Paper No.336/2021

STOCHASTIC MATCHED FIELD PROCESSING USING DIRECTED RIEMANNIAN DISTANCE

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***Abstract**: The Matched Field Processing (MFP) plays a crucial role in modern passive SONAR system since its effectiveness of underwater source localization. Their applications are but not limited to floating boat detecting, submarine detecting, fish finding as well as ocean environmental parameters determining. In the past, the stochastic matched field processing (SMFP) are derived on the basis of Riemannian distances (RDs) which were calculated using isometric mappings (IMs). In this paper, a new STMP is provided using directed RDs which were obtained by solving the geodesic equations directly instead of using IMs. In addition, we exploit the symmetric property of Riemannian manifold to solve the geodesic equation in the simple manner. The performance of the proposed STMP outperformed to that of standard algorithm at the expense of a little more of computation.*

***Keywords:** Matched Field Processing, Riemannian Distance, Geodesics, Christofell symbol*

DOI:

1. INTRODUCTION

The pioneers of Matched Field Processing (MFP) are A. Tolstoy and A. B. Baggeroer as in [1-4]. Whereas the book of A. Tolstoy [1] discussed many issues of the MFP such as popular processors, pitfalls, and the applications, the paper of A. B. Baggeroer [3] is in a form of overview document. The most valuable contribution of those authors is pointed out that MTP is an effective way of underwater source localization [1-4]. The main applications of MFP are floating ship detection, submarine detection in military section and fish finding in civilization. Besides, the trend of determining environmental parameters such as sound speed profile, bottom topography and array tilt is also developed.

Since the MFP is a model based approach, it required not only a suitable simulated model of underwater sound propagation but also a collected data from a sensor array which could be vertical, horizontal or even towed array. This difficulty overcame because sensor data were provided by SACLANTC center by the introduction in [5]. In order to increase its reliability and resolution some methods such as empirical mode decomposition, adaptive MFP, compressive MFP and especially stochastic MFP using Riemannian geometry have been introduced recently [6-9].

The fact that stochastic Matched Field Processing (SMTP) are derived on the basis of Riemannian distance (RDs) which were calculated using isometric mappings (IMs) [9-10].

If an IM does not exist, we must solve geodesic equations directly to find the geodesic distance. Normally, this task leads to solve a system of second-order differential equations. However, it is usually very difficult to solve the system of differential equations analytically [11].

Fortunately, if a manifold is symmetrical (for instance, in $\boldsymbol{R^3}$, there are two kinds of underwater sound propagation, namely, Spherical or Cylindrical spreading in which the former having two symmetrical coordinates ($\boldsymbol{\theta}$, $\boldsymbol{ø}$) and the letter having one symmetrical coordinate ($\boldsymbol{ø}$)) we can exploit the symmetric of the manifold to simplify our task. Here, we introduce "Killing vector" or conserved quantity which will be used to find the solution of geodesic equations.

Then a new Stochastic Matched Field Processing (SMFP) is defined using the directed geodesic distance in order to locate the true source position in a more practical manner.

In addition, not only close-form of SMFP is derived but also the SMFP is verified by simulations in which ocean variability is taken into account. The performance of the proposed SMFP is verified in simulations, the true source could be detected if 20 modeled field replicas and 10 replicas of SONAR array data were used. The performance of the proposed STMP outperformed to that of standard algorithm at the expense of a little more of computation.

The paper is organized as follows. Part 2 introduces the measurement of directed Riemannian distance. The stochastic Riemannian matched field processing is described in Part 3. Some simulations are given in Part 4. Finally, we conclude the paper in Part 5.

2. DIRECTED RIEMANNIAN DISTANCE

2.1 CSDM MATRIX MANIFOLD

An CSDM manifold ($\boldsymbol{M,g_m}$) is a manifold $\boldsymbol{M}$ which consists of CSDM matrices and is equipped with inner product (Riemannian metric) $\boldsymbol{g_m}$ on the tangent space $\boldsymbol{T_M(m)}$. Given the inner product $\boldsymbol{g_m}$ on $\boldsymbol{T_M(m)}$, each point m that varies smoothly from point to point in the sense that if $\boldsymbol{X}$ and $\boldsymbol{Y}$ are differentiable vector fields on $\boldsymbol{M}$, then $m \mapsto g_m(X|_m, Y|_m)$ is a smooth function.

2.2 THE SYMMETRIC OF HERMITIAN POSITIVE DEFINITE MATRICES

The fact that the data collected usually in form of CSDMs which are not random but Hermit and positive definite, form a manifold that each CSDM is a point on it.

Let ***P*** be the set of Hermitian ***nxn*** complex matrices. Let denote the subset of ***P*** of positive definite matrices and give it the subspace topology. This set carries the structure of a vector space ***M*** over ***C*** under usual addition or multiplication.

As is known, every finite dimensional vector space ***M*** over ***C*** is locally compact when equipped with the topology induced by any norm.

Therefore, $M = \{A \in P_1, \det A > 0\}$, by continuity of the determinant, it follows that *M* is open in the locally compact. If ***M*** is simply connected, compact type (positive definite or nonnegative sectional curvature), it can belong to a simply connected Riemannian symmetric space. The other simply connected Riemannian symmetric space can be Euclidean or non-compact types.

Now, $M = \{A \in P_1, \det A > 0\}$, ***M*** is said to be simply connected, compact and symmetric.

2.3 GEODESIC EQUATIONS

According to [11], geodesic equations are equivalent to the system of differential equations as follows

$$\begin{cases} \Gamma_{11}^1 (u')^2 + 2\Gamma_{12}^1 u'v' + \Gamma_{22}^1 (u')^2 + u'' = 0 \\ \Gamma_{11}^2 (u')^2 + 2\Gamma_{12}^2 u'v' + \Gamma_{22}^2 (u')^2 + u'' = 0 \end{cases} \tag{1}$$

where

Γ_{ij}^k are Christofell symbols and ***u***,

v are local coordinates.

2.4 SYMMETRIC AND KILLING VECTOR

Solving a system of second-order ordinary differential equations can be easy for simple metrics, but quickly become very difficult for more interesting cases. Here we exploit the symmetric of a manifold to simplify our tasks.

The simplest symmetries can be found by observing if the metric is independent of any of its coordinates. We can define a vector field for each symmetry such that, at every point, a vector points along the direction in which the metric does not change due to that symmetric. This is called a "Killing vector", after the German mathematician Wilherm Killing.

For example, if we have a metric independent of $\boldsymbol{x^1}$, the killing vector of the manifold in $\boldsymbol{R^3}$ associated with that symmetry is

$$\xi^\alpha = (1,0,0) \tag{2}$$

The Riemannian distance between two point (m_a, m_b) is given by

$$L = \sqrt{p_{ij} x^i x^j} \tag{3}$$

where

p_{ij} is the Riemannian metric of the surface.

So, the Euler-Lagrange equation become

$$\frac{d}{d\sigma}\left(\frac{\partial L}{\partial (dx^1 / d\sigma)}\right) = 0 \tag{4}$$

This means that the quantity inside the derivative is constant along the geodesic.

Now,

$$\begin{aligned} \frac{\partial L}{\partial (dx^1 / d\sigma)} &= -p_{1\beta} \frac{1}{L}\frac{dx^\beta}{d\sigma} \\ &= -p_{\alpha\beta}\xi^\alpha u^\beta = -\boldsymbol{\xi}.\mathbf{u} = C^{te} \end{aligned} \tag{5}$$

where

ξ^α is a killing vector and

u^β is a velocity.

Thus $\boldsymbol{\xi}.\mathbf{u}$ is a conserved quantity. We can exploit this to solve geodesic equations.

2.5 GEODESIC EQUATION OF SPHERICAL SPREADING OF UNDERWATER SOUND WAVE

As is known, there are two kinds of underwater sound propagation, namely, Spherical and Cylindrical spreading. In this paper, the former is used. Therefore, let us introduce the methodology of computing the geodesic equation by calculating the geodesic on the surface of the 3D sphere.

The radius of a 3D sphere is a constant $\boldsymbol{R=C^{te}}$, therefore the Spherical coordinates are $(x^1, x^2) = (\theta, \phi)$, the Christofell symbols are

$$\Gamma_{ij}^1 = \begin{bmatrix} 0 & 0 \\ 0 & -\sin\theta\cos\theta \end{bmatrix}, \Gamma_{ij}^2 = \begin{bmatrix} 0 & \cot g\theta \\ \cot g\theta & 0 \end{bmatrix} \tag{6}$$

The geodesic equations are obtained from (6)

$$\begin{cases} \theta'' - \sin\theta.\cos\theta.(\phi')^2 = 0 \\ \phi'' + 2\cot g\theta.\theta'.\phi' = 0 \end{cases} \tag{7}$$

This couple of second-order ordinary differential equation is called geodesic equations.

The Riemannian distance between two points on the surface of 3D sphere can be written as

$$L = R\sqrt{(d\theta)^2 + (d\phi)^2} \tag{8}$$

The first derivative of (8) give us the velocity, ***u***, as follows

$$\mathbf{u} = \frac{\partial L}{\partial s} = (R^2 \frac{d\theta}{ds}, R^2 \frac{d\phi}{ds}) \tag{9}$$

If we divide both side of (8) by ds we obtain

$$1 = R^2 (\frac{d\theta}{ds})^2 + R^2 (\frac{d\phi}{ds})^2 = \mathbf{u}.\mathbf{u} \tag{10}$$

Since the metric is independent of Ø, we can choose the Killing vector as $\boldsymbol{\xi} = (0,1)$.

Therefore, the conserved quantity is

$$\boldsymbol{\xi}.\mathbf{u} = R^2 \frac{d\phi}{ds} = b = C^{te} \tag{11}$$

From (10),(11), we deduce

$$\begin{aligned} \phi' &= \frac{b}{R^2} \\ \theta' &= \frac{\sqrt{(R^2 - b^2)}}{R^2} \end{aligned} \tag{12}$$

If we set ***R=1***, $\phi' = b = C^{te}, \theta' = \sqrt{1-b^2}$.

Now, the parameterization of the geodesic on the surface of 3D sphere can be written as

$$(x^1, x^2, x^3) = (R = 1, \theta = \cos t, \phi = \sin t) \tag{13}$$

It is exactly the equation of Great circle. Generally, the Great circle equation in Cartesian coordinates are

$$\begin{cases} x = R\cos(\theta)\cos[\cos^{-1}(\cot g\theta)] \\ y = R\sin(\theta)\cos[\cos^{-1}(\cot g\theta)] \\ z = R\cos\theta \end{cases} \quad (14)$$

where
R is radius of the sphere.

3. STOCHASTIC MATCHED FIELD PROCESSING

An acoustic pressure field on a vertical array of N sensors with locations $p_a = (r_a, z_a), a = \overline{1,N}$ and from the true source coordinate is $p_s = (r_s, z_s)$ given by

$$F_{p_s}(p_s, p_a) = S.G(p_s, p_a) + W(p_a) \quad (15)$$

where
S is a spectral component of the source,
G is Green function which is calculated by Normal mode model and
W represents uncorrelated additive ambient noise.

The cross-spectral density matrix is written as

$$\bar{R}_{p_s} = \sum_{m=1}^{M} \left[F_{p_s}\right]_m \left[F_{p_s}\right]_m^H \quad (16)$$

Normalization of CSDM using Frobenius norm, we have

$$R_{p_s} = \frac{\bar{R}_{p_s}}{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{M} \left|(\bar{R}_{p_s})_{mn}\right|^2}} \quad (17)$$

The Frobenius norm define that $\|A\|_F^2 = \sum_{ij} a_{ij} = tr(AA^H)$

where
A_{ij} is element of matrix
A and H is the transpose conjugate [12]. The corresponding normalization of CSDM of modeled field replica from estimated source coordinate $p = (\hat{r}, \hat{z})$ denoted by R_p.

The matched field processor based on Riemannian Geometry is received by obtaining the space coordinates of modeled field replicas which are scanning over all modeled field replicas position $\hat{p} = (\hat{r}, \hat{z})$ with a subject constraint of minimization of specific Riemannian distance.

According to (14) a new stochastic matched field processors which are based on directed Riemannian distance is defined as follows

First step:

Without loss the generality, the Riemannian matched field processor based on Riemannian geometry is received by obtaining the space coordinates of data replicas which are scanning over all modeled field replicas position $\hat{\mathbf{p}} = (\hat{r}, \hat{z})$ with a subject constraint of minimization of Riemannian distance as follows

$$(\hat{r}, \hat{z}) = \arg\min_{\mathbf{p}} \sqrt{tr(\mathbf{R}_{\mathbf{p}_s}) + tr(\mathbf{R}_{\mathbf{p}}) - 2\,\mathrm{tr}(\mathbf{R}_{\mathbf{p}_s}\mathbf{R}_{\mathbf{p}})} \quad (18)$$

Second step:

Now, on the basis of the outcome of directed Riemannian distance (Part 2), we found that the geodesic distance of Spherical spreading is preferred to Great circle distance. This mean that

$$\begin{aligned} & d_{\min} = \min(d_1, d_{Great\ Circle}) \\ & d_1 = \sqrt{tr(\mathbf{R}_{\mathbf{p}_s}) + tr(\mathbf{R}_{\hat{\mathbf{p}}_s}) - 2tr(\mathbf{R}_{\mathbf{p}_s}\mathbf{R}_{\hat{\mathbf{p}}_s})} \\ & d_{Great\ Circle} = R\sqrt{(f(\theta) - f(\hat{\theta}))^2 + (\cos\theta - \cos\hat{\theta})^2} \\ & f(\theta) = \sin\theta.\sin[\cos^{-1}(\cot g\theta)] \\ & (\hat{r}_s, \hat{z}_s) = \arg\min_{\hat{\mathbf{p}}}(d_1, d_{Great\ Circle}) \end{aligned} \quad (19)$$

where
R radius of sphere
θ parameterization of modeled data
$\hat{\theta}$ parameterization of measurement data

In another way, the diagram of the proposed SMFP which is distinguished to other SMFP is shown in Fig. 1 as follows

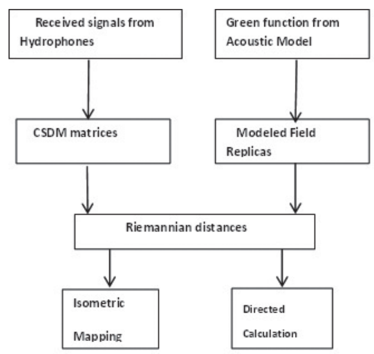


Fig. 1: Classification of Stochastic Matched Field Processes

4. SIMULATIONS

4.1 ACOUSTIC MODEL

The acoustic model in this paper using Normal mode model, in this case the acoustic pressure from [13] is given by

$$F(r,z) = \frac{i}{\rho(z_s)\sqrt{8\pi r}} e^{-i\frac{\pi}{4}} \sum_{m=1}^{M} \psi_m(z_s)\psi_m(z) \frac{e^{ik_m r}}{\sqrt{k_m}} \quad (20)$$

where
r is range,
z is depth,
z_s is the depth of the source,
ρ is sea water density,
ψ_m is amplitude of m^{th} mode, and
k_m is m^{th} eigenvalue (wavenumber).

4.2 INPUT ACOUSTIC DATA

Passive array data SONAR from SACLANTC1993 North Elba experiment available in Internet was used for processing [14]. The vertical underwater acoustic array data was collected in shallow-water off the Italia west coast by the NATO SACLANT Center in La Spezia, Italy. The original SACLANT time series has been converted to a series of MATLAB .mat files each of which contains a matrix "dat" that is 48 sensors by 64K data points long. Each file represents about 1 minute of data. The vertical array consists of 48 hydrophones with spacing 2 m between elements at total aperture length 94 m (18.7 m to 112.7 m in depth). The source emitted PRN signal with center frequency of 170 Hz.

The Sound Speed Profile (SSP) from [14] is described in Fig. 2.

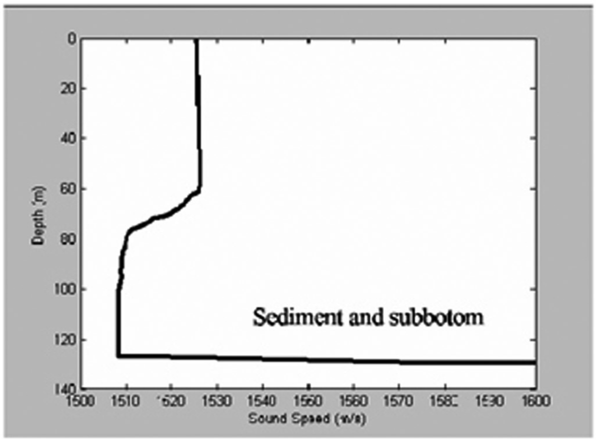


Fig. 2: SSP of SACLANTC 1993 North Elba

4.3 SIMULATION RESULTS OF GREAT CIRCLE ON THE SURFACE OF A SPHERE

We simulate a 3D sphere and its geodesic. The great circle between two fix points ($\boldsymbol{m}_a$, $\boldsymbol{m}_b$) on the surface of the 3D sphere (red line) is shown Fig. 3 as follow

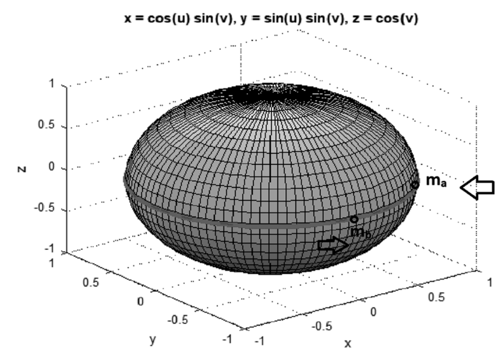


Fig. 3: The great circle (red line) between two fix points on the surface of the 3D sphere.

4.4 SIMULATION RESULTS OF STOCHASTIC MATCHED FIELD PROCESSING

Fig. 4 is obtained from conventional MFP in which only one modeled field and one replica of SONAR array data were used. It should be noted that the data is from SACLANTC and SNR level is -3 dB and the number of snapshot is greater than 20 samples It can be seen that the true source can be detected at depth of 60 m and range of 6000 m.

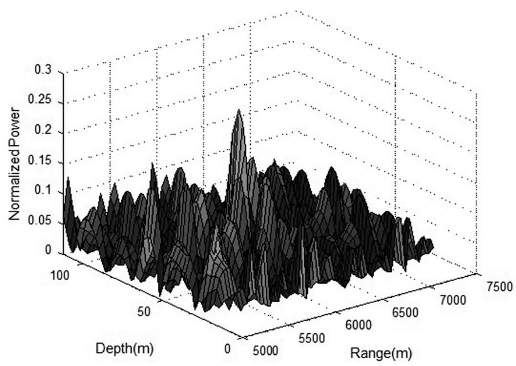
To illustrate the efficient of the methodology (part 3), we take some simulations. Here the ocean environment is considered as an uncertain environment. So twenty modeled field replicas are obtained from variable sound speeds that changed to depth according to SSP as depicted in Fig. 2. Each simulation uses 10 replicas of SONAR array data. Fig. 5 are the results of the proposed stochastic MFP.

The similarity between the conventional MTP and the proposed stochastic MFP is both method could detect exactly the location of the underwater source. However, the proposed stochastic MFP could work in an uncertain ocean environment where there are a lot of modeled field replicas as well as replicas of SONAR array data whereas the conventional could not. It means that, the conventional MFP is very sensitive to an uncertain ocean environment, if there is a small changing of the replicas, one could not detect the true underwater source precisely.

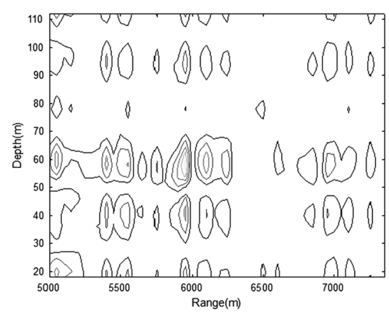
The difference between the two methods is shown by comparing the Fig. 4 and Fig. 5, the performance of conventional MFP is worse than that of the proposed stochastic MFP since in conventional MFP beside the true source location there are some other lower peaks whereas in the proposed MFP almost the lower peaks are suppressed.

The proposed stochastic MFP can be applied to the arbitrary Riemannian manifold, not necessary in R3 as in other Riemannian MFPs. It is because in those Riemannian MFPs the isometric mapping existences are required. The complexity of the proposed stochastic MFP is a little bit more than that other Riemannian MFP since it required the second step in part 3. However, almost SONAR systems now a day are supported by powerful microprocessors, so the speed of computation of the second step is only in few seconds.

The fact that, MFP is a model based approach so it depends on the underwater sound mechanism. In this paper, the spherical spreading is assumed. In future, we may justify the other type of sound spreading, namely cylindrical spreading and if possible, we may implement the proposed STMP into a real SONAR system.

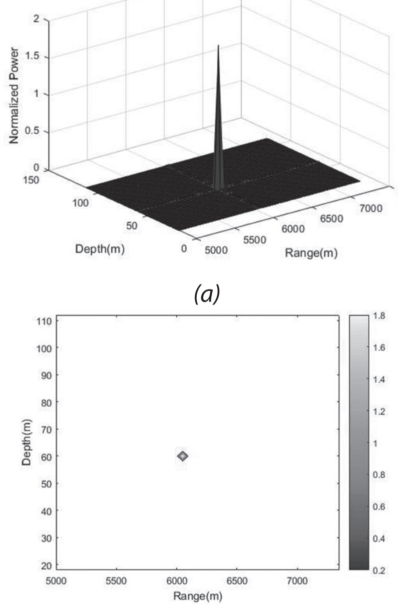


(a)



(b)

Fig. 4: Ambiguity surface of conventional matched field processing (one modeled field and one data replica, SNR=-3dB, No of snapshot>20 samples): 4a) in 3 dimensions 4b) in 2 dimensions.



(a)

(b)

Fig. 5: Riemannian ambiguity surface for 20 modeled field replicas and 10 data replicas, SNR=-3dB, No of snapshot>20 samples: 5a) in 3 dimensions 5b) in 2 dimensions.

5. CONCLUSION

In this paper, we introduce a close form of new SMFP in which the directed Riemannian distance is calculated. With the assumption of Spherical spreading of underwater sound propagation, we found that the geodesic path in the fashion of Great circle. The performance of the proposed SMFP is verified in simulations, the true source could be detected if 20 modeled field replicas and 10 replicas of SONAR array data were used. The performance of the proposed STMP outperformed to that of standard algorithm at the expense of a little more of computation. In future, one may justify the other type of sound spreading, namely cylindrical spreading. The main applications of the proposed algorithm are floating ship localization, submarine localization in military section and fish finding in civilization. Besides, the trend of determining environmental parameters such as sound speed profile, bottom topography and array tilt is also developed. A SONAR system which is embedded the proposed algorithm is suggested for the ships of Czech Republic Navy or cargo ships in commercial use.

ACKNOWLEDGEMENT

We would like to thanks SACLANTC for providing access of SONAR array data.

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