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# Antenna without Phase Center for DOA Estimation in Compressive Array Processing

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**Abstract**

*Recently, Compressive Sensing (CS) has been applied to array signal processing. In theory, Direction-of-Arrival (DOA) estimation based on CS recovery can work well in correlated environments. However, a large number of sensors (i.e., linear measurements) are still needed for CS recovery. To improve on this, we propose a new CS-based DOA estimation method with a recently designed antenna structure called the Asymmetric Antenna without Phase Center (Asym-AWPC). The best reconstruction is achieved by solving the $l_1$-norm optimization problem, which is cast as an $l_1$-regularized least-squares program. Simulated results indicate the effectiveness of the proposed CS-based Asym-AWPC DOA estimator in a multipath environment over a recent Asym-AWPC DOA estimator but using the Multiple Signal Classification (MUSIC) rather than CS. Further improvement on the resolution can be achieved by tuning the degree of asymmetry in designing the Asym-AWPC.*

**Keywords:** *Array processing, direction of arrival, multiple signal classification (MUSIC), uniform circular array, antenna-without-phase-center, multipath, compressive sensing, less sensors than sources*

## 1. Introduction

Direction-of-Arrival (DOA) estimation always plays a key role in radar, navigation, or wireless communications. Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotation Invariance Techniques (ESPRIT) are well-known subspace-based DOA techniques. However, these algorithms only work well if the signals are uncorrelated. Meanwhile, wireless environments are complex due to the inherent multipath characteristic. This characteristic causes the signals to be *correlated* (*i.e.*, some signals are scaled and delayed versions of an original signal) or even *coherent* (*i.e.*, some signals are the same as the origin signal) [1]. In such environments, these subspace-based methods either perform poorly or, worse, fail because the covariance matrix of the source signals becomes rank deficient [2].

Several preprocessing techniques such as Forward-Backward Averaging, Toeplitz Completion, Forward-Backward Spatial Smoothing are used to improve the rank but their ability is limited to several array geometries such as Nonuniform Linear Array [2], Uniform Linear Array [3], and some special Uniform Circular Arrays [4]. Moreover, these techniques are only applied for cases where the number of sources is small [2].

Compressive Sensing (CS) is a signal processing technique for efficiently acquiring and reconstructing a sparse or compressible signal with fewer samples than the Nyquist-Shannon theorem [5-8]. The last few years have seen a tremendous progress in CS theory and applications. It can be applied for array signal processing in both time and spatial domains

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[6]. For DOA estimation, a CS-based method has been proposed in [9] and has some initial advantages of compressing data in the time and spatial domains. It is limited to working only with uncorrelated signals. Another CS-based method is able to operate with single-snapshot correlated signals [10]. However, the required number of sensors is rather large; for example, about 30 sensors are needed for estimating 5 signals. This paper focuses on dealing with correlated and coherent signals, and thus we are interested in the application of CS for DOA estimation.

The theory of CS shows that under certain conditions of source sparsity and system incoherence, a sparse source signal can be reconstructed from a limited number of linear measurements of the source [6, 11]. Among several well-known algorithms of CS, $l_1$ minimization has been demonstrated effective for exact reconstruction [5, 10]. In $360^0$ DOA estimation, according to CS, the signal length, $D_s$, is at least 360 (if the resolution is $1^0$), which is much larger than the number of the sources to be estimated. Therefore, in DOA estimation, the signal in the spatial domain can be considered sparse, justifying the use of CS.

The other conditions in order to apply CS for DOA estimation are Restricted Isometry Property (RIP) and incoherence. Both RIP and incoherence can be obtained by designing the measurement matrix $\mathbf{\Phi}$ to be random; *i.e.*, measurements are merely randomly weighted linear combinations of the sparse or compressible signal. In the case where the sparsity basis matrix $\mathbf{\Psi}$ is an identity matrix, $\mathbf{\Phi}$ always is incoherence with $\mathbf{\Psi}$ and RIP is satisfied if random Gaussian measurements $M = \mathcal{O}(D \log D_s/D)$ [5].

Taking an example where the CS-based DOA estimator would operate for a maximum of 6 sources, we would then need an antenna array geometry with at least $M = \mathcal{O}(6 \log 360/6) = \mathcal{O}(11)$ sensors. If we use a conventional array, such as the Uniform Linear Array (ULA), the size of the array for the CS-based DOA estimator is about $11\,\lambda/2$ where $\lambda$ is the wavelength. Moreover, the observations must be multiple by a random matrix as in [12] to satisfy RIP and incoherence conditions. Meanwhile, one would only need to use 7-sensor array to estimate 6 uncorrelated sources using MUSIC.

Based on the above analysis, our aim is to make use of the advantage of CS for DOA estimation to reduce hardware complexity greatly in terms of the antenna size, the RF front-end circuit number, and memory for storing entries of the random matrix. This is realized by proposing to use the recently proposed antenna structure called Asymmetric Antenna without Phase Center (Asym-AWPC) [13], rather than using the conventional ULA. The Asym-AWPC is optimized for working in the $360^0$ range with some desirable properties (ambiguity-free, compact, and array isotropic). In addition, DOA estimation using the Asym-AWPC has been proposed in [13], with the MUSIC algorithm. However, it only works well in uncorrelated environments. We propose in this paper a DOA estimator based on the Asym-AWPC and uses CS that is able to work in correlated environments.

The paper is organized as follows. In Section 2, we review a recent DOA estimation method using the Asym-AWPC antenna and the MUSIC algorithm in [13], abbreviated by Asym-AWPC-MUSIC. In Section 3, we propose a new DOA estimation method using the Asym-AWPC antenna and the CS algorithm, abbreviated by Asym-AWPC-CS. The resolution of Asym-AWPC-CS is further improved in Section 4.

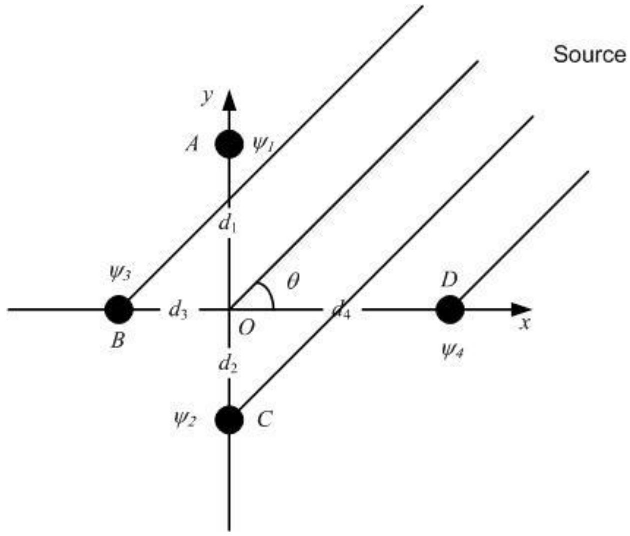



**Figure 1. Asym-AWPC Structure with 4 Dipoles**

## 2. DOA Estimation based on Asym-AWPC and MUSIC

### 2.1. Asym-AWPC

An Asym-AWPC includes four dipoles *A*, *B*, *C* and *D* as shown in Figure 1. The distances between the *A*, *C*, *B* and *D* dipoles and the origin are respectively $d_1$, $d_2$, $d_3$ and $d_4$. The structure is asymmetric in the sense that $d_1 \neq d_2$, or $d_3 \neq d_4$. According to antenna theory, the total electric field of the sensors in the antenna array is expressed by

$$E(\theta) = -\frac{jk}{4\pi}\frac{e^{-jkR_0}}{R_0}|I_0|\beta(\theta), \tag{1}$$

where $k$ is the wave number, $R_0$ is the distance between the origin and the source, $|I_0|$ is the amplitude of the current of each sensor, $\theta$ is the direction of propagation, and $\beta(\theta)$ is the array factor (AF). The AF is given by

$$\beta(\theta) = e^{j\psi_1}e^{-jkd_1\sin\theta} + e^{j\psi_2}e^{jkd_2\sin\theta} + e^{j\psi_3}e^{jkd_3\cos\theta} + e^{j\psi_4}e^{-jkd_4\cos\theta}, \tag{2}$$

where $\psi_1 = 0^0, \psi_2 = 180^0, \psi_3 = 90^0$, and $\psi_4 = 270^0$ are the phases of the currents at *A*, *C*, *B* and *D*, respectively. The amplitude pattern, $G(\theta)$and the phase pattern, $\Upsilon(\theta)$, of the Asym-AWPC are obtained by

$$G(\theta) = \sqrt{\Re^2\{\beta(\theta)\} + \Im^2\{\beta(\theta)\}}, \tag{3}$$

$$\Upsilon(\theta) = \angle\beta(\theta), \tag{4}$$

where

$$\Re\{\beta(\theta)\} = -2\sin\left[\frac{k}{2}(d_1+d_2)\sin\theta\right]\sin\left[\frac{k}{2}(d_1-d_2)\sin\theta\right] \\ -2\sin\left[\frac{k}{2}(d_3+d_4)\cos\theta\right]\cos\left[\frac{k}{2}(d_3-d_4)\cos\theta\right], \tag{5}$$

$$\Im\{\beta(\theta)\} = -2\sin\left[\frac{k}{2}(d_1+d_2)\sin\theta\right]\cos\left[\frac{k}{2}(d_1-d_2)\sin\theta\right]$$ (6)

$$-2\sin\left[\frac{k}{2}(d_3+d_4)\cos\theta\right]\sin\left[\frac{k}{2}(d_3-d_4)\cos\theta\right].$$

The Asym-AWPC is proposed in [13] to resolve the ambiguity, which is the similarity of two or more steering vectors corresponding to widely separated angles in the array manifold. The ambiguity can be checked by:

$$\gamma(\theta_1,\theta_2) \triangleq \frac{|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)|}{\|\mathbf{a}(\theta_1)\|\|\mathbf{a}(\theta_2)\|},$$ (7)

where $\mathbf{a}(\theta_1)$ and $\mathbf{a}(\theta_2)$ are two arbitrary steering vectors at directions $\theta_1$, and $\theta_2$. If $\mathbf{a}(\theta_1)$ and $\mathbf{a}(\theta_2)$ are co-linear then $\gamma(\theta_1,\theta_2)=1$ and if they are orthogonal, meaning that $|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)|=0$, and hence $\gamma(\theta_1,\theta_2)=0$. The array geometry is ambiguity-free if $\gamma(\theta_1,\theta_2)<1$. The performance is improved as $\gamma$ gets smaller.

Besides, it is desirable to decrease mutual coupling while keeping the antenna size small. Therefore, the following geometrical configuration should be selected [13]:

$$(d_1,d_2,d_3,d_4) = \left(\lambda/4\,,\lambda/4\,,\sqrt{3}\,\lambda/4\,,\left(\sqrt{3}/4+\Delta d\right)\lambda\right).$$ (8)

Figure 2(a) plots $\gamma$ of the Asym-AWPC with $\Delta d = 0.6$. The result shows that $\gamma(\theta_1,\theta_2)<1$ except when $\theta_1=\theta_2$; that means the antenna has no ambiguity.

### 2.2. Data Model

Consider $D$ narrowband zero-mean Gaussian sources $s_1(t), s_2(t), \ldots, s_D(t)$ impinging on the Asym-AWPC, assuming that the elevation angle is equal to $90^0$. The antenna is rotated in $M$ steps in the clockwise direction. At step $m$, for $m=0,\ldots,M-1$, the received signal is modeled as

$$x_m(t) = \sum_{i=1}^{D} s_i(t)G(\theta_i+m\Delta\theta)e^{j\Upsilon(\theta_i+m\Delta\theta)} + n_m(t),$$ (9)

where $\theta_i$ is the incident angle of the $i$-th source, $\Delta\theta$ is the antenna rotation angle, and $n_m(t)$ is the spatially zero-mean white Gaussian noise with variance of $\sigma^2$, statistically independent of the sources. In matrix form, the data model becomes

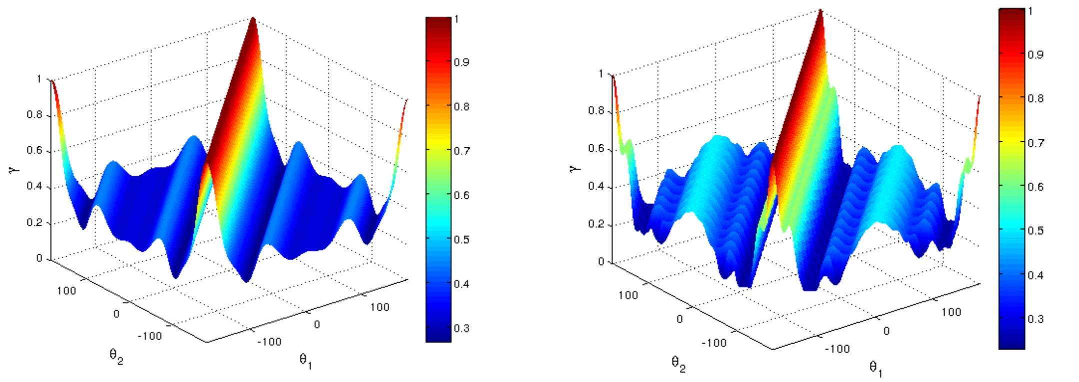
$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t),$$ (10)

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_D(t)]^T$ is the source vector, $\mathbf{n}(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T$ is the noise vector, $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$ is the received vector, and $\mathbf{A}(\boldsymbol{\theta})$ is the steering matrix defined by

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_D)].$$ (11)

In (11), $\mathbf{a}(\theta_i)$ is the steering vector associated with the $i$-th source and is given by

$$\mathbf{a}(\theta_i) = \begin{bmatrix} G(\theta_i)e^{j\Upsilon(\theta_i)} \\ \vdots \\ G(\theta_i+(M-1)\Delta\theta)e^{j\Upsilon(\theta_i+(M-1)\Delta\theta)} \end{bmatrix}.$$ (12)




(a) (b)

**Figure 2. Ambiguity Checking for Asym-AWPC with: (a)** $\Delta d = 0.6$**, (b)** $\Delta d = 1.5$

The well-known MUSIC algorithm, based on exploiting the eigenstructure of the spatial covariance matrix of the output vector, was proposed by Schmidt in 1979. MUSIC can provide information about the number of incident signals, the strength, and DOA of each signal with very high resolution. However, it requires accurate array calibration [14].

The spatial covariance matrix of the output vector is expressed as

$$\mathbf{R_x} = \mathbf{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{R_s}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (13)$$

where $\mathbf{E}\{\cdot\}$ denotes the statistical expectation operator, and $\mathbf{R_s}$ is the source covariance matrix. In practice, the spatial covariance matrix is estimated by the following sample spatial covariance matrix: $\widehat{\mathbf{R}} = \frac{1}{K}\sum_{t=1}^{K}\mathbf{x}(t)\mathbf{x}^H(t)$, where $t = 1, \dots, K$ and $K$ is called the number of snapshots.

Next, this covariance matrix is eigen-decomposed as $\widehat{\mathbf{R}} = \widehat{\mathbf{U}}\widehat{\mathbf{\Lambda}}\widehat{\mathbf{U}}^H$, where $\widehat{\mathbf{U}}$ contains the eigenvectors and $\widehat{\mathbf{\Lambda}} = \mathrm{diag}\{\lambda_1, \lambda_2, \dots, \lambda_M\}$ is a diagonal matrix satisfying $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D > \lambda_{D+1} = \cdots = \lambda_M = \sigma^2$. The MUSIC spatial spectrum is then obtained by

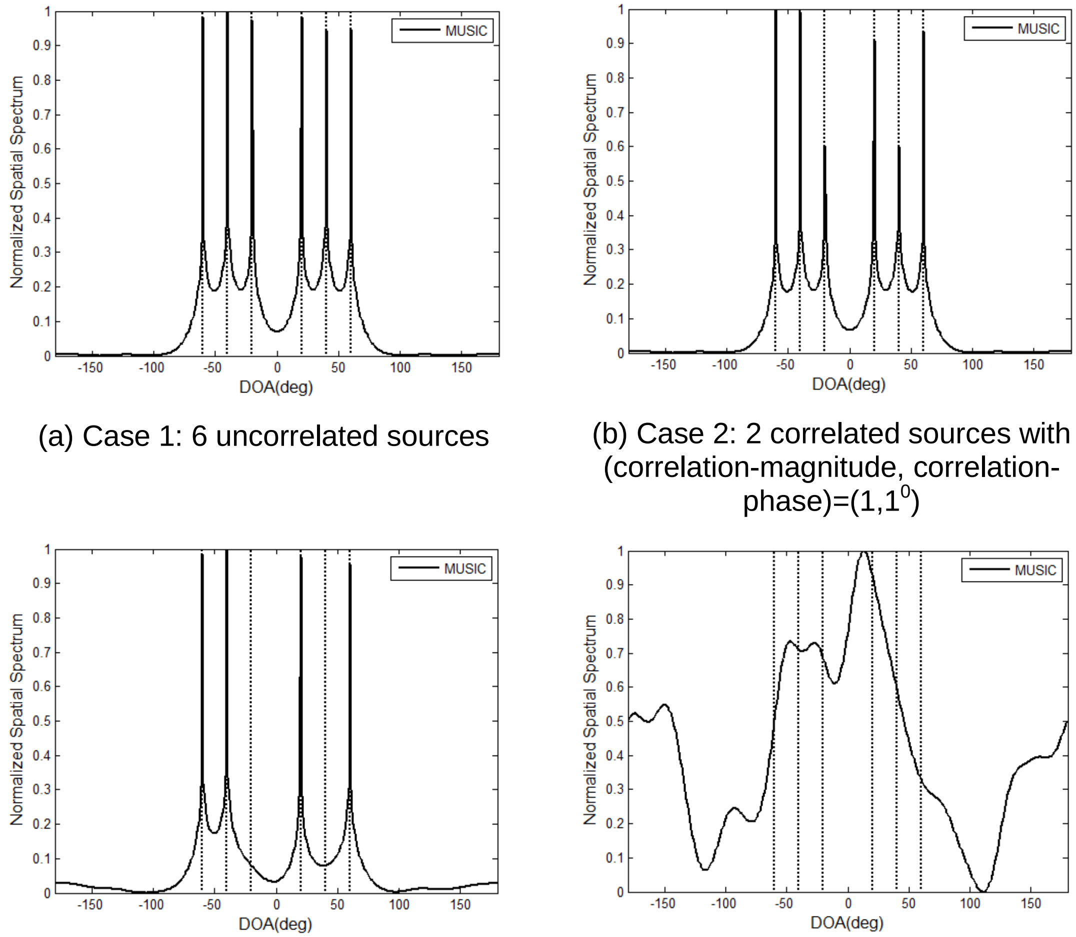
$$P_{\mathrm{MUSIC}}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\widehat{\mathbf{U}}_n^H\widehat{\mathbf{U}}_n\mathbf{a}(\theta)}, \quad (14)$$

where $\widehat{\mathbf{U}}_n$ is the noise source matrix which is formed by the last $M - D$ columns of $\widehat{\mathbf{U}}$, corresponding to $M - D$ eigenvalues $\lambda_{D+1}, \dots, \lambda_M$. The orthogonality between $\mathbf{a}^H(\theta)$ and $\widehat{\mathbf{U}}_n$ will minimize the denominator of (14). Therefore, the $D$ largest peaks in the MUSIC spatial spectrum correspond to the DOAs of the signals impinging on the antenna.

### 2.3. Results and Discussions

Although MUSIC is a well-known super-resolution algorithm, its performance decreases if sources are correlated. Some following numerical examples demonstrate operation of Asym-AWPC-MUSIC in multipath environment. Six sources are presented at azimuth ($-60^0$, $-40^0$, $-20^0$, $20^0$, $40^0$, $60^0$) and the signal-to-noise ratios (SNRs) are all equal to 25dB. The snapshot number is 10. The Asym-AWPC with $\Delta d = 0.6$ is rotated with $M = 17$ and $\Delta\theta = 360/17$. The sources are set in four cases:




(a) Case 1: 6 uncorrelated sources

(b) Case 2: 2 correlated sources with (correlation-magnitude, correlation-phase)=(1,1$^0$)

(c) Case 3: 2 coherent sources

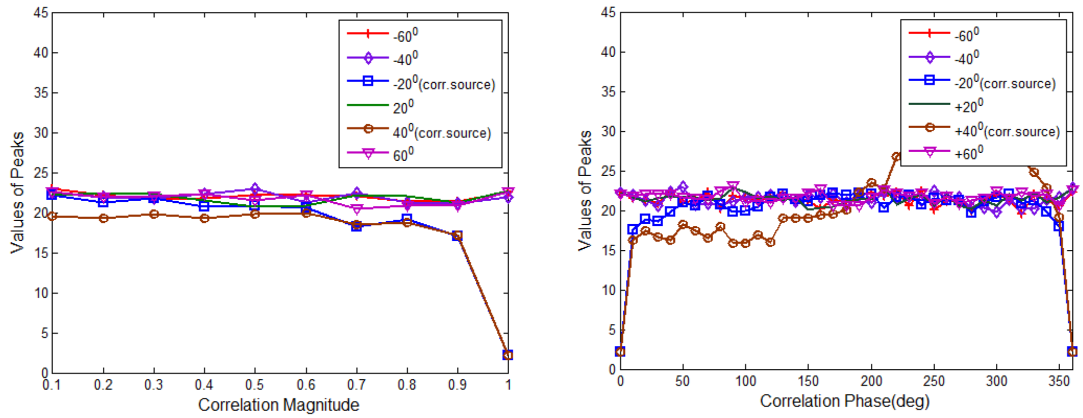
(d) Case 4: 6 coherent sources

**Figure 3. Asym-AWPC-MUSIC in Multipath Environment with** $\Delta d = 0.6$

- Case 1: all 6 sources are uncorrelated.
- Case 2: sources -20$^0$ and 40$^0$ are correlated, the others are uncorrelated.
- Case 3: sources -20$^0$ and 40$^0$ are coherent, the others are uncorrelated.
- Case 4: all 6 sources are coherent.

The DOA estimation results are shown in Figure 3. In all the figures, the dashed vertical lines present the true DOAs. The results show that the uncorrelated sources (-60$^0$, -40$^0$, 20$^0$, 60$^0$) are always revealed by high sharp peaks in all cases while the correlated sources (-20$^0$, 40$^0$) depend on the correlation coefficient in terms of the correlation-magnitude and the correlation-phase. Figure 4(a) provides information about values of the MUSIC peaks versus the correlation-magnitude (the correlation-phase is equal to 0$^0$). Values of the peaks which correspond to the four uncorrelated sources are always high and steady whereas those of the other two correlated sources decrease slightly in the interval [0.1, 0.9] and drop suddenly in




(a) Values of MUSIC peaks versus correlation-magnitude

(b) Values of MUSIC peaks versus correlation-phase

**Figure 4. Values of MUSIC Peaks versus Correlation Coefficient**

the interval (0.9,1]. Figure 4(b) shows the trend of values of the MUSIC peaks versus correlation-phase (correlation-magnitude is 1). The values of uncorrelated peaks are also high and steady in all range whereas those of the others decrease sharply close to $0^0$ and $360^0$. Hence, we can conclude that the performance of Asym-AWPC-MUSIC degrades in a multipath environment. Worse, Asym-AWPC-MUSIC even fails if the sources are coherent.

## 3. DOA Estimation based on Asym-AWPC and CS

As previously explained, the required number of sensors and the RF front-end are rather large and the hardware complexity increases because of storing entries of random matrix measurements of CS. In this section, the number of sensors is reduced to four by using the Asym-AWPC.

### 3.1. Data Model

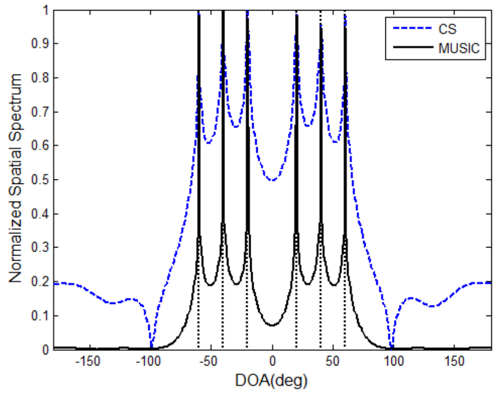
Let $\boldsymbol{\theta}_s = \left(\theta_1, \dots, \theta_{D_s}\right)$ be a set of angles, $D_s$ be the total number of angles we want to scan, $\boldsymbol{\theta} \in \boldsymbol{\theta}_s$. Using Asym-AWPC with the steering vector given by (12), we define an angle scanning matrix of size $M \times D_s$ as $\mathbf{A}(\boldsymbol{\theta}_s) = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}\left(\theta_{D_s}\right)\right]$, where $M$ is the number of spatial samples, corresponding to the number of rotation steps of Asym-AWPC. We also define an $D_s \times 1$ sparse vector $\mathbf{z}(t) = \left[z_1(t), z_2(t), \dots, z_{D_s}(t)\right]^T$, with $D$ nonzero coefficient $z(t) = s(t)$ at positions corresponding to the $D$ sources, and zero coefficients at the remaining $D_s - D$ positions. Therefore, the signal model of (10) can be rewritten as

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta}_s)\mathbf{z}(t) + \mathbf{n}(t). \tag{15}$$

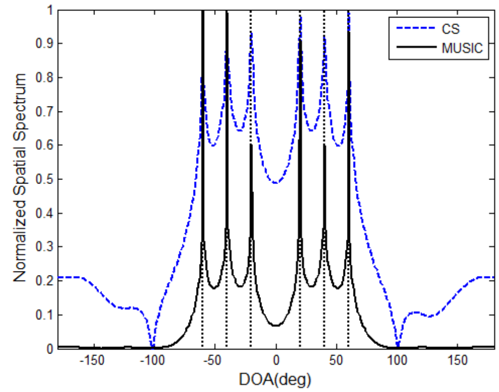
Once $\hat{\mathbf{z}}$ has been estimated, the CS spatial spectrum of CS recovery is expressed by [12]:

$$P_{\text{CS}}(\theta_i) = \frac{1}{K}\sum_{k=1}^{K} \hat{z}_{\theta_i}(k), \tag{16}$$

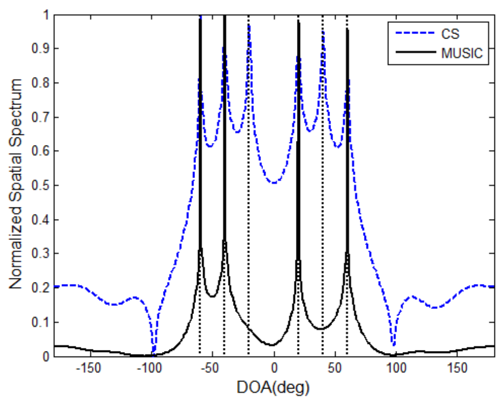
where $i = 1, \dots, D_s$.



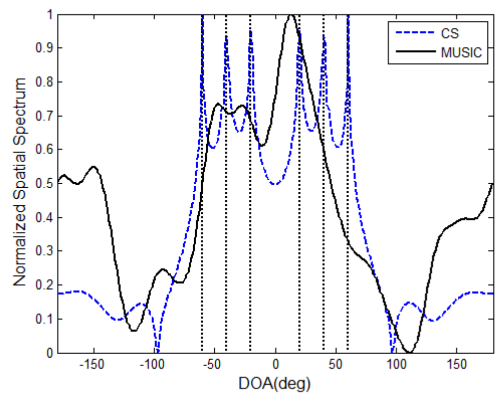

(a) Case 1: 6 uncorrelated sources




(b) Case 2: 2 correlated sources with (correlation magnitude, correlation phase)=(1,1$^0$)




(c) Case 3: 2 coherent sources




(d) Case 4: 6 coherent sources

**Figure 5. Asym-AWPC-CS and Asym-AWPC-MUSIC with $\Delta d = 0.6$ in Multipath Environment for DOA Estimation of 6 Sources**

### 3.2. Reconstruction Algorithm: $l_1$-Regularized Least Squares

Many CS-based reconstruction algorithms, which are based on optimization, have been proposed. In this paper we choose the $l_1$-optimization in (17) because of high accuracy [5, 6]:

$$\hat{\mathbf{z}} = \operatorname{argmin}\|\acute{\mathbf{z}}\|_1 \quad \text{subject to} \quad \mathbf{A}\acute{\mathbf{z}} = \mathbf{x}, \tag{17}$$

where $\|\mathbf{u}\|_p = (\sum_i |u_i|^p)^{1/p}$ denotes the $l_p$ norm of vector $\mathbf{u}$. Problem (17) is also cast as an $l_1$-Regularized Least-Squares program which is solved by several standard methods such as interior-point methods [15]. With $l_1$-regularized least squares, we solve an optimization of the form

$$\min\|\mathbf{Az} - \mathbf{x}\|_2^2 + \eta\|\mathbf{z}\|_1, \tag{18}$$

where $\|\mathbf{z}\|_1 = \sum_{i=1}^{D_s}|z_i|$, $\mathbf{A} \in \mathbb{C}^{M\times D_s}$ is the measurement matrix, $\mathbf{z}$ is an arbitrary vector in $\mathbb{C}^{D_s}$, $\mathbf{x} \in \mathbb{C}^M$ is observation vector and $\eta > 0$ is the regularization parameter [15].

The Truncated Newton Interior-Point method, described in detail in [15], is chosen to solve (18) because of good convergence rate. The performance of the Truncated Newton Interior-Point method depends on $\eta$ and mutual coherence $\mu$, which is defined, with an arbitrary measurement matrix, as [16]

$$\mu = \max_{\theta_1 \neq \theta_2} \frac{|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)|}{\|\mathbf{a}(\theta_1)\|\|\mathbf{a}(\theta_2)\|}. \tag{19}$$

The Truncated Newton Interior-Point method is slow when $\eta$ is too small (gives a not very sparse solution) and $\mu$ is close to 1 [16].

### 3.3. Results and Discussions

The performance of Asym-AWPC-CS is compared to that of Asym-AWPC-MUSIC in this section. The simulation scenarios are the same as those in Section 2.3. The results are shown in Figure 5. The dashed lines show the results of the MUSIC algorithm while the solid lines show those of the CS algorithm. Values of peaks of the Asym-AWPC-CS are steady in all cases while those of Asym-AWPC-MUSIC decrease when the sources are correlated. This is due to the fact that DOA estimation using CS does not depend on correlation of sources. However, in Figures 5(a), 5(b), and 5(c), we also see that the peaks resolved by the Asym-AWPC-MUSIC are sharper than those of the Asym-AWPC-CS. Therefore, the Asym-AWPC-CS is a promising method for a compact DOA estimator in a multipath environment, but the resolution need be improved.

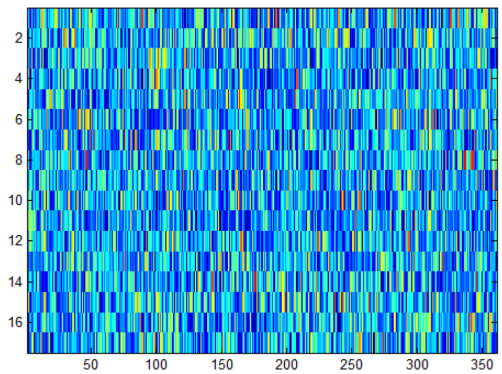
## 4. Improved Resolution by Decreasing Mutual Coherence

### 4.1. Asym-AWPC Measurement Matrix Characteristics

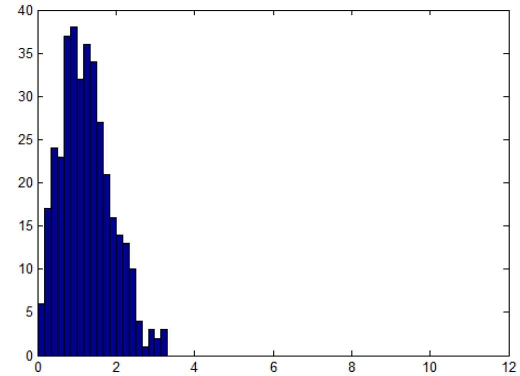
In this section, we will consider measurement matrix characteristics of the Asym-AWPC in terms of orthogonality, statistical distribution, and mutual coherence. The orthogonality of the columns of the measurement matrix is proved in Section 2.1. However, the orthogonal level depends on the configuration of the Asym-AWPC. Figure 2(a) and Figure 2(b) present the orthogonality of all pairs of steering vectors of Asym-AWPC with $\Delta d = 0.6$ and $\Delta d = 1.5$, respectively.

In CS, the measurement matrix is often designed to be random. The statistical distribution of the measurement matrix is important because it affects the solution obtained by CS. The measurement matrix constructed by the Asym-AWPC is however deterministic. We examine the variation of the values of the measurement matrix for three different cases: (i) normal distribution, (ii) Asym-AWPC with $\Delta d = 0.6$, and (iii) UCA, as shown in Figure 6. All matrices have the same size of $17 \times 360$. The left hand side shows the scale-data-and-display-as-image (SDDI) of the three matrices and the right hand side is the histograms of a specific row of the matrices (row 12 was chosen randomly). The results indicate that there is no variation in the values for the chosen row when the matrix is designed in accordance with the UCA structure. This explains the reason why we cannot apply CS for the UCA.

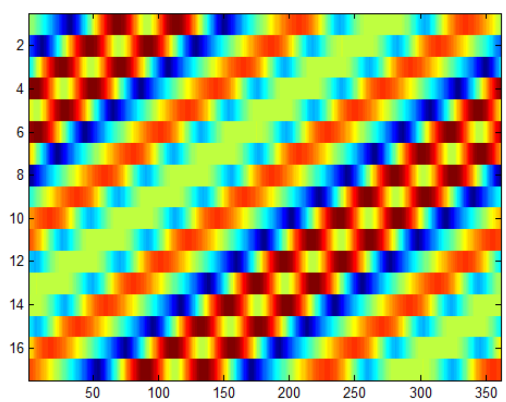
In the general case, the mutual coherence of a measurement matrix is expressed by (19) in which the function to be maximized is the same as that in (7). The ambiguity checking factor $\gamma(\theta_1, \theta_2)$ defined by (7) is shown in Figure 2. The value of (7) is really high if $|\theta_1 - \theta_2|$ is close to $0^0$ or $360^0$. That means, the mutual coherence in DOA estimation mainly depends on



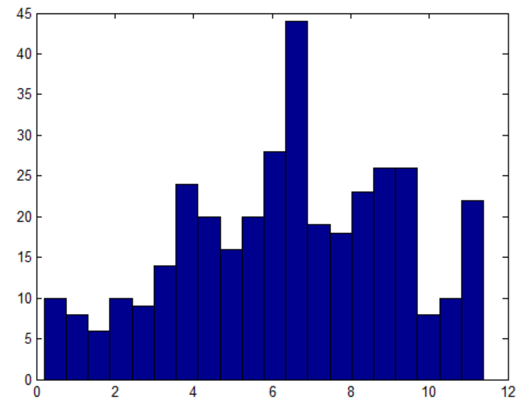

(a) SDDI of normal distribution



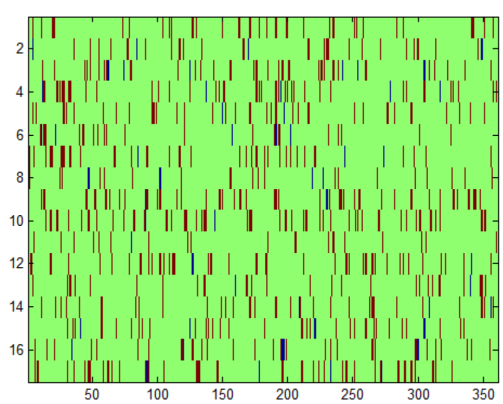

(b) HIS of normal distribution



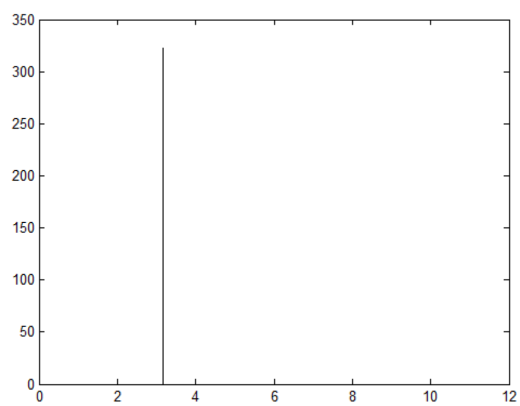

(c) SDDI of Asym-AWPC with $\Delta d = 0.6$




(d) HIS of Asym-AWPC with $\Delta d = 0.6$




(e) SDDI of UCA




(f) HIS of UCA

**Figure 6. Scale-data-and-display-as-images (SDDI) and Histograms (HIS) at row 12 of the Absolute Measurement Matrices**

the resolution (closely spaced angles). Therefore, the mutual coherence in DOA estimation should change as follows:

$$\mu = \max_{|\theta_1 - \theta_2| \geq \epsilon} \frac{|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)|}{\|\mathbf{a}(\theta_1)\|\|\mathbf{a}(\theta_2)\|} \quad (20)$$

where $\epsilon$ is the resolution of the system. Figure 7 displays the mutual coherence in the range [-175$^0$,175$^0$] with $\epsilon = 5^0$ versus the asymmetry factor $\Delta d$ of the Asym-AWPC. At $\epsilon = 5^0$, $\mu$ gets smaller as $\Delta d$ increases.

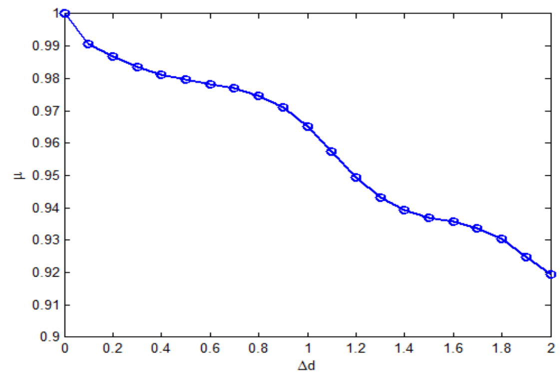
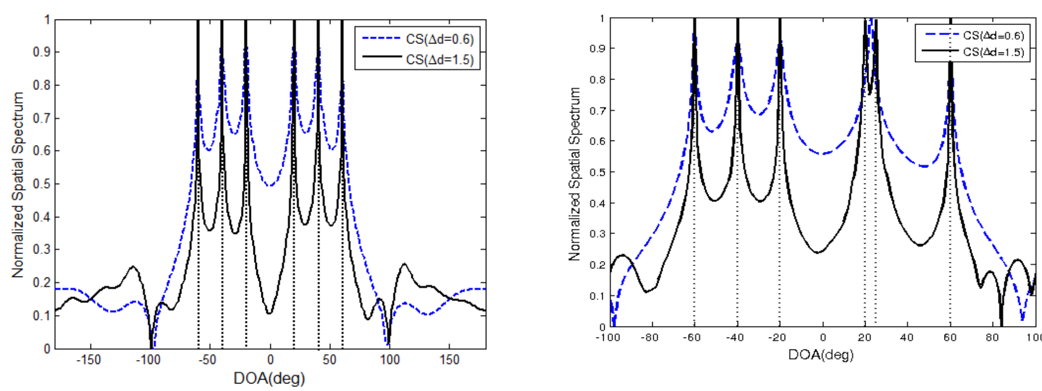



**Figure 7. Mutual Coherence in** [-175$^0$, 175$^0$] **Range with** $\epsilon = 5^0$

### 4.2. Results and Discussions

The spatial spectrum of the Asym-AWPC-CS with $\Delta d = 0.6$ and $\Delta d = 1.5$ are presented in Figure 8 in forms of dashed and solid line, respectively. Figure 8(a) indicates that the sharpness of the peaks when $\Delta d = 0.6$ is worse than those when $\Delta d = 1.5$. The resolution ability is shown in Figure 8(b) with true angles (-60$^0$, -40$^0$, -20$^0$, 20$^0$, 25$^0$, 60$^0$). The two close peaks at 20$^0$ and 25$^0$ can be resolved by $\Delta d = 1.5$ but $\Delta d = 0.6$.




(a) $\Delta d = 0.6$

(b) $\Delta d = 1.5$

**Figure 8. j Spectrum of CS with** $\Delta d = 0.6$ **and** $\Delta d = 1.5$

## 5. Conclusions

In this paper we have investigated DOA estimation in a multipath environment wherein the number of sensors is small, even less than that of the sources. This is done by combining the special antenna structure Asym-AWPC and CS algorithm. The obtained estimation results are good for all cases in the wireless environment while those of the MUSIC algorithm degrade or even fail for cases in which the sources are highly correlated or coherent. The resolution of the Asym-AWPC-CS spectrum can be improved if we design the Asym-AWPC with a larger asymmetry factor $\Delta d$ that effective makes the mutual coherence smaller.

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