Nonlinear dynamic and stability analysis of ES-FGM plates on elastic foundation

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1. INTRODUCTION

Since its first introduction in 1984 by a group of material scientists in Japan, **functionally graded materials (FGMs)** have attracted considerable attention in many engineering applications namely extremely high temperature resistant materials. Eccentrically stiffened FGM (ES-FGM) was firstly studied in 2011 [1]. After that, ES-FGM was attracted and studied more by many authors; however, their studies mainly used classical theory and did not study the effects of temperature on stiffeners .

Solution:

- using the first order shear deformation plate theory and stress function
- With both FGM plates and stiffeners having temperature – dependent properties.

objective: This paper presents an analytical approach to investigate the nonlinear dynamic response and stability analysis of centrically stiffened FGM plates on elastic foundation using both of the first order shear deformation plate theory and stress function. The FGM plates are assumed to rest on elastic foundation and subjected to mechanical, thermal loads in thermal environment.

2. ES-FGM plates

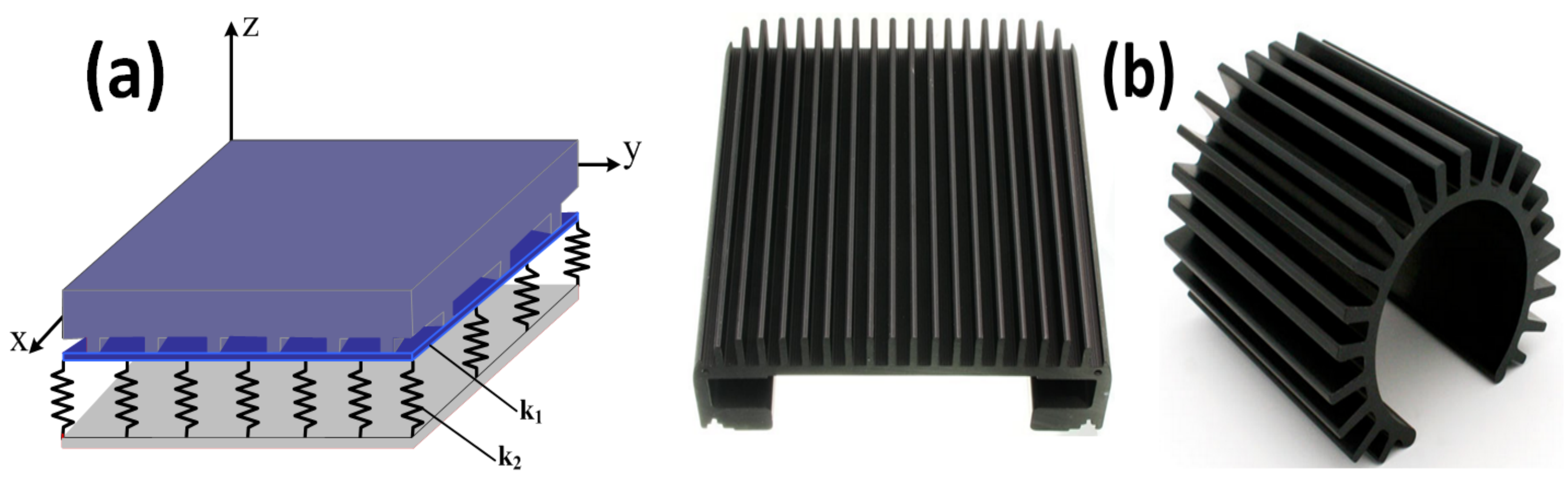

Figure 1 (a) Geometry and coordinate system of an eccentrically stiffened FGM plates on elastic foundation. (b)Different heat sinks made of stiffened ceramic FGM materials

Consider a ceramic–metal eccentrically stiffened moderately FGM plate of length a, width b and thickness h resting on an elastic foundation, a coordinate system (x, y, z) is established, in which (x, y) plane is on the middle surface of the plate and z is the thickness direction $-h/2 \le z \le h/2$ as shown in Fig. 1.

The Young's modulus and thermal expansion coefficient can be expressed in the form:

$$\begin{bmatrix} E(z) \\ \alpha(z) \end{bmatrix} = \begin{bmatrix} E_m \\ \alpha_m \end{bmatrix} + \begin{bmatrix} E_{cm} \\ \alpha_{cm} \end{bmatrix} \times \left(\frac{2z+h}{2h}\right)^N ,$$

3. THEORETICAL RESULTS AND DISCUSSION

The nonlinear motion equations of plate based on the first order shear deformation plate theory

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \frac{\partial^2 v}{\partial t^2} + I_1 \frac{\partial^2 \phi_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} - K_1 w + K_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + q = I_0 \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v}{\partial t^2}$$

The force and moment of the ES-FGM plate

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ z\sigma_{xx} \\ z\sigma_{yy} \\ z\tau_{xy} \end{Bmatrix} dz + \begin{Bmatrix} \frac{1}{l_x}\int_{A_x} \sigma^s_{xx} dA_x \\ \frac{1}{l_y}\int_{A_y} \sigma^s_{yy} dA_y \\ 0 \\ \frac{1}{l_x}\int_{A_x} z\sigma^s_{xx} dA_x \\ \frac{1}{l_y}\int_{A_y} z\sigma^s_{yy} dA_y \\ 0 \end{Bmatrix}$$

It is apparent that T-D material properties make the FGM plate considerably weaker under thermal load.

It is clear that the stiffeners can enhance the thermal loading capacity for the imperfect and perfect moderately FGM plates

It can be seen that the amplitude of the nonlinear dynamic response of eccentrically stiffened FGM plate increases when increasing the power law index N.

Nonlinear analysis of stability [2]

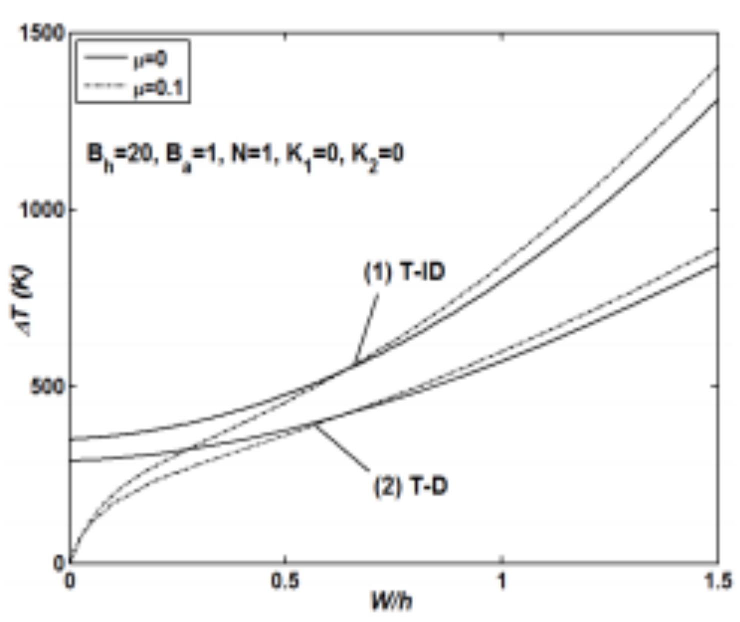

Fig. 2. Effect of thermal

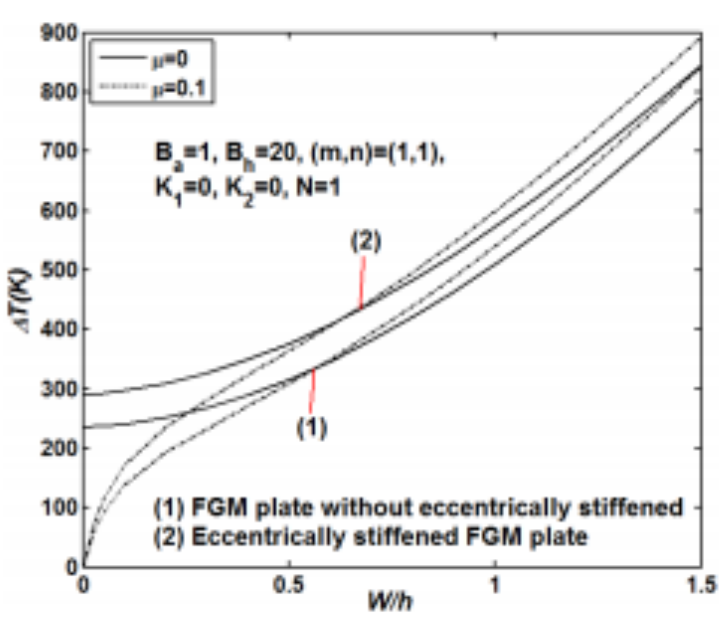

Fig. 3. Effect of eccentric stiffeners

Nonlinear dynamic response [3]

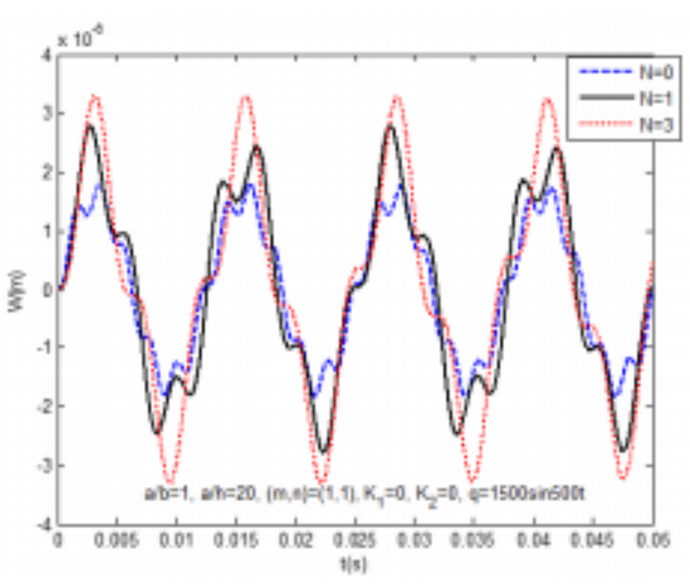

Fig.4. Effect of power law index

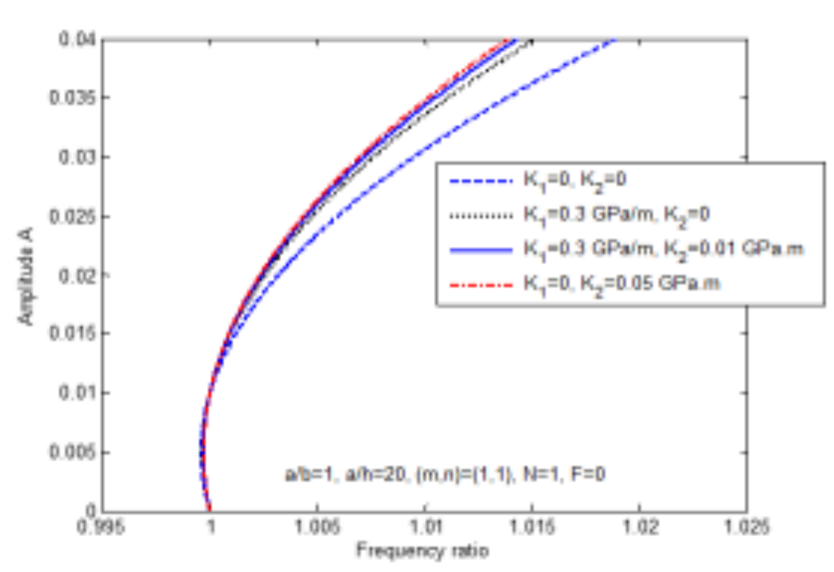

Fig. 5. Effect of the elastic foundations

4. CONCLUSIONS

- The stiffeners have an obvious influence on the loading ability of moderately thick FGM plates.
- The elastic foundation parameters strongly affect the buckling and post-buckling and dynamic response.
- The material properties depending on temperature have an obvious impact on the static nonlinear behavior.

5. REFERENCES

[1] Bich DH, Nam VH, Phuong NT (2011) Vietnam Journal of Mechanics, VAST, 33 (3): 131-147.

[2] Cong PH, An PTN, Duc ND (2015). Sci Eng Compos Mater 2015; doi 10.1515/secm-2015-0225.

[3] Duc et al. (2015). Journal of Sandwich Structures and Materials,doi:10.1177/1099636215602142