On Optimization of Antennas without Phase Center for DOA estimation

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***Abstract*—Antennas without phase center (AWPC) are applied to resolve the less-sensors-than-sources problem in direction-of-arrival (DOA) estimation using subspace methods, typically, the well-known multiple signal classification (MUSIC) algorithm. This paper focuses on optimization of some design parameters of such antennas, including the rotation angle, the rotation step number and distances of two dipole couples, in order to improve the ambiguity and accuracy of all estimators which use AWPC structure. These optimization problems are formulated and solved by using Cramer-Rao bound (CRB) and ambiguity checking criteria. Specially, the parameters were optimized to avoid issues related to the ambiguity for half space localization problem while minimizing the CRB for improving the DOA estimation accuracy.**

***Keywords*– Direction of Arrival (DOA), Antenna without Phase Center (AWPC), Multiple Signal Classification (MUSIC), Cramer-Rao Bound (CRB).**

I. Introduction

Direction-of-arrival (DOA) estimation is a well established topic in array signal processing in which directions of emitting sources are estimated from signals collected at a set of sensors [1][2]. Many interesting problems arise within this framework, including the one related to antenna design and optimization considered in this paper.

A class of antennas "*without Phase Center*" have been proposed in [3] for DOA estimation in one incident source case. A phase center of an antenna is the center of a sphere, which considers with the surface of constant phase within a sector of one beam of directional pattern. AWPC structures do not have such phase centers. The MUSIC estimator using AWPC with linear phase pattern was proposed in [4] while extension of the estimator to non-linear phase pattern one as a sensor in uniform circular array (UCA) was proposed in [5]. Unlike normal antenna arrays, such as uniform linear array, UCA, wherein the steering vector is dependent on the spatial locations of the array sensors, in DOA estimation system using AWPC the steering vector is built based on the radiation pattern caused by the electrical rotation of the antenna in steps.

The locations of the sensors in antenna array design in general or in AWPC design in particular are important for DOA estimation because they affect the ambiguity and the estimation accuracy. This paper concentrates on optimization of AWPC sensor locations.

Ambiguity occurs when two or more steering vectors corresponding to widely separated directions are similar [6]. In AWPC structure, this depends on antenna rotation parameters, which are related to the rotation angle and the rotation step number, and radiation pattern, which is related to the distances of two couples dipoles in AWPC. In this paper, we first optimizes the antenna rotation parameters based on the Cramer-Rao bound (CRB) criterion. CRB is a well-known bound which expresses the minimum achievable variance on estimating parameters of any unbiased estimator [7]. In array processing, it depends on the observations (the number of snapshots) and the measurements (Signal-to-Noise ratio (SNR)), the positions of sources, the locations of sensors, but does not depend on a given estimation algorithm [8]. Next, given the optimized antenna rotation parameters, we introduce the CRB and the ambiguity-checking function for all small and moderate-size AWPC structures to find optimal distances of two couples of dipoles. The optimization problem is too complex to find closed-form solution. Therefore, we opt to carry out an exhaustive numerical 2D search. We can afford this exhaustive search since optimization is achieved once for all.

The paper is organized as follows. Section II presents the AWPC structure and the data model. Section III introduces the CRB and the ambiguity-checking function. Section IV shows the optimization solutions for antenna rotation parameters and antenna configuration. Section V provides a numerical example that demonstrates the efficientiveness of the optimized AWPC. Section VI concludes the paper.

II. Antenna without Phase Center and Data Model for DOA Estimation

A. DOA Estimation using the Antenna without Phase Center

The AWPC is illustrated in Fig. 1. The distance between dipoles I-1 and I-2 of the first couple I is called d_1, and that for the second couple is d_2. The dipole couples are perpendicular to each other, i.e, $d_1 \perp d_2$. The relative phases of I-1, II-2 and I-2 with respect to that of II-1 are 90°, 180° and 270°, respectively. Under the above conditions for locations of the dipoles and the feeding points, the amplitude pattern, $G(\theta)$, and the phase pattern, $\Phi(\theta)$, of the AWPC are given by [3]

978-1-4673-2493-9/12/$31.00

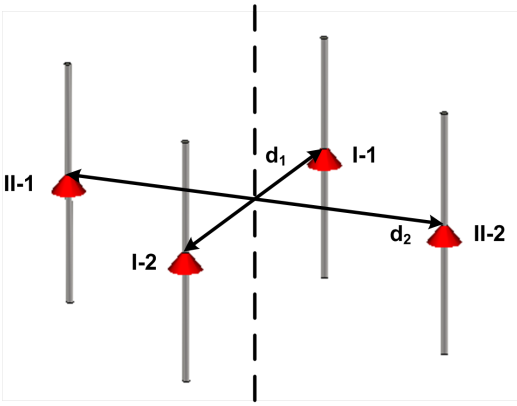
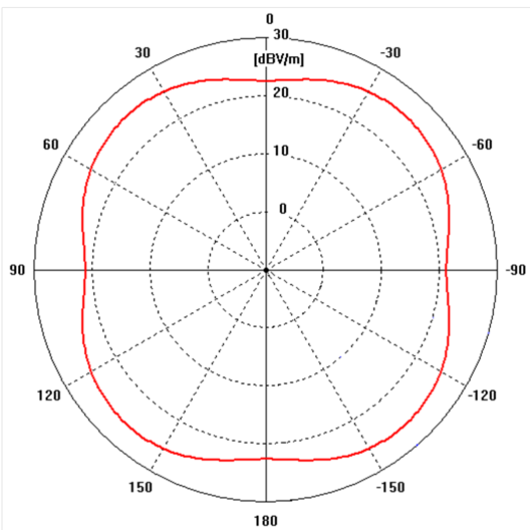
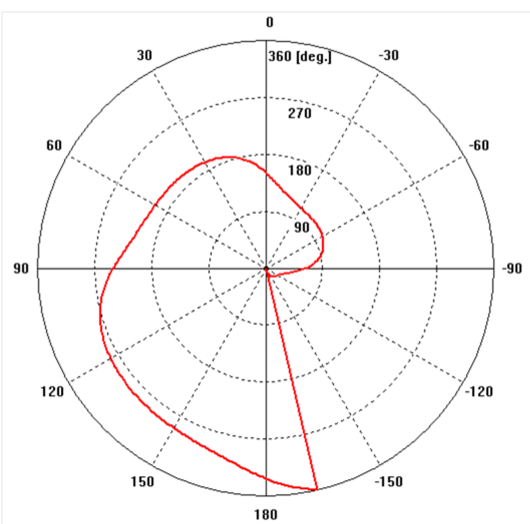


Fig. 1. Structure of the Antenna without Phase Center.



(a)



(b)

Fig. 2. AWPC radiation pattern for $(d_1, d_2) = (0.6, 0.7)\lambda$: (a) Amplitude pattern and (b) Phase pattern.

$$G(\theta) = \sqrt{\sin^2\left(\frac{kd_1}{2}\cos\theta\right) + \sin^2\left(\frac{kd_2}{2}\sin\theta\right)}, \quad (1)$$

$$\Phi(\theta) = \angle\left(\sin\left(\frac{kd_1}{2}\cos\theta\right), \sin\left(\frac{kd_2}{2}\sin\theta\right)\right), \quad (2)$$

where θ is the direction of propagation, k is the wave number of the carrier, and $\angle$ denotes the phase of a complex number. The radiation pattern is rotated by constants of the phases of the signals at all the feeding points. As an illustration, Fig. 2 shows the radiation pattern for $(d_1, d_2) = (0.6, 0.7)\lambda$, where λ is the wavelength.

B. Data model and Analysis

Assume that elevation angle is equal to 90°, consider D uncorrelated, narrowband, zero-mean Gaussian sources, impinging on the antenna: $s_1(t), s_2(t), \ldots, s_D(t)$. The antenna is rotated in M steps in the clockwise direction. At step m $(m = 0, \ldots, M-1)$, the received signal is modeled as

$$x_m(t) = \sum_{i=1}^{D} s_i(t)G(\theta_i + m\Delta\theta)e^{j\Phi(\theta_i + m\Delta\theta)} + n_m(t), \quad (3)$$

where θ_i is the incident angle of the i-th source, $\Delta\theta$ is the antenna rotation angle, and $n_m(t)$ is the spatially zero-mean white Gaussian noise with variance of σ^2 and is statistically independent of the sources. In matrix form, the data model becomes

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \quad (4)$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_D(t)]^T$ is the vector of the sources, $\mathbf{n}(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T$ is the noise vector, $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$ is the received vector, and $\mathbf{A}(\theta)$ is the steering matrix defined by

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_D)], \quad (5)$$

In (5), $\mathbf{a}(\theta_i)$ is the steering vector associated with the i-th source.

$$\mathbf{a}(\theta_i) = \begin{bmatrix} G(\theta_i)e^{j\Phi(\theta_i)} \\ G(\theta_i + \Delta\theta)e^{j\Phi(\theta_i + \Delta\theta)} \\ \vdots \\ G(\theta_i + (M-1)\Delta\theta)e^{j\Phi(\theta_i + (M-1)\Delta\theta)} \end{bmatrix}, \quad (6)$$

The spatial covariance matrix of the output vector is expressed as

$$\begin{aligned} \mathbf{R_x} &= \mathbf{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} \\ &= \mathbf{A}\mathbf{R_s}\mathbf{A}^H + \sigma^2\mathbf{I}, \end{aligned} \quad (7)$$

where $\mathbf{E}\{\cdot\}$ denoted statistical expectation, $\mathbf{R_s}$ is the source covariance matrix.

III. CRB and Ambiguity-Checking Function

This section provides definitions and analysis of the CRB and the ambiguity-checking function, to be used in Section IV.

A. CRB

1) Single-Source CRB: Assume that $\mathbf{p}$ is a parameter vector of an unbiased estimator, according to [10] the mean-square error (MSE) is given by

$$\text{MSE}(\hat{p}_i) = \mathbf{E}\{(\hat{p}_i - p_i)^2\} \geq \text{CRB}(p_i), \tag{8}$$

where p_i and $\hat{p}_i$ denote the i-th element of $\mathbf{p}$ and its estimate $\hat{\mathbf{p}}_i$, repsectively, and

$$\text{CRB}(p_i) = [\mathbf{J}^{-1}]_{ii}, \tag{9}$$

where $\mathbf{J}$ is the Fisher information matrix.
In the single-source case, assuming Gaussian source signal with given N_s independent samples of a zero mean Gaussian process $\mathbf{x}$ whose statistics depend on the parameter vector $\mathbf{p} = \theta$, CRB are given by:

$$\text{CRB}(\theta) = \mathbf{J}^{-1}, \tag{10}$$

where

$$\begin{aligned} \mathbf{J} &= N_s \cdot \text{trace}\left\{\mathbf{R}_\mathbf{x}^{-1}\frac{\partial \mathbf{R}_\mathbf{x}}{\partial \theta}\mathbf{R}_\mathbf{x}^{-1}\frac{\partial \mathbf{R}_\mathbf{x}}{\partial \theta}\right\} \\ &= \frac{2\,\text{SNR}^2}{(1+\text{SNR}\,|\mathbf{a}|^2)^2}[2(\Re(\mathbf{a}^H\dot{\mathbf{a}}_\theta))^2 \\ &+ (1+\text{SNR}\,|\mathbf{a}|^2)(|\mathbf{a}|^2|\dot{\mathbf{a}}|^2 - |\mathbf{a}^H\dot{\mathbf{a}_\theta}|^2)], \end{aligned} \tag{11}$$

where $\dot{\mathbf{a}}_\theta = \frac{\partial \mathbf{a}}{\partial \theta}$.
For simplicity, from now, CRB is computed with $N_s = 1$ only because the results for $N_s > 1$ can be obtained by dividing the CRB values by N_s.

2) Averaged CRB (ACRB): This definition has been used in [8] to compare the performances of antenna arrays with the same aperture and applied for one emitting source case.

$$\text{ACRB} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{J}^{-1}d\theta, \tag{12}$$

where $\mathbf{J}$ is given by equation (11).

B. Ambiguity-Checking Function (ACF)

In subspace-based direction finding methods, estimated DOAs are obtained from the steering vectors. Therefore, large estimation errors occur when widely separated angles correspond to co-linear steering vectors.

To check this error, we use the following criterion:

$$\text{ACF}(\theta_1, \theta_2) = 1 - \frac{|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)|^2}{\|\mathbf{a}(\theta_1)\|^2\|\mathbf{a}(\theta_2)\|^2}, \tag{13}$$

which represents a similarity measure of two steering vectors at directions θ_1 and θ_2 [9]. When the steering vectors are co-linear, ACF is equal to zero.

Also, if $\mathbf{a}(\theta_1)$ is orthogonal to $\mathbf{a}(\theta_2)$, that means $|\mathbf{a}^H(\theta_1)\mathbf{a}(\theta_2)| = 0$, ACF reaches its maximum value of 1.

The two previous criteria (CRB and ACF) are used next as target function in the optimization problems of the antenna design parameters.

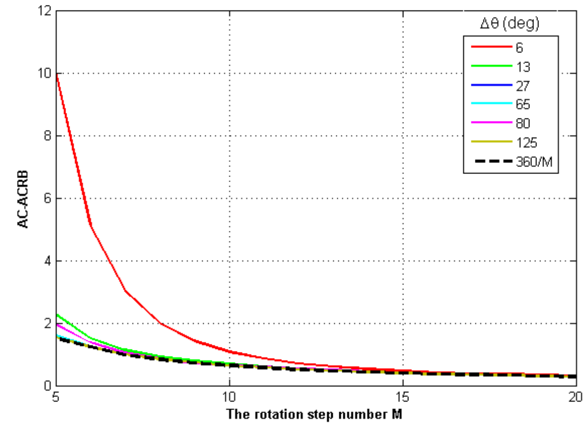


Fig. 3. AC-ACRB versus the number of antenna rotation steps M

IV. Optimization of the AWPC Design Parameters

The performance of a DOA estimator depends highly on the form of the steering vector. In our case, there are three factors impacting our steering vector: antenna rotation angle $\Delta\theta$, number of antenna rotation steps M and antenna configuration (d_1, d_2). Therefore, the optimization of these parameters is the purpose of this section. Direct and full optimization is too complex and not always possible without specific constraints. For these reasons, we proposed a simplified approach that leads to quasi-optimal, or at least good, design solutions by using: (i) numerical optimization instead analytical one, and (ii) separate optimization for different design parameters.

A. Optimization of the antenna rotation parameters: rotation angle $\Delta\theta$ and rotation step number M

In this case, we assume that only one emitting source is impinging on the antenna. To optimize $\Delta\theta$, we introduce the following performance measurement, called the antenna configuration and averaged CRB (AC-ACRB)

$$\text{AC-ACRB} = \frac{1}{K}\sum_{(d_1,d_2)}\text{ACRB}, \tag{14}$$

where K is the number of couples of (d_1, d_2). With some random given $\Delta\theta$ values, we plot AC-ACRB versus the number of antenna rotation steps M in Fig. 3. It is observed that:

- The MSE of DOA estimation decreases as M increases, and all curves converge to the same MSE level when M increases. When $M > 15$, the decreasing of the MSE becomes negligible. We suggest choosing M_{opt} around 15 for a good-enough trade-off between the computational cost and the estimation accuracy.
- The choice $\Delta\theta = 360/M$ leads to the lowest averaged MSE. So, we can assert that this is the optimal value of the antenna rotation angle that we are interested in; that is, $\Delta\theta = 2\pi/M$.

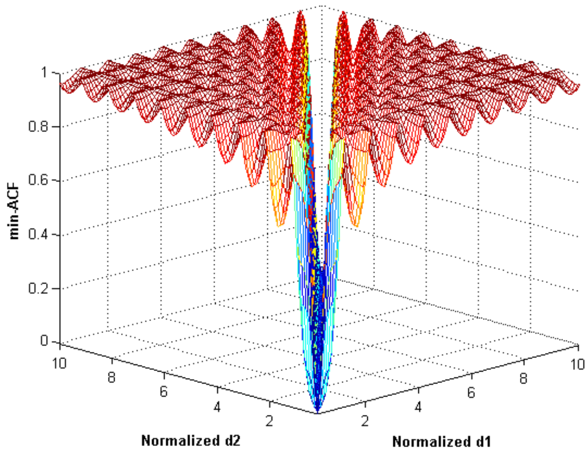


Fig. 4. min-ACF versus d_1, d_2 normalized to λ

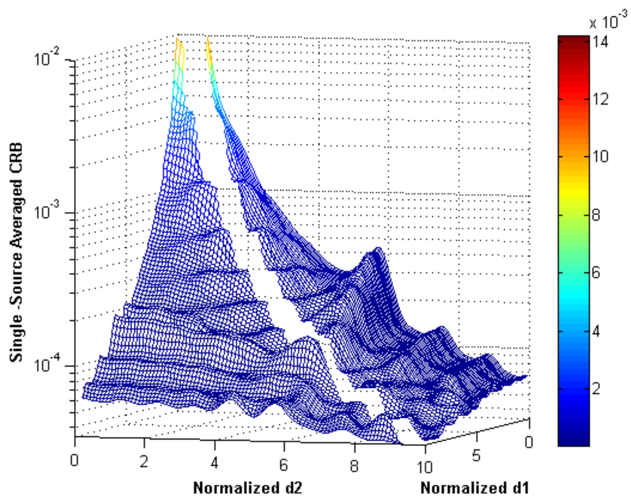


Fig. 5. Single-source Averaged CRB versus d_1, d_2 normalized to λ

B. Optimization of the antenna configuration (d_1, d_2)

The optimization of antenna configuration (d_1, d_2) is obtained by decreasing ambiguity error and increasing estimation accuracy. The optimal values selected above are used in the optimization of (d_1, d_2).

1) Decreasing ambiguity error:
In this part, we use (13) to check the ambiguity for both following cases.
By observing that

$$\begin{cases} G(\theta+\pi) = G(\theta) \\ \Phi(\theta+\pi) = \Phi(\theta) + \pi, \end{cases} \tag{15}$$

due to the natural symmetry of the antenna, we have a π-ambiguity error for all (d_1, d_2). Hence, the AWPC structure does allow only half space localization.
Also, if $d_1 = d_2$ we have another antenna symmetry axis and in that case

$$\begin{cases} G(\theta+\pi/2) = G(\theta) \\ \Phi(\theta+\pi/2) = \Phi(\theta) - \pi/2, \end{cases} \tag{16}$$

which leads to a $\pi/2$-ambiguity error.
The latter can be observed visually in Fig. 4, representing the minimum value of ACF versus (d_1, d_2) for the angle pairs $(\theta_1 \in (\pi/2, -\pi/2, \theta_2 = \theta_1 + \pi/2)$. As we can see, this value is equal to zero if $d_1 = d_2$, and not if otherwise. We also observe that if $|d_1 - d_2| > 0.5\lambda$ then the minimum value of ACF is greater than 0.6. We can assert that the antenna has no ambiguity problem for the half space localization if we choose $|d_1 - d_2| > 0.5\lambda$.

2) Increasing estimation accuracy:
The averaged CRB for single-source case versus (d_1, d_2), shown in Fig. 5, is used to study the DOA estimation accuracy for a given antenna configuration. We observe that the smaller the values of (d_1, d_2) are the higher the CRB.
We also observe that the optimal value of the single-source CRB under the constraints $|d_1 - d_2| > 0.5\lambda$ and $0 \le d_1, d_2 \le 10\lambda$ is obtained for $(d_1, d_2) = (9.4, 10)\lambda$. However, the optimal choice for (d_1, d_2) in practice would depend on the "*authorized*" maximal size of the antenna for the considered application.

V. NUMERICAL SIMULATIONS

In this section, we provide a numerical example that demonstrate efficientiveness of the optimized AWPC with $M = 17$, $\Delta\theta = 2\pi/M$ and $(d_1, d_2) = (5.2, 2.3)\lambda$, correspond to ACF $= 1$ and ACRB $\approx 1.9 * 10^{-5}$. The results are shown in Fig. 6. The dashed lines present origin DOAs while the solid lines present estimate DOA spectrum. Here, we show the spectrum of the MUSIC estimator for two different antena configurations ($d_1 = d_2$) and ($|d_1 - d_2| > 0.5\lambda$), the MUSIC algorithm is described in Table I in details. Six sources are presented at azimuth (-60°, -40°, -20°, 20°, 40°, 60°) and SNRs of 25dB. The MUSIC estimator was applied to 1000 random data snapshots.

The first subfigure shows MUSIC spatial spectrum for the AWPC with $d_1 = d_2 = 2.3\lambda$. This configuration has steering vectors that are colinear at $(\theta_1, \theta_2 = \theta_1 \pm \pi/2)$ ($\pi/2$-ambiguity error) and $(\theta_1, \theta_2 = \theta_1 \pm \pi)$ (π-ambiguity error) and therefore, beside six origin peaks at (-60°, -40°, -20°, 20°, 40°, 60°), twenty four ghost peaks appear at (30°, 50°, 70°, 110°, 130°, 150°), (-150°, -130°, -110°,-70°, -50°, -30°), (120°, 140°, 160°, -20°, -40°, -60°), and (20°, 40°, 60°, -160°, -140°, -120°), correspond to $(\theta_1, \theta_2 = \theta_1 + \pi/2)$, $(\theta_1, \theta_2 = \theta_1 - \pi/2)$, $(\theta_1, \theta_2 = \theta_1 + \pi)$, and $(\theta_1, \theta_2 = \theta_1 - \pi)$.

The second subfigure shows MUSIC spatial spectrum for the AWPC with $d_1 = 5.2\lambda, d_2 = 2.3\lambda$. This configuration has steering vectors that are colinear only at $(\theta_1, \theta_2 = \theta_1 \pm \pi)$ (π-ambiguity error) and therefore, in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ space, the estimate DOAs are the same as the origin DOAs at (-60°, -40°, -20°, 20°, 40°, 60°).

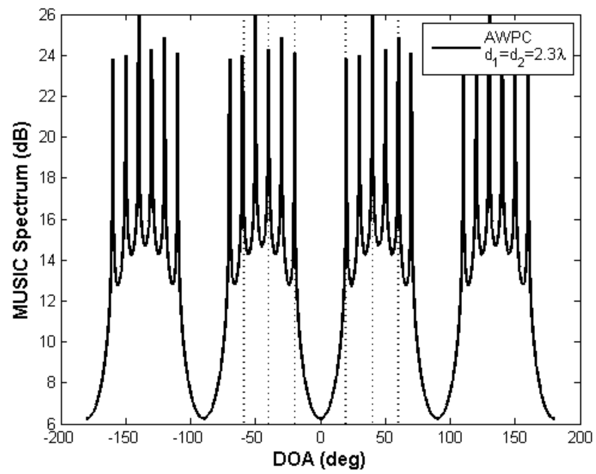
TABLE I
MUSIC ALGORITHM

Compute Spatial Sample Covariance Matrix
$\hat{\mathbf{R}} = \frac{1}{K}\sum_{i=1}^{K}\mathbf{x}(t)\mathbf{x}^H(t)$,
where $t = 1, \ldots, K$ and K is called the number of snapshots.
Eigendecomposition of $\hat{\mathbf{R}}$
$\hat{\mathbf{R}} = \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^H$,
where $\hat{\mathbf{U}}$ is eigenvectors and $\hat{\mathbf{\Lambda}} = diag\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ is a diagonal matrix of real eigenvalues ordered such as $\{\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M > 0\}$, in which $\{\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_D > \sigma^2\}$ and $\{\lambda_{D+1} = \ldots = \lambda_M = \sigma^2\}$.
Determine D incident sources and noise eigenvectors $\hat{\mathbf{U}}_n$
Assumed that $\hat{\mathbf{R}}$ is full rank, based on $M - D$ eigenvalues are equal to σ^2 in $\hat{\mathbf{\Lambda}}$, determine D incident sources and noise eigenvectors $\hat{\mathbf{U}}_n$ (corresponding to $M - D$ eigenvalues are equal to σ^2).
Plot Spatial Spectrum of MUSIC
$P_M(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\mathbf{U}}_n^H\hat{\mathbf{U}}_n\mathbf{a}(\theta)}$,

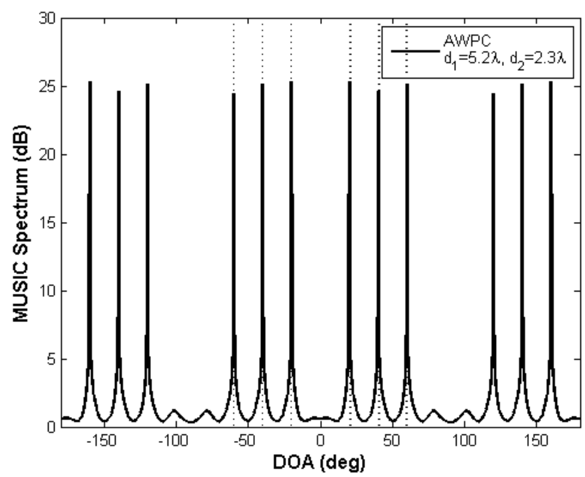
In shorts, the optimized AWPC with $M = 17$, $\Delta\theta = 2\pi/M$ and $(d_1, d_2) = (5.2, 2.3)\lambda$ has no-ambiguity and high accuracy for a half space localization, as shown in Fig. 6b.

VI. CONCLUSIONS

In this paper we have considered the problem of general AWPC design-parameter optimization for DOA estimation using arbitrary estimator. The design parameters (antenna rotation angle, atenna rotation step number and the antenna dipoles distances) were optimized in such a way that we avoid ambiguity issues for a half space localization problem while minimizing the CRB for improving the DOA estimation accuracy. This work can be extended to the multiple-sensor case as well as consider other desired properties like isotropy or antenna resolution.



(a)



(b)

Fig. 6. MUSIC spectrum of the Antenna without Phase Center: (a) AWPC ($d_1 = d_2$), (b) AWPC ($|d_1 - d_2| > 0.5\lambda$).

ACKNOWLEDGMENT

This work is supported by Vietnam National University Hanoi under the TRIG.B project and VNU University of Engineering and Technology under the CN (2012) project.

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