Designing Kalman Filters for Integration of Inertial Navigation System and Global Positioning System

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Designing Kalman Filters for Integration of Inertial Navigation System and Global Positioning System

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***Abstract*-- Due to the strong growth of MEMS technology, the Inertial Navigation System (INS) is widely applied to navigation and guidance of aircraft movements. However, there are existing errors in the accelerometer and gyroscope signals that cause unacceptable drifts. To minimize these effects on the INS system, a GPS is usually employed simultaneously with an INS in order to increase the dimension of the system; the desired parameters are estimated by the Kalman filtering technique applied to the enlarged system. In this paper, we present a structure consisting of a Kalman Filter (KF) of which the input is the difference between the noisy INS output and the noisy GPS output; the output of KF is finally introduced into the unaided INS system. In this structure, INS errors are compensated by a feedforward and a feedback loop. The overall performance of the system is analyzed by simulation; and results show that this technique substantially improves the quality of the navigation and guidance systems.**

I. INTRODUCTION

The demand of navigation and guidance has been urgent for many years. In fact, GPS have been employed widely in many applications while INS are daily used in controlling flight dynamics. Nowadays, with the strong growth of Micro-Electro-Mechanical-System (MEMS) technology, there appears a new trend in navigation and guidance domain[5]: it consists of the integration of INS and GPS altogether. Integrating these two methods can improve the performance of the system and reduce concurrently the disadvantages of both INS and GPS [1].

II. INS and GPS

A. Inertial Navigation System (INS)

An INS often consists of three accelerometers and three gyroscopes in order to measure the accelerations in three dimensions and the rotation rates around three axes. The strongly development of MEMS technology has been a stimulus to widen the application area of INS. Today, an Inertial Measurement Unit (IMU) even has got only one tri-axial gyroscope and one tri-axial accelerometer [6], [7].

There are two typical INS systems: gimble system and strapdown system [3]. The strapdown INS system has been used more popular than gimble system. The strapdown system is mostly based on the MEMS technology that is relatively inexpensive and compact. In strapdown system, accelerometers and gyroscopes are fixed to body frame of the aircraft. Signals from these sensors are processed in order to obtain three Euler angles. The results are corrected by gravity acceleration and Earth rotation velocity.

The orientation of the aircraft is determined by three Euler angles ψ, θ, ϕ (see Fig. 1). The following equation shows the relation between velocities of roll, pitch, yaw (p, q and r) and three Euler angles:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{1}$$

Then, we integrate Equ. (1) to obtain three Euler angles.

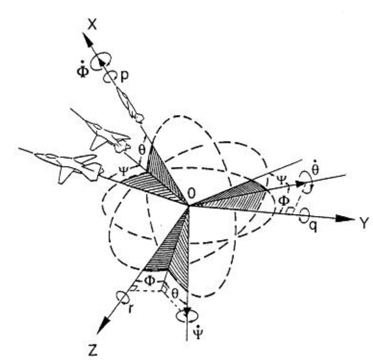

Figure 1. Three Euler angles

Three accelerations (a_x, a_y and a_z) along three axes of body frame relate to three velocities (U, V and W) in the Earth fixed frame by following equation:

$$\dot{U} = a_x + V.r - \mathrm{W.q} + \mathrm{g}.\sin\theta$$
$$\dot{V} = a_y - U.r + \mathrm{W.p} - \mathrm{g}.\cos\theta.\sin\phi \qquad (2)$$
$$\dot{W} = a_z - U.q + \mathrm{W.p} - \mathrm{g}.\cos\theta.\cos\phi$$

After integrating Equ. (2), we can obtain U, V and W. By using Direct Cosine Matrix (DCM), we can convert the movement from Earth fixed frame to navigation frame:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = DCM^T \begin{bmatrix} \mathrm{U} \\ \mathrm{V} \\ \mathrm{W} \end{bmatrix} \qquad (3)$$

Where:

$$DCM = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

We can position the aircraft by integrating the Equ. (3). Then, we can obtain the latitude, longitude and height of the aircraft by following equations:

$$\dot{\lambda} = \frac{V_N}{R_{earth}} \qquad \dot{\mu} = \frac{V_E}{R_{earth}\cos\lambda} \qquad \dot{H} = -V_D \qquad (4)$$

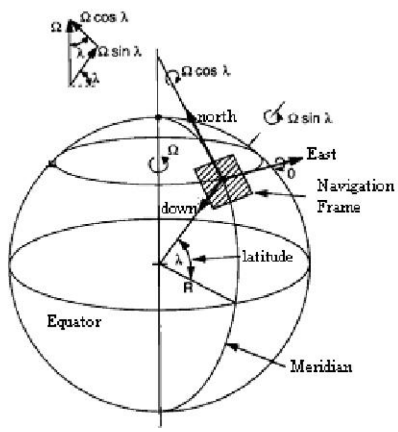

Figure 2. Navigation frame

There are many kinds of errors in the INS. Most of the errors in the INS are caused by sensor imperfections. Most common errors in strapdown INS can be summarized in Table 1.

Table 1. Common errors in the INS

Type	***Errors***
Alignment	Roll, pitch, yaw
Gyroscope	Bias, drift and scale factor errors
Accelerometer	Bias, scale factor error
Random noise	Random error

A discrete INS algorithm was developed in our lab in order to develop a completed Inertial Measurement Unit (IMU). The detail of this discrete algorithm will not be presented in this paper.

B. Global Positioning System (GPS)

The GPS consists of 24 satellites which fly above the surface of the Earth at the height 19.200 km in order to acquire the position of the aircraft (latitude, longitude and height) [2]. Radio signals often hardly transmit over solid buildings, tunnels…To get the correct position of the aircraft; it requires at least four satellites. They can separate the GPS into three segments (see Fig. 3):

- Space segment: consists of 24 satellites.
- Control segment: ground control
- User segment: receiver (civil and military purposes)

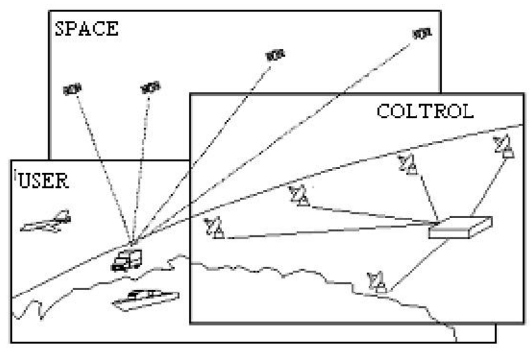

Figure 3. Structure of the GPS

Errors in the GPS mostly caused by six following factors (not including Selective Availability error):

- Ephemeris data
- Satellite clock
- Multipath reflection
- Atmospheric delay
- Random measurement noise
- Receiver (include software)

III. INTEGRATION OF INS AND GPS USING KALMAN FILTERING

The INS system has two main advantages when comparing with other navigation system: self-contained ability and high accuracy for short term navigation. The serious problem of the INS caused by accumulation of gyroscope and accelerometer errors. Therefore, in long-term navigation applications, the INS works with the aid of other systems such as radio navigation systems (Loran, Tacan), satellite navigation systems (GPS, GLONASS). The important advantage of these systems is stable performance. Consequently, there is a great need for integration of INS and one of these systems. The integration of INS and GPS is considered an optimal combination. The heart of integrated system is Kalman algorithm [4].

The Kalman filter is a multiple input, multiple output digital filter that can optimally estimate in real time the states of the system based on its noisy outputs. These states are position errors (latitude, longitude, height) and velocity errors (V_N, V_E, V_D) of the INS. The GPS output is used as a tool to estimate the error in the INS and to correct the error as much as possible. We call this is the GPS-aided INS system configuration [8], 9]. Looking at the state space model:

$$\dot{x} = Fx + Gu \qquad (5)$$

Where:

$$F = \begin{bmatrix} F_{rr} & F_{rv} \\ F_{vr} & F_{vv} \end{bmatrix} \quad G = \begin{bmatrix} 0_{(3\times3)} & 0_{(3\times3)} \\ C_b^n & 0_{(3\times3)} \end{bmatrix} \quad u = \begin{bmatrix} \delta f^b \\ \delta\omega_{ib}^b \end{bmatrix}$$

where f^b is the acceleration of the aircraft in the body frame, F is the dynamic matrix (obtained by partial derivatives), x is the state vector, G is the design matrix and u is the forcing function.
Because our navigation system works in real time mode, we convert equation 5 to its discrete time form:

$$x_{k+1} = \Phi_k x_k + w_k \quad (6)$$

where Φ_k is the state transition matrix, w_k is the driven response at t_{k+1} due to the presence of input white noise during time interval (t_k, t_{k+1}).
Because this time interval is short (it means that the update rate of the INS is high), we can approximate Φ_k as:

$$\Phi_k = e^{(F\Delta t)} \approx I + F\Delta t \quad (7)$$

and the covariance matrix associated with w_k is

$$Q_k = E\left[w_k w_k^T\right] \approx \Phi_k G Q G^T \Phi_k^T \Delta t \quad (8)$$

where

$$Q = diag\left[\sigma_{ax}^2 \quad \sigma_{ay}^2 \quad \sigma_{az}^2 \quad \sigma_{\omega x}^2 \quad \sigma_{\omega x}^2 \quad \sigma_{\omega x}^2\right]$$

Looking at the observation equation:

$$z_k = H_k x_k + v_k \quad (9)$$

where

$$z_k = \begin{pmatrix} \lambda_{INS} - \lambda_{GPS} \\ \mu_{INS} - \mu_{GPS} \\ h_{INS} - h_{GPS} \end{pmatrix}, H_k = \left(I_{3\times3} \middle| 0_{3\times3} \middle| 0_{3\times3}\right) \text{ and}$$

$R_k = E\left[v_k v_k^T\right] = diag\left[\ \sigma_\lambda^2 \quad \sigma_\mu^2 \quad \sigma_h^2 \ \right]$ is the covariance matrix for v_k.

In this system, we assume that the process noise w_k and the measurement noise v_k are uncorrelated. Figure 4 shows the KF loop implemented after we imitate the estimates of the state vector and the covariance matrix.

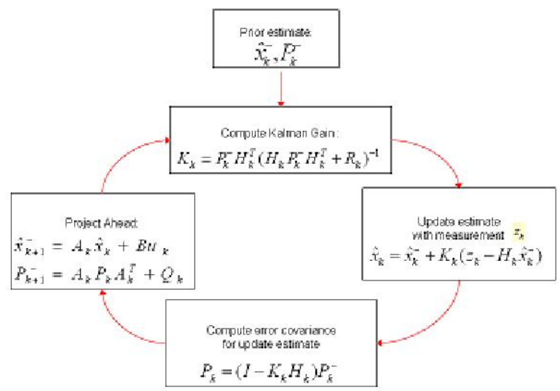

Figure 4. Kalman Filter loop

Fig. 5 illustrates an open loop (or feedforward) configuration. Its advantage is that provides a rapid filter response. Alternatively, the configuration in Fig. 6 is a closed loop one. This configuration is more complex than the open loop one but it can provide better performance in the exist of nonlinear effects. In our project, the feedforward configuration is chosen in order to provide fast performance.

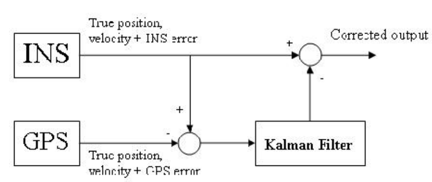

Figure 5. Feedforward configuration

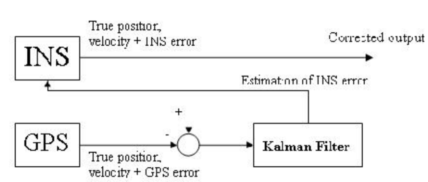

Figure 6. Feedback configuration

IV. SIMULATION RESULTS

As mentioned above, six states were considered and Kalman filter was used to estimate the INS errors. The INS can give out correct values of the position by subtracting the noisy INS to the estimation of the INS errors.

The trajectory mentioned in this paper was obtained by Aerosim Blockset, which is one of Flight Dynamic Control (FDC) programs. The standard deviation of the sensors chosen was 30 mGal. The standard deviation of the GPS was 20 m. The simulation time is 100 s, the INS update period is 0.02 s, the GPS update period is 1 s and the Kalman filter takes 0.5 s to update the estimated results.

Figure 7-9 show the output of our programs. Fig. 7 and Fig. 8 show the position of the aircraft along North and East direction on the Earth instead of the latitude and the longitude. The reason is that we can prevent numerical instabilities in calculating the Kalman gain.

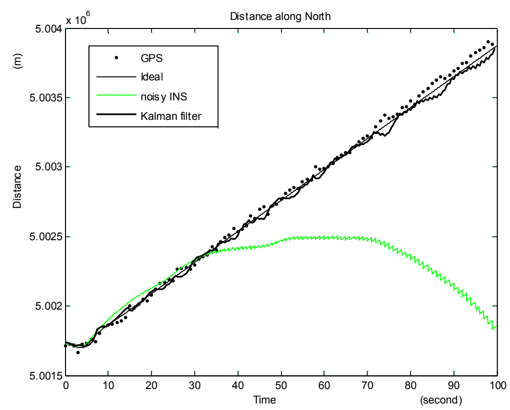

Figure 7. Distance along North

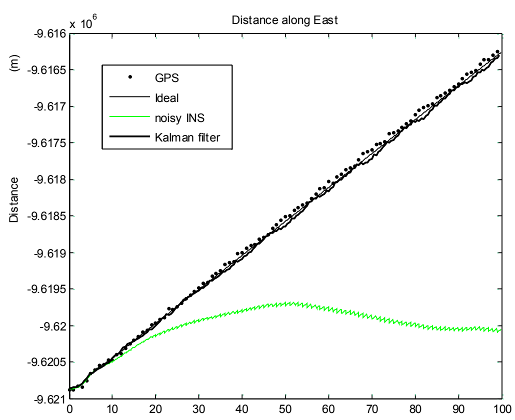

Figure 8. Distance along East

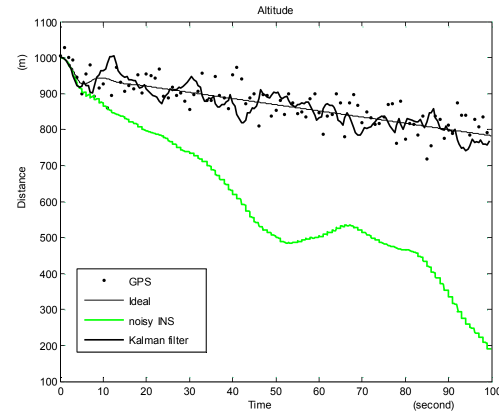

Figure 9. Altitude of the aircraft

We can see that the un-aided INS deviates from the ideal trajectory by a large distance. The Kalman filter can trace the ideal trajectory quite well but it is not much better than the GPS. But in case the SA is introduced or the aircraft is flying in difficult environments, the accuracy of the Kalman is still better than the GPS (see Fig. 10). Obviously, the Kalman could not provide the accuracy as it can perform before.

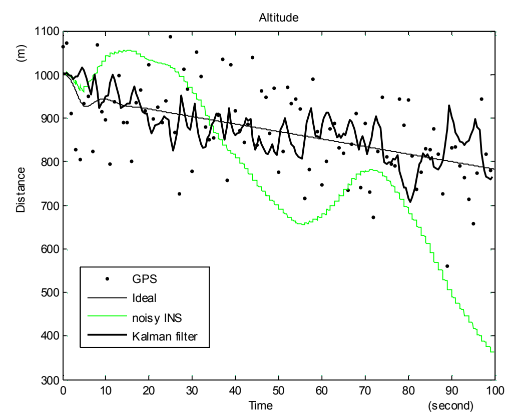

Figure 10. Altitude of the aircraft in case deviation of the GPS is increased.

V. CONCLUSIONS

In this paper, a six state Kalman filter was proposed to be used in order to enhance the quality of a combined GPS and INS system. Simulation results show that this proposed system works quite well for several different environments. In fact, the Kalman filter tremendously improves accuracies compared to the GPS and INS when operating alone as individual navigation systems. The accuracy of estimated parameters is improved by increasing the dimension of the systems' states space. Obviously, by its simplicity, this model can be embedded easily into a real time system.

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