DOA ESTIMATION SYSTEMS IN CORRELATED ENVIRONMENT: ASYM-AWPC-CS AND ASYM-AWPC-SML

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1. Introduction

The need for Direction Of Arrival (DOA) estimation arises in many engineering applications including wireless communications, radar, radio astronomy, and sonar, navigation, tracking of various objects, rescue and other emergency assistance devices.

Asymmetric-Antenna without Phase Center (Asym-AWPC) has been studied to resolve ambiguity and restricted source number problems in Direction Of Arrival (DOA) estimation system and combined with Compressive Sensing (CS) to work well in correlated environment. The big drawback of the system is complex computation, compared to some traditional algorithms such as MUltiple SIgnal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT). Besides, Stochastic Maximum Likelihood (SML) is also a well-known algorithm in multipath environment with characteristics of consistency and statistical efficiency.

This project focuses on comparing performance of Asym-AWPC-CS and that of Asym-AWPC-SML in terms of estimation error and computation time.

2. The Asym-AWPC Structure

The Asym-AWPC illustrated in Fig. 1 consists of 4 dipoles A, B, C, D away a distance of d_1, d_2, d_3, and d_4, respectively from the origin. The structure is asymmetric in the sense that $d_1 \neq d_2$ or $d_3 \neq d_4$.

To gain some characteristics: no whole-space ambiguity, compact and scalar array, the Asym-AWPC is optimized with:

$$(d_1, d_2, d_3, d_4) = (\lambda/4, \lambda/4, \sqrt{3})\,\lambda/4, (\sqrt{3}/4 + 0.6\lambda).$$

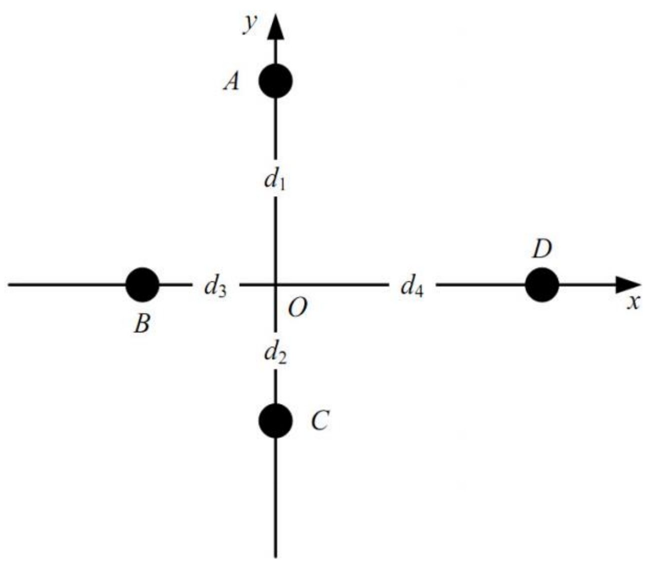

Fig.1: The Asym-AWPC structure

The amplitude pattern, $G(\theta)$, and the phase pattern, $\Phi(\theta)$, of the Asym-AWPC are given by:

$$G(\theta) = \sqrt{\Re^2\{\beta(\theta)\} + \Im^2\{\beta(\theta)\}} \quad (1)$$

$$\Phi(\theta) = \angle\beta(\theta) \quad (2)$$

where $\angle$ denotes the phase of a complex number, and

$$\Re\{\beta(\theta)\} = -2\sin\left[\frac{k}{2}(d_1 + d_2)sin\theta\right]\sin\left[\frac{k}{2}(d_1 - d_2)sin\theta\right] - 2\sin\left[\frac{k}{2}(d_3 + d_4)cos\theta\right]\cos\left[\frac{k}{2}(d_3 - d_4)cos\theta\right] \quad (3)$$

$$\Im\{\beta(\theta)\} = -2\sin\left[\frac{k}{2}(d_1 + d_2)sin\theta\right]\cos\left[\frac{k}{2}(d_1 - d_2)sin\theta\right] - 2\sin\left[\frac{k}{2}(d_3 + d_4)cos\theta\right]\sin\left[\frac{k}{2}(d_3 - d_4)cos\theta\right] \quad (4)$$

$$\beta(\theta) = e^{j\psi_1}e^{-jkd_1 sin\theta} + e^{j\psi_2}e^{-jkd_2 sin\theta} + e^{j\psi_3}e^{-jkd_3 cos\theta} + e^{j\psi_4}e^{-jkd_4 cos\theta} \quad (5)$$

with $\psi_1 = 0^0,\ \psi_2 = 180^0,\ \psi_3 = 90^0,\ and\ \psi_4 = 270^0$ are respectively phases of the currents of dipoles A, B, C, and D.

3. The Asym-AWPC associated with CS

CS is based on solving linear programming from limited measurements and sparsity of root.

The data model:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta}_s)\mathbf{z}(t) + \mathbf{n}(t) \quad (6)$$

The spatial space of CS is given by:

$$P_{CS}(\theta_i) = \frac{1}{K}\sum_{k=1}^{K}\hat{z}_{\theta_i}(k) \quad (7)$$

To solve (7), use l_1-regularized least squares due to its accuracy that solve the optimization problem below:

$$\min\|\mathbf{Az} - \mathbf{x}\|_2^2 + \lambda\|\mathbf{z}\|_1 \quad (8)$$

where $\|\mathbf{z}\|_1 = \sum_{i=1}^{D_s}|z_i|$, $\mathbf{A} \in \mathcal{R}^{M\times D_s}$ is data matrix, $\mathbf{z} \in \mathcal{R}^{D_s}$ is vector of unknowns, $\mathbf{x} \in \mathcal{R}^{M}$ is vector of observations, $\lambda > 0$ is regularization parameter.

4. The Asym-AWPC associated with SML

SML has characteristics: consistency and statistical efficiency.

The data model:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})s(t) + \mathbf{n}(t) \quad (9)$$

Denotes: K is the number of snapshot.

$\mathbf{R_n} = \sigma^2\mathbf{I}$ is noise covariance matrix.

$\hat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k)$ is signal covariance matrix

The SML function can be written as:

$$\boldsymbol{\theta}_{\mathrm{SML}} = arg\left\{\min_{\boldsymbol{\theta}} \log\left|\mathbf{A}(\boldsymbol{\theta})\hat{\mathbf{P}}_{\mathrm{SML}}\mathbf{A}^H(\boldsymbol{\theta}) + \hat{\sigma}^2_{\mathrm{SML}}(\boldsymbol{\theta})\mathbf{I}\right|\right\} \quad (10)$$

where:

$$\hat{\mathbf{P}}_{\mathrm{SML}}(\boldsymbol{\theta}) = \mathbf{A}^{\dagger}(\boldsymbol{\theta})(\hat{\mathbf{R}} - \hat{\sigma}^2_{\mathrm{SML}}(\boldsymbol{\theta})\mathbf{I})\mathbf{A}^{\dagger H}(\boldsymbol{\theta}) \quad (11)$$

$$\hat{\sigma}^2_{\mathrm{SML}}(\boldsymbol{\theta}) = \frac{1}{M-D}trace\{\mathbf{I} - \mathbf{A}(\boldsymbol{\theta})\mathbf{A}^{\dagger}(\boldsymbol{\theta})\hat{\mathbf{R}}\} \quad (12)$$

$$\mathbf{A}^{\dagger}(\boldsymbol{\theta}) = \left(\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta})\right)^{-1}\mathbf{A}^H(\boldsymbol{\theta}) \quad (13)$$

with $\mathbf{A}(\boldsymbol{\theta})$ is the steering matrix

5. Simulation Results and Analysis

Scenario: single snapshot, SNR = 20 dB, correlated environment.
Software: MATLAB

With this scenario, the two algorithms can estimate exactly the DOA angle with high resolution as illustrated in Fig. 2 and Fig. 3.

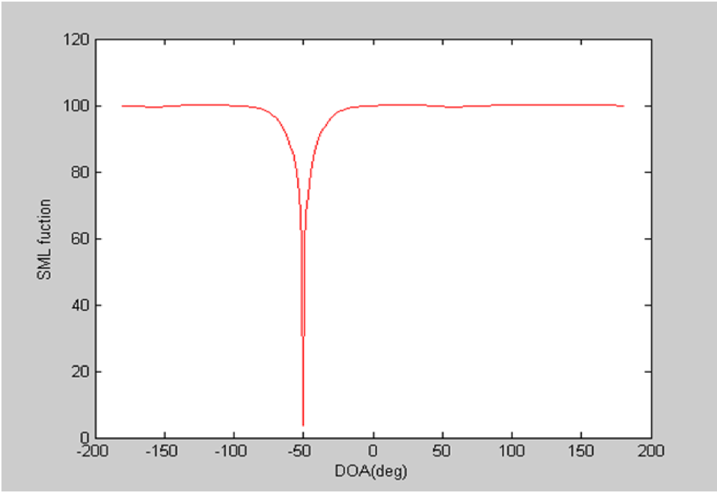

Fig. 2: Asym-AWPC-SML at -50º

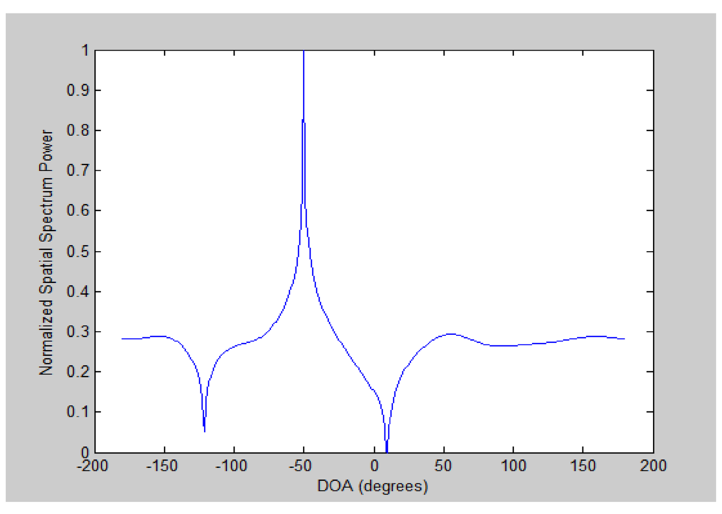

Fig. 3: Asym-AWPC-CS at -50º

Again, the two methods can estimate accurately the DOA angles with high resolution.

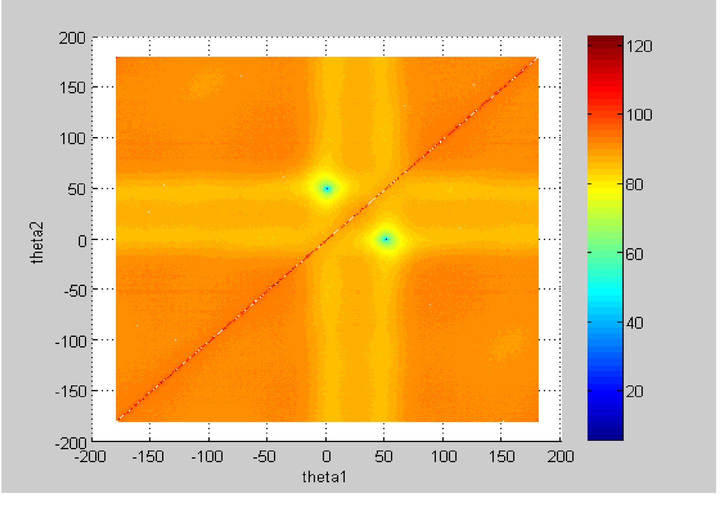

Fig.4: Asym-AWPC-SML at (0º, 50º)

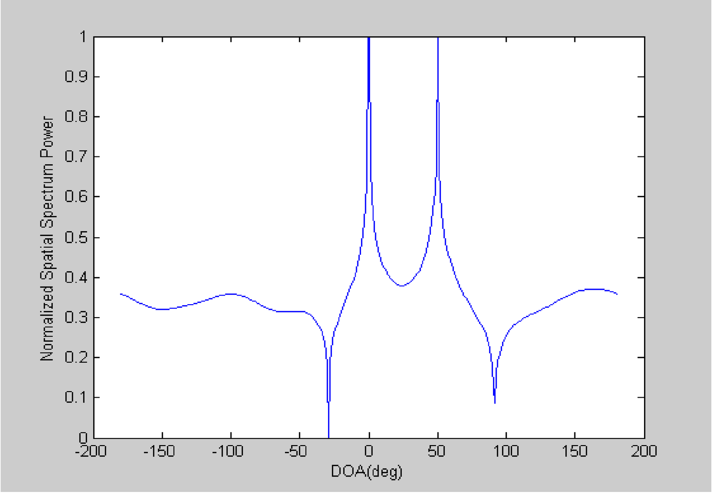

Fig.5: Asym-AWPC-CS at (0º, 50º)

In case there exist 3 targets, as shown in Fig.6 and Fig. 7, the output of both methods are accurate, but SML cannot give a visualized figure result as CS does.

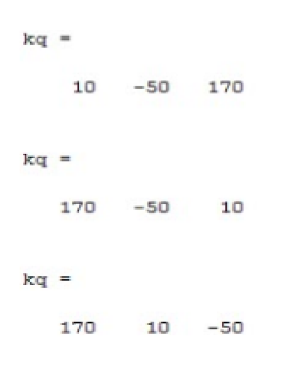

Fig.6: Asym-AWPC-SML at (-50º, 10º, 170º)

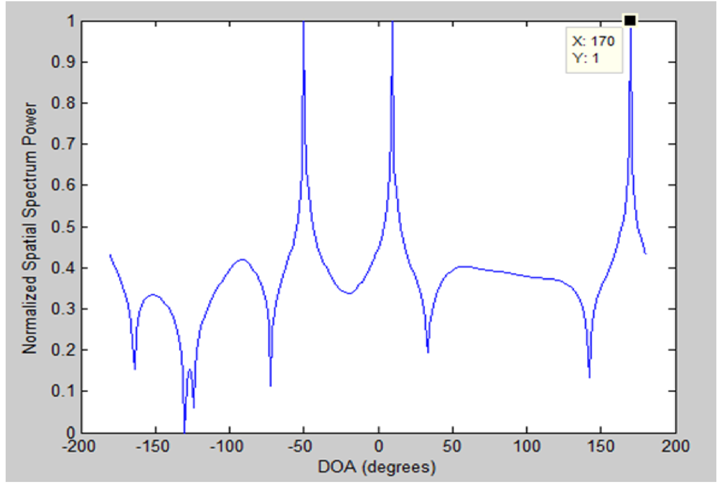

Fig.7: Asym-AWPC-CS at (-50º, 10º, 170º)

Take 50 trials for each SNR value in 2 cases: Single snapshot and 10 snapshots. RMSE is calculated as:

$$RMSE = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(\theta_i - \hat{\theta}_i)^2} \quad (14)$$

where θ_i is true angles and $\hat{\theta}_i$ is estimate angles.

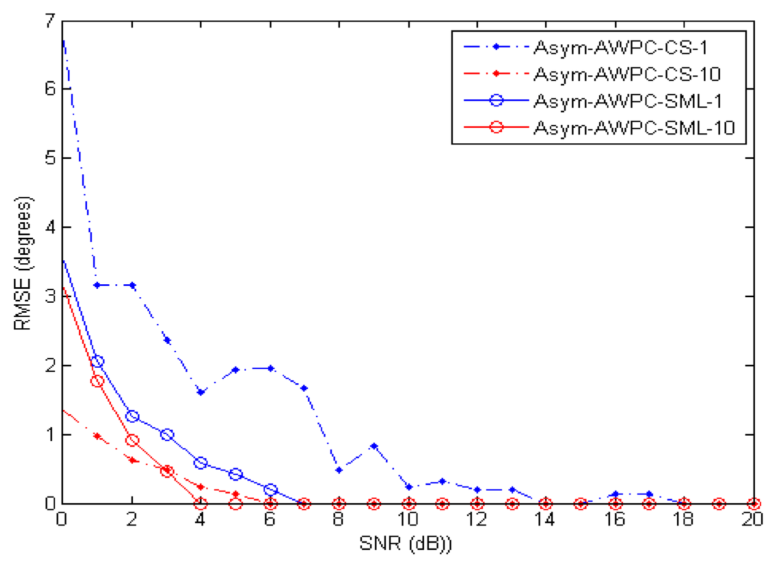

Fig.8: RMSE in degrees at (-50º)

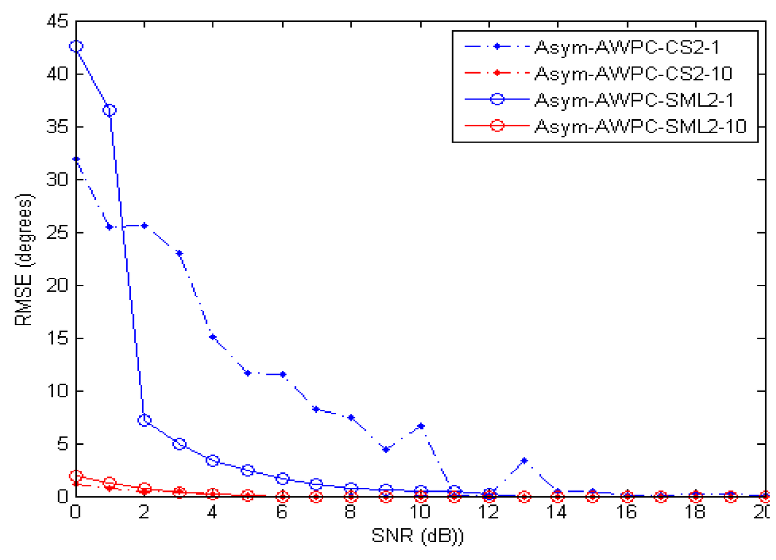

Fig.9: RMSE in degrees at (-30º, 50º)

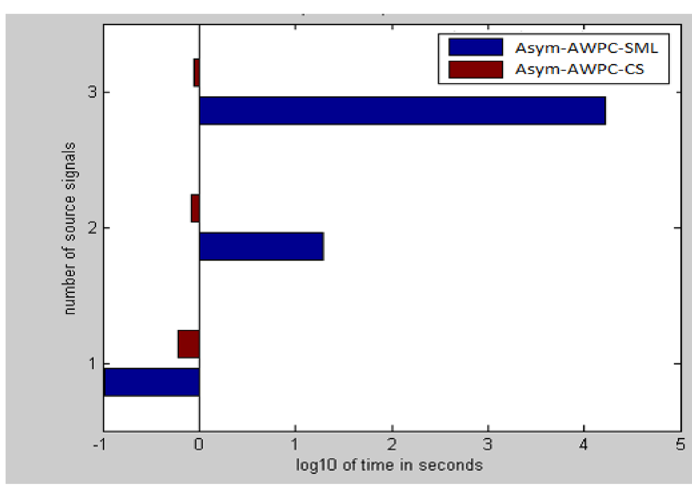

Fig.10: Comparison of computing time

In general, Asym-AWPC-SML performs better, with lower RMSE in both cases. However, Asym-AWPC-CS takes much shorter computing time than Asym-AWPC-SML

6. Conclusions

In this project, we have compared performance of CS with that of SML, in terms of RMSE and computing time, in correlated environment. The simulation results show that CS is better choice for balancing time cost and accuracy.

Hanoi, April 2016