible substitute in many cases, especially when the worker wants to be off in several days, which requires more than one colleague for replacement. Even if the substitute is willing to accept the absent worker's demand, he can break some work rules or have to come back to work on his day-off. As a consequence, many disruptions occur and the scheduler has to change the origin schedule using his experiences with manual efforts. This task makes him quite stressful since workers often ask for being off right before the day they want to be absent.

During the rescheduling process, the station wishes to keep the original schedule as much as possible to reduce disruptions. More specifically, reducing the number of affected employees and the number of changed shifts is preferred. The scheduler also wants to restrict the changes within a position, e.g. if a worker reports a sudden absence, the new schedule should affect his colleagues in the same position only.

### IV. SOLUTION METHOD

In this section, we propose a mixed integer program to mathematically model the problem from which we solve it with a MIP solver.

#### Mixed integer programming model

#### Sets

$N$	Set of workers (71 people)
$N_S$	Set of shift supervisors;
$N_G$	Set of group leaders;
$N_C$	Set of central control operators;
$N_M$	Set of machine hall supervisors;
$N_{SG}$	Set of spillway and intake gate supervisors;
$N_{E1}$	Set of auxiliary type-1 equipment operators;
$N_{E2}$	Set of auxiliary type-2 equipment operators;
$N_{GIS}$	Set of GIS supervisors;
$G$	Set of groups;
$D$	Set of days in the considering month
$D_a$	Set of days in the rescheduling period
$O$	Set of unexpected days-off
$J$	Set of works: Supervising ( $W$ ), Cleaning 1 ( $C_1$ ), Cleaning 2 ( $C_2$ ), Administrative ( $A$ ), Emergency exercise ( $T$ )
$S$	Set of 8-hour shifts in a day: day shift ( $K_1$ ), evening shift ( $K_2$ ), night shift ( $K_3$ ); $T^* = C_1 \cup C_2 \cup A \cup T$ set of works that must be executed on $K_1$ .
$B$	Set of workers in pre-selected group doing task $T$
$Sun$	Set of Sundays in the considering month

#### Parameters

$w$	Worker who wants to be off unexpectedly
$d$	The first day in days-off sequence
$n_{off}$	Total number of days-off in the considering month
$n_W$	Maximum length of same-supervising-shift sequence
$n_S$	Maximum length of working-day sequence
$p_{ikt}$	Binary coefficient representing day-off preferences of worker $i$ $p_{ikt} = 1$ if worker $i$ is absent in shift $k$ of day $t$ and 0 otherwise
$e$	Length of long vacation for a worker
$ga_{gkt}$	Binary parameter, equals to: 1 if group $g$ is assigned supervising job in shift $k$ of day $t$ , 0 otherwise;
$o_i$	Length of final day-off sequence of worker $i$ in previous month
$des_i$	Destination of vacation for worker $i$ $des_i = 1$ presents for Ha Noi while 2 presents for Son La
$m_T$	Emergency response exercise in typical schedule
$\alpha, \beta, \gamma$	Weighted coefficients for objective function
$a_{ijkt}$	Typical schedule of all workers: $a_{ijkt} = 1$ if worker $i$ is assigned task $j$ shift $k$ on day $t$ , 0 otherwise

#### Variables

$x_{ijkt}$	Binary variable, equals to: 1 if worker $i$ is assigned work $j$ shift $k$ on day $t$ 0 otherwise
$l_{it}$	Binary variable, equals to: 1 if day $t$ is the first day in $e$ -day-off sequence of worker $i$ 0 otherwise
$z_{1t}, z_{2t}$	Binary variable, equals to: 1 if cleaning 1,2 ( $C_1, C_2$ ) is done in day $t$ 0 otherwise
$z_{3t}$	Binary variable, equals to: 1 if emergency exercise day $T$ is done in day $t$ 0 otherwise;
$u_{gkt}$	Integer variables representing the number of workers in group $g$ that do supervising job together in shift $k$ day $t$ ;
$c_i$	Binary variable, equals to: 1 if $o_i$ final days-off of previous month are linked with $e - o_i$ first days-off of this month
$y_{ikt}$	Binary variable, equals to: 1 if shift $k$ of worker $i$ on day $t$ is changed and 0 otherwise;
$h_i$	Binary variable, equals to: 1 if worker $i$ is affected by new schedule and 0 otherwise;

#### Constraints

#### Accept for unexpected days-off

1) Worker  $w$  is absent as requested:

$$\sum_{j \in J} \sum_{k \in S} x_{wjkd} = 0 \quad \forall d \in O$$

2) All the typical schedules before day  $d$  are kept as origin:

$$x_{ijkd} = a_{ijkd} \quad \forall i \in N, \forall j \in J, \forall k \in S, \forall t \in D \setminus D_a$$

**Works and workforce:**

3) All works  $T^*$  must be performed in a day shift  $K_1$ :

$$\sum_{j \in J \setminus \{W\}} \sum_{k \in S \setminus \{K_1\}} \sum_{t \in D} x_{ijkd} = 0 \quad \forall i \in N$$

4) Each  $W$  shift must have exact number of workers at exact positions as following:

$$\sum_{i \in N_S} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_G} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_C} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_{E1}} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_{E2}} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_{SG}} x_{iWkt} = 2 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_M} x_{iWkt} = 3 \quad \forall k \in S, \forall t \in D_a$$

$$\sum_{i \in N_{GIS}} x_{iWkt} = 1 \quad \forall k \in S, \forall t \in D_a$$

5) There are at least 2 workers that do administrative works (A) a day:

$$\sum_{i \in N} x_{iAK_1t} \geq 2 \quad \forall t \in D$$

6) Cleaning 1 ( $C_1$ ) is performed everyday except sundays, 4 workers are required each time; ( $M$  is a large enough number):

$$\sum_{t \in D} z_{1t} = \|\mathbf{D}\| - \|\mathbf{Sun}\|$$

$$z_{1t} = 0 \quad \forall t \in Sun$$

$$z_{1t} * M \geq \sum_{i \in N} x_{iC_1K_1t} \quad \forall t \in D$$

$$\sum_{i \in N} x_{iC_1K_1t} = 4 * z_{1t} \quad \forall t \in D$$

7) The number of days that cleaning 2 ( $C_2$ ) is performed must be greater or equal to 15 and not greater than the number of working days of a worker per month, 4 - 5 workers are required each time; ( $M$  is a large enough number):

$$15 \leq \sum_{t \in D} z_{2t} \leq \|\mathbf{D}\| - n_{off}$$

$$z_{2t} * M \geq \sum_{i \in N} x_{iC_2K_1t} \quad \forall t \in D$$

$$\sum_{i \in N} x_{iC_2K_1t} - 5 \leq (1 - z_{2t}) * M \quad \forall t \in D$$

$$4 - \sum_{i \in N} x_{iC_2K_1t} \leq (1 - z_{2t}) * M \quad \forall t \in D$$

8) Reassign (if necessary) emergency response exercise day ( $T$ ) for set of workers  $B$  on day  $m_T$ ; ( $M$  is a large enough number):

$$\sum_{t \in D} z_{3t} = 1 \quad \forall t \in D_a : m_T \geq d$$

$$z_{3t} * M \geq \sum_{i \in B} x_{iTK_1t} \quad \forall t \in D_a : m_T \geq d$$

$$\sum_{i \in B} x_{iTK_1t} = z_{3t} * \|\mathbf{B}\| \quad \forall t \in D_a : m_T \geq d$$

$$\sum_{i \notin B} \sum_{t \in D} x_{iTK_1t} = 0$$

**Work rules:**

9) The length of working day sequence of a worker is not greater than  $n_S$ :

$$\sum_{t=b}^{b+n_S} \sum_{j \in J} \sum_{k \in S} x_{ijkd} \leq n_S, \quad \forall i \in N, b = 1, 2, \dots, \|\mathbf{D}\| - n_S$$

10) The length of same supervising shift sequence of a worker is not greater than  $n_W$ :

$$\sum_{t=b}^{b+n_W} x_{iWkt} \leq n_W, \quad \forall i \in N, \forall k \in S, b \in [1, \|\mathbf{D}\| - n_W]$$

11) The number of working days of any worker except  $w$  in a month is equal to the number of days subtracted to the number of days off in that month:

$$\sum_{j \in J} \sum_{k \in S} \sum_{t \in D} x_{ijkd} = \|\mathbf{D}\| - n_{off} \quad \forall i \in N$$

12) Minimum break time between 2 consecutive working shifts is 16 hours:

$$\forall i \in N, \forall t \in D$$

$$\sum_{k \in S} \sum_{j \in J} x_{ijkd} \leq 1$$

$$\sum_{j \in J} x_{ijK_2t} + \sum_{j \in J} x_{ijK_3t} + \sum_{j \in J} x_{ijK_1t+1} \leq 1$$

$$\sum_{j \in J} x_{ijK_3t} + \sum_{j \in J} x_{ijK_1t+1} + \sum_{j \in J} x_{ijK_2t+1} \leq 1$$

**Preference and day-off:**

13) Accepted workers' preferences must be ensured:

$$(1 - p_{ikt}) * M \geq \sum_{j \in J} x_{ijkt}, \forall i \in N, k \in S, t \in D$$

14) Allocate long vacations for workers who have no preference:

a) The whole  $e$  days is in the scheduling month:

$$\sum_{t=b}^{b+e-1} \sum_{j \in J} \sum_{k \in S} x_{ijkt} \leq (1 - l_{it}) * M, \\ \forall i \in N, b = 1, 2, \dots, \|\mathbf{D}\| - e - 1$$

$$\sum_{t=b}^{b+e-1} \sum_{j \in J} \sum_{k \in S} x_{ijkt} \geq 1 - l_{it}, \\ \forall i \in N, b = 1, 2, \dots, \|\mathbf{D}\| - e - 1$$

$$c_i = 0 \quad \forall i \in N : o_i \notin [1, e - 1]$$

b)  $o_i$  previous month days off of worker  $i$  is linked to  $e - o_i$  first days of the current scheduling month:

$$\sum_{t=1}^{e-o_i} \sum_{j \in J} \sum_{k \in S} x_{ijkt} \leq (1 - c_i) * M, \\ \forall i \in N : o_i \in [1, e - 1]$$

c) If a worker is off on first  $e - o_i$  days, he might not have a long vacation:

$$\sum_{t=1}^{\|\mathbf{D}\| - e + 1} l_{it} \leq (1 - c_i) * M, \forall i \in N : o_i \in [1, e - 1]$$

$$\sum_{t=1}^{\|\mathbf{D}\| - e + 1} l_{it} \geq 1 - c_i, \forall i \in N : o_i \in [1, e - 1]$$

15) Shift assignment before and after a worker's long vacation:

a) Should not assign shift  $K_1$  (or day-off) for workers after long vacation:

$$\sum_{j \in J} x_{ijK_1(e-o_i+1)} \leq (1 - c_i) * M, \\ \forall i \in N : o_i \in [1, e - 1]$$

$$\sum_{j \in J} \sum_{t=1}^{|D|-e} x_{ijK_1(t+e)} \leq (1 - l_{it}) * M, \\ \forall i \in N : o_i \notin [1, e - 1]$$

b) Should assign shift  $K_1$  (or day-off) for workers in Ha Noi and shift  $K_1$  or  $K_2$  (or day-off) for workers in Son La before their long vacation:

$$\sum_{t=2}^{|D|-e+1} \sum_{j \in J} \sum_{k=des_i+1}^3 x_{ijk(t-1)} \leq (1 - l_{it}) * M, \\ \forall i \in N : o_i \notin [1, e - 1]$$

**Variables' definition constraints:**

16) The following constraints define variables  $y_{ikt}$ :

$$|\sum_{j \in J} x_{ijkt} - \sum_{j \in J} a_{ijkt}| = y_{ikt}, \forall i \in N, t \in D_a, k \in S$$

17) The following constraints define variables  $h_i$ :

$$h_i \leq \sum_{j \in J} \sum_{k \in S} \sum_{t \in D_a} |x_{ijkt} - a_{ijkt}| \leq h_i * M, \forall i \in N$$

18) The following constraints define variables  $u_{gkt}$ :

$$\sum_{i \in G} x_{iWkt} \geq u_{gkt} * ga_{gkt}, \forall g \in G, \forall k \in S, \forall t \in D_a$$

**Objective function**

Maximize

$$\alpha * \sum_{g \in G} \sum_{k \in S} \sum_{t \in D} ga_{gkt} * u_{gkt} - \beta * \sum_{i \in N} \sum_{k \in S} \sum_{t \in D_a} y_{ikt} - \gamma * \sum_{i \in N} h_i$$

The objective function has three terms. The first relates to the original objective of the scheduling problem: maximizing the number of workers in the same group doing supervising task together. The second minimizes number of shifts which are changed in rescheduling process. The third specifies that the number of workers whose original schedules are changed is minimized.

**V. EXPERIMENTAL RESULTS**

This section reports and analyses computational results of our research. The experiments have been run on a personal computer with Intel(R) Core(TM) i5-2450M CPU @ 2.50GHz processor and 6.00 GB RAM. CPLEX 12.6.2 is used to solve the mixed integer programming model described above. The maximum running time for each test is set to 180 seconds.

We test our method on the real data (preferences, original schedule) that were collected at Lai Chau hydropower station. We randomly generate an unexpected days-off sequence for all the workers. The length of these sequences can be short (in range from 1 to 3 days) or long (4 to 6 days). The starting of day-off sequence is randomly selected in the beginning (from day 1 to day 10), the middle (from day 11 to day 20), or the end of the month (from day 21 to day 30). For each length and starting-date combination, five instances have been randomly generated. Due to the length restriction of the paper, we only report the results of May 2017. In total, we have  $5 \times 2 \times 3 \times 71 = 2130$  tests. But we eliminate ones in which the long

vacation in the original schedule contains the unexpected day-off sequence of a worker because we have nothing to do and the schedule remains unchanged in these tests. Finally, there are 1861 tests left. We believe that, by this way, our generated instances will cover every situation that can occur in practice.

Table II presents our results with all three coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  being set to 1, i.e. we consider three terms of the objective have the same priority. Columns "Feas sol" represent the percentage of tests in which we can find a feasible solution. Because the main objective of the rescheduling problem is minimizing disruptions, we only report the second and third terms (denoted T2 and T3 respectively) of the objective function. Gaps of the solutions obtained by CPLEX are represented in Columns "Gap".

As can be seen in Table II, the obtained solutions have relatively small gaps, less than 1 % on average. This shows the performance of our MIP model which can deal with real-world instances quickly. In found feasible solutions, no more than 14 shifts and 5 workers are averagedly affected by the rescheduling process. Other further observations can be derived. The instances seem to be more feasible if the unexpected day-off sequence is shorter. This is because the shorter sequence causes less affect on the current schedule. The similar phenomenon can also be observed if the rescheduling process takes place early in the considering month. This can be easy to explain. If the sudden event occurs lately in the month, there is less opportunity to find replacements and the problem tends to be more infeasible. Finally, during the solution process, we observe that the infeasibility often occurs in the case where a worker asks for an absence while his normal days-off requirement has already been fully satisfied.

Fig. 1 illustrates a found solution when worker s5 reports a sudden absence in three days 17, 18, and 19. Seven workers (s1, s2, s3, s4, s5, s6, c7) have to change their schedules and 12 shifts are affected (s2 in days 18, 19; s3 in days 18, 20, 24, 26; s4 in days 24, 25; s5 in days 18, 19, 26, 30).

Table II. Report with  $\alpha = \beta = \gamma = 1$  and the running time limit: 180s

Period	Day-off sequence											
	1-3 days						4-6 days					
	# of tests	Solved (%)	T2	T3	Time (s)	Gap (%)	# of tests	Solved (%)	T2	T3	Time (s)	Gap (%)
1-10	281	87.54	11.63	3.76	38.25	0.71	315	71.11	11.99	3.43	37.14	0.45
11-20	281	62.99	8.28	2.58	13.03	0.08	325	34.46	13.98	4.36	1.73	0
21-31	316	34.49	9.85	3	1.73	0	343	8.75	8.73	2.93	1.51	0

### VI. CONCLUSIONS

In this research, we study the rescheduling problem arising in Lai Chau hydropower station. We analyzed the process from which we formulate mathematically the problem. A method based on mixed integer programming is proposed to solve the problem. The computational results on real data show that our MIP model can provide very good solutions (if any exists) with relatively small gaps for all real instances in less than 180 seconds. Our method serves as a support tool for the scheduler dealing with the complex scheduling problem. It also helps the station with good reschedules which still satisfy

all the constraints while reduce the disarrangement as much as possible.



Fig. 1. Illustration of a solution.

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