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Solving the multi-vehicle multi-covering tour problem

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ABSTRACT

The well-known multi-vehicle covering tour problem (m-CTP) involves finding a minimum-length set of vehicle routes passing through a subset of vertices, subject to constraints on the length of each route and the number of vertices that it contains, such that each vertex not included in any route is covered. Here, a vertex is considered as covered if it lies within a given distance of at least a vertex of a route. This article introduces a generalized variant of the m-CTP that we called the multi-vehicle multi-covering Tour Problem (mm-CTP). In the mm-CTP, a vertex must be covered at least not only once but several times. Three variants of the problem are considered. The binary mm-CTP where a vertex is visited at most once, the mm-CTP without overnight where revisiting a vertex is allowed only after passing through another vertex and the mm-CTP with overnight where revisiting a vertex is permitted without any restrictions. We first propose graph transformations to convert the last two variants into the binary one and focus mostly on solving this variant. A special case of the problem is then formulated as an integer linear program and a branch-and-cut algorithm is developed. We also develop a Genetic Algorithm (GA) that provides high-quality solutions for the problem. Extensive computational results on the new problem mm-CTP as well as its other special cases show the performance of our methods. In particular, our GA outperforms the current best metaheuristics proposed for a wide class of CTP problems.

1. Introduction

The Vehicle Routing Problem (VRP) is one of the most popular optimization problems. Its objective is to find the optimal set of routes for a fleet of vehicles in order to visit all the customers (Toth et al., 2014). However, in a number of real-world routing applications, we do not need to visit every customer but a subset of them to fulfil their demands. Many variants of the VRP have been introduced and studied in the literature to deal with such situations. For example, the Generalized Vehicle Routing Problem (GVRP) was studied in Bektas et al. (2011) and Hà et al. (2014), the Capacitated Team Orienting Problem (CTOP) and Profitable Vehicle Routing Problem (PVRP) was in Archetti et al. (2009), etc. Interested readers are recommended to Toth et al. (2014) for more details on different variants of the VRP and solution methods.

Another related variant called the multi-vehicle Covering Tour Problem (m-CTP) was introduced in Hachicha et al. (2000). In the m-CTP, among all vertices in a graph, we must select a subset of them to visit such that covering constraints are met. We now give the formal description and applications of the problem and summarize the work related to m-CTP in the literature.

1.1. Multi-vehicle covering tour problem

The m-CTP is defined on an undirected graph $G = (V \cup W, E_1 \cup E_2)$, where $V \cup W$ is the vertex set and $E_1 \cup E_2$ is the edge set. $V = \{v_0, \ldots, v_{n-1}\}$ is the set of n vertices that can be visited, $T \subset V$ is a set of vertices that must be visited and $W = \{w_1, w_2, \ldots, w_l\}$ is the set of l vertices that must be covered. Vertex $v_0 \in T$ is the depot, where m identical vehicles are located. m can be a predefined number or a decision variable. However, in this paper, we consider the case where m is a variable. A length (or cost) c_{ij} is associated with each edge of $E_1 = \{(v_i, v_j) : v_i, v_j \in V, i < j\}$ and a distance d_{ij} is associated with each edge of $E_2 = \{(v_i, v_j) : v_i \in V \setminus T, v_j \in W\}$. The m-CTP consists in finding at most m vehicle routes such that the total cost is minimized and the following constraints are satisfied:

- Each route begins and ends at the depot;
- Each vertex of T is visited exactly once while each vertex of $V \backslash T$ is visited at most once;

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- Each vertex w_j of W is covered by at least a route, i.e., lies within a distance r of at least one vertex of $V\backslash T$ that is visited, where r is the covering radius;
- The number of vertices on each route (excluding the depot) is less than a given value p;
- The length of each route does not exceed a fixed value q.

Main applications of the m-CTP are to model problems concerned with the design of bilevel transportation networks such as the construction of routes for mobile health-care teams (Hodgson et al., 1998; Swaddiwudhipong et al., 1995) and urban patrolling teams (Oliveira et al., 2013), and the location of post boxes (Labbé and Laporte, 1986), banking agencies, milk collection points (Simms, 1989), and relief centers (Doerner and Hartl, 2008). In these applications, a number of distribution centers must be located among a set of candidate sites in such a way that all customers are within reasonable distance from at least one center and that the cost of delivery routes is minimized. As an example, in the disaster relief problem (Doerner and Hartl, 2008), after a disaster the health care organizations have to supply the affected populations with food, water, and medicine. The relief vehicles (e.g., mobile hospitals) stop at several locations and the populations must visit one of the vehicle stops. The health care organizations have to choose appropriate stops among potential locations so that all populations can reach one of these stops within acceptable time and the transportation cost passing through chosen stops is minimized.

The number of published works on the CTPs has been limited despite their numerous potential applications. A special case of the one-vehicle version (1-CTP) was first defined and introduced in Current (1981). They called it the "Covering Salesman Problem (CSP)" and did not distinguish between the visited and covered vertices (i.e. $V \equiv W$). Recently, the CSP was solved by an integer linear programming-based heuristic in Salari and Azimi (2012) and an ant colony optimization algorithm in Salari et al. (2015). The 1-CTP was solved exactly by a branch-and-cut algorithm and approximatively by a heuristic in Gendreau et al. (1997). The authors of Baldacci et al. (2005) used a two-commodity flow formulation and developed a scatter search algorithm.

For the multi-vehicle version (m-CTP), Hachicha et al. (2000) introduced a three-index vehicle flow formulation and three heuristics inspired from classical algorithms: Clarke and Wright (1964), the sweep algorithm (Gillett and Miller, 1974), and the route-first/cluster-second method (Beasley, 1983). The three heuristics were compared to each other, and the optimality gap is therefore unknown. Recently, Jozefowiez (2014) proposed a branch-and-price algorithm. It was based on a column generation approach in which the master problem was a simple set covering problem, and the pricing problem was formulated similarly to the 1-CTP model of Gendreau et al. (1997).

Another exact approach was proposed in Hà et al. (2013) to solve the m-CTP without the length constraints on each route. The problem was called m-CTP-p and the method was a branch-and-cut algorithm based on two-commodity flow formulation strengthen by valid inequalities. Computational results for a set of instances with up to 200 vertices where the tour contains up to 100 vertices showed that, although less general, it outperformed the algorithm of Jozefowiez (2014) in the same context. Also in Hà et al. (2013), a metaheuristic, which is a hybrid of the Greedy Randomized Adaptive Search Procedure (GRASP) and Evolutionary Local Search (ELS), was introduced. The algorithm seemed to be very efficient for solving the m-CTP-p since it provided very good solutions which were within 1.45% of optimality for the considered test instances. Although the authors claimed that their algorithm could solve the general problem m-CTP, they did not report results for it.

Kammoun et al. (2017) solved the m-CTP-p using Variable Neighborhood Search (VNS) heuristic based on Variable Neighborhood Descent (VND) method. Their results outperformed those of Hà et al. (2013). However, the VNS worked only on the special case m-CTP-p and could not solve the general m-CTP.

More recently, the multi-depot covering tour problem was introduced and studied in Allahyari et al. (2015). Two mixed integer programming formulations and a hybrid metaheuristic combining GRASP, iterated local search, and simulated annealing were developed. Flores-Garza et al. (2015) introduced the multi-vehicle cumulative covering tour problem whose motivation arises from humanitarian logistics. In this problem, the goal is not to minimize the cost but the sum of arrival times at visited nodes. A mixed integer linear formulation and a GRASP were proposed for the problem (Flores-Garza et al., 2015).

1.2. Multi-vehicle multi-covering tour problem

All of the earlier generalizations of the m-CTP assume that once a node is covered, its entire demand can be serviced. However, in many real-world situations this is not necessarily the case. For instance, in the disaster relief problem mentioned above, if the demand of some areas is too large and cannot be satisfied by a single coverage, multiple coverages are needed. Consequently, rather than assuming that a node's demand is completely serviced when one of its covering vertices is visited, we generalize the m-CTP by specifying the coverage demand u_k which denotes the number of times a node w_k should be covered. In other words, node w_k must be covered u_k times by visits to nodes that can cover node w_k. A similar generalization for the CSP can be found in Golden et al. (2012) where the authors generalized the CSP without depot by considering that each vertex is covered at least several times. The problem was called the Generalized Covering Salesman Problem (GCSP) and was solved by local search-based heuristics.

In this article, we study the multi-vehicle multi-covering tour problem (mm-CTP) which generalizes the m-CTP in the same way. The problem is defined exactly as in the m-CTP except that each vertex w_j of W is now covered at least u_j times by the routes, i.e., lies within a distance r of at least u_j vertices of V that are visited. The mm-CTP is clearly NP-hard since it reduces to a m-CTP when $u_k = 1, \quad \forall w_k \in W$, or to a GCSP when the capacity constraints are relaxed, i.e., $p, q = \infty$ and $W \equiv V$.

As proposed in Golden et al. (2012), we also define three variants of the mm-CTP. The first, called binary mm-CTP, enforces that each node can be visited by the routes at most once. In the second one, visiting a node v_i more than once is possible, but an overnight stay is not allowed (i.e., to revisit a node v_i, the tour has to visit another node before it can return to v_i). Finally, in the third variant, the tour can visit each node more than once consecutively. In the following, we show that the last two variants can be reduced to the first one by appropriate graph transformations:

- Reduction of the second variant. Let W_i be the set of vertices covered by vertex v_i, we construct a new graph $G_1 = (V_1, E_1)$ by adding, for each node $v_i \in V\backslash T$, co_i copies of v_i to the graph G where $co_i = \max_{w_j \in W_i} u_j - 1$. Let C_i be the set that contains the node v_i and its copies. The length of the new edges is defined as follows. The length of edges whose two endpoints in C_i is set to a very large number in order to forbid the revisit to the node v_i. The length of edges linking a copy of node v_i to a node $v_k \in V_1 \backslash C_i$ is equal to the length of those linking v_i to v_k.
- Reduction of the third variant. We build a new graph G_2 in a similar way as for G_1, except that the length of edges whose two endpoints in C_i is set to zero in order to allow the revisit to node v_i.

The Fig. 1 illustrates our graph transformations on an example with $|V| = 3$ and $|W| = 2$. The number above a node of W represents its number of required coverages.

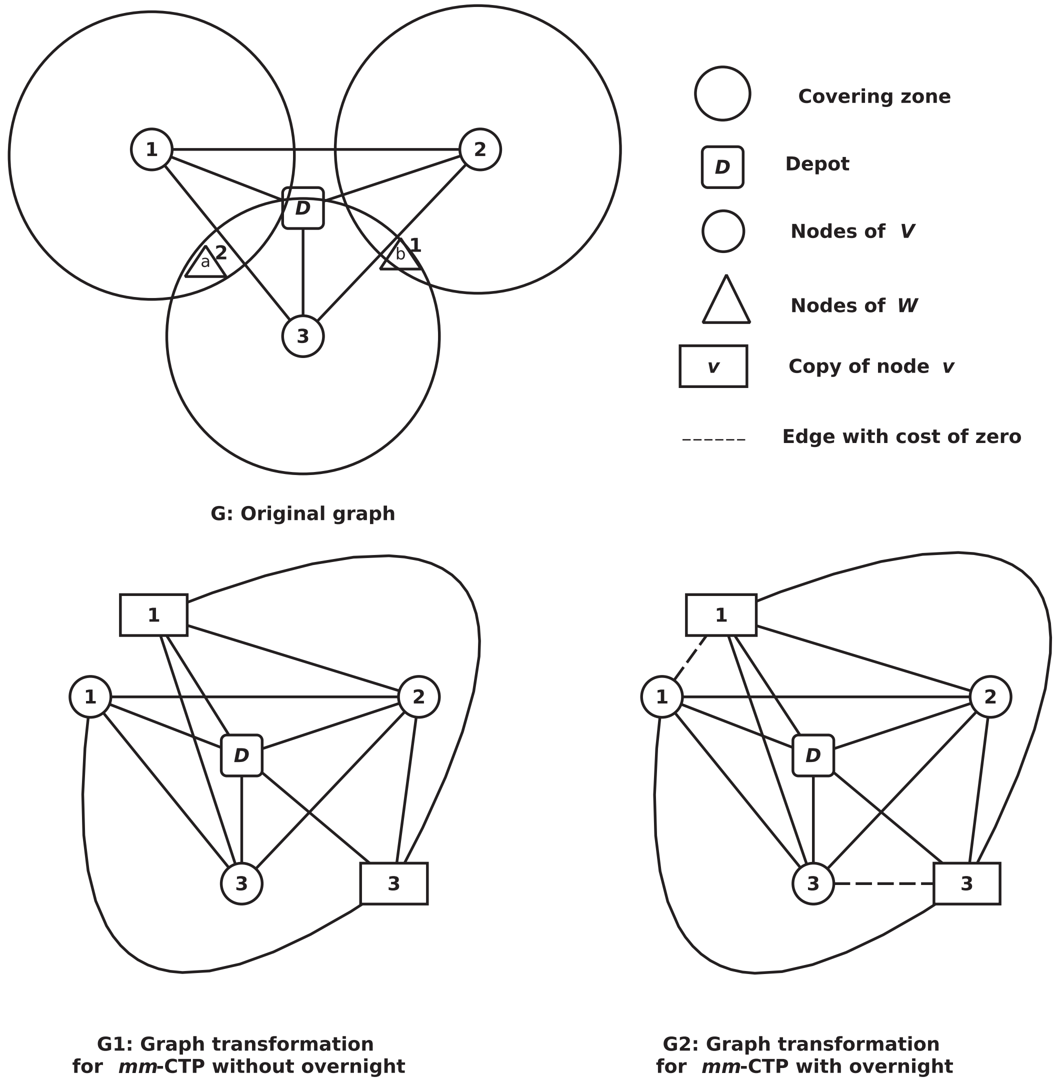

Fig. 1. Example of graph transformations.

Since the two last variants can be reduced to the first one, we mostly focus on solving the first variant. However, we also report and analyse computational results for the remaining variants. The main contributions of the article are:

- We introduce the *mm*-CTP, a new variant of the CTPs which generalizes several existing problems.
- We propose an exact method for a special case of the variant
- We propose a GA-based metaheuristic for the general problem (*mm*-CTP).
- We conducted extensive computational experiments and the results show that our exact method can solve problem instances up to 50 vertices of V and our metaheuristic gives very high quality solutions. More remarkably, the new genetic algorithm outperforms current state-of-the-art algorithms, namely, GRASP-ELS (Hà et al., 2013) on all six variants of *mm*-CTP, and VNS (Kammoun et al., 2017) on the *m*-CTP-*p* variant.

The remainder of the paper is organized as follows. Section 2 describes our problem formulation, several valid inequalities and the branch-and-cut algorithm. Our metaheuristic is presented in Section 3. Section 4 discusses the computational results, and Section 5 summarizes our conclusions.

2. Mathematical formulation and exact method

To formulate the *mm*-CTP, one can adapt the formulation with three-index variables proposed for the *m*-CTP by Hachicha et al. (2000). However, branch-and-cut algorithms based on three-index formulations are only capable of solving very small-size instances because of symmetries among vehicle indices. In this section, we describe an integer programming formulation with two-index variables to solve a special case of the *mm*-CTP in which the constraints on the length of each route are relaxed (i.e. $q = +\infty$). We name the problem *mm*-CTP-*p* and develop a branch-and-cut algorithm based on the mathematical formulation which can solve to optimality instances with up to 50 vertices of V. Solutions of the branch-and-cut algorithm are used to analyse the complexity of the problem as well as the performance of the metaheuristics.

The idea underlying the formulation was first introduced in Finke et al. (1984) for the traveling salesman problem (TSP). Langevin et al. (1993) extended this approach to solve the TSP with time windows. Baldacci et al. (2004) used this method to derive a new formulation and a branch-and-cut for the VRP, and Baldacci et al. (2005) adapted it to formulate the 1-CTP without the capacity constraints. Our formulation is an extension of the model proposed for the *m*-CTP-*p* in Hà et al. (2013).

The original graph G is first extended to $\overline{G} = (\overline{V} \cup W, \bar{E_1} \cup E_2)$ by adding a new vertex v_n, which is a copy of the depot v_0. We have $\overline{V} = V \cup \{v_n\}$, $V' = \overline{V} \setminus \{v_0, v_n\}$, $\overline{E} = E_1 \cup \{(v_i, v_n), v_i \in V'\}$, and $c_{in} = c_{0i}$, $\forall\ v_i \in V'$.

This formulation requires two flow variables, f_{ij} and f_{ji}, to represent an edge of a feasible *mm*-CTP solution along which the vehicle initially carries a load of p units. When a vehicle travels from v_i to v_j, flow f_{ij} represents the number of vertices that can still be visited and flow f_{ji} represents the number of vertices already visited (i.e., $f_{ji} = p - f_{ij}$).

Let x_{ij} be a $0-1$ variable equal to 1 if edge $\{v_i,\ v_j\}$ is used in the solution and 0 otherwise. Let y_i be a binary variable that indicates the presence of vertex v_i in the solution. We set the binary coefficients λ_{ik} equal to 1 if and only if $w_k \in W$ can be covered by $v_i \in V \backslash T$. Then *mm*-CTP can be stated as:

$$\text{Minimize} \quad \sum_{\{v_i, v_j\} \in \overline{E}} c_{ij} x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{v_i \in V \backslash \{v_0\}} \lambda_{ik} y_i \geq u_k, \qquad \forall w_k \in W \tag{2}$$

$$\sum_{v_i \in \overline{V}, i<k} x_{ik} + \sum_{v_j \in \overline{V}, j>k} x_{kj} = 2y_k, \qquad \forall v_k \in V' \tag{3}$$

$$\sum_{v_j \in \overline{V}} \left(f_{ji} - f_{ij}\right) = 2y_i, \qquad \forall v_i \in V' \tag{4}$$

$$\sum_{v_j \in V'} f_{0j} = \sum_{v_i \in V'} y_i \tag{5}$$

$$\sum_{j \in V'} f_{nj} = mp \tag{6}$$

$$f_{ij} + f_{ji} = p x_{ij}, \qquad \forall \left\{v_i, v_j\right\} \in \overline{E} \tag{7}$$

$$f_{ij} \geq 0,\ f_{ji} \geq 0, \qquad \forall \left\{v_i, v_j\right\} \in \overline{E} \tag{8}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall \left\{v_i, v_j\right\} \in \overline{E} \tag{9}$$

$$y_i \in \{0, 1\}, \qquad \forall v_i \in V' \tag{10}$$

$$m \in \mathbb{N} \tag{11}$$

where m is the decision variable indicating the number of used vehicles in the solution.

The objective (1) is to minimize the total travel cost. Constraints (2) ensure that every customer w_k of W is covered at least u_k times, while constraints (3) enforce that each vertex of V' is visited at most once. Constraints (4)–(7) define the flow variables. Specifically, constraints (4) state that the inflow minus the outflow at each vertex $v_i \in V'$ is equal to 2 if v_i is used and to 0 otherwise. The outflow at the source vertex v_0 (5) is equal to the total demand of the vertices that are used in the solution, and the inflow at the sink v_n (6) corresponds to the total capacity of the vehicle fleet. Constraint (7) is derived from the definition of the flow variables. Constraints (8)–(11) define the variables.

The linear relaxation of *mm*-CTP can be strengthened by the addition of valid inequalities. The following valid inequalities are directly derived from the definition of the binary variables x_{ij} and y_i:

$$x_{ij} \leq y_i \text{ and } x_{ij} \leq y_j \ (v_i \text{ or } v_j \in V \setminus T) \tag{12}$$

The following flow inequalities, which were introduced in Baldacci et al. (2005) are also valid for the problem:

$$f_{ij} \geq x_{ij}, f_{ji} \geq x_{ji} \text{ if } v_i, v_j \neq v_0 \text{ and } v_i, v_j \neq v_n. \tag{13}$$

Dominance inequalities can also be derived, based on covering considerations. Let v_i, $v_j \in V \backslash \{v_0\}$, vertex v_i is said to dominate v_j if v_i can cover all the vertices of W that v_j can cover. Define W_j the set of all vertices covered by v_j. Then the following dominance constraints follow immediately:

$$y_i + y_j \leq \max_{w_k \in W_j} u_k \tag{14}$$

In the dominance inequalities (15), this dominance relation is extended to three vertices where a set of two vertices v_i, v_t dominates vertex v_j. We have

$$y_i + y_j + y_t \leq 1 + \max_{w_k \in W_j} u_k \tag{15}$$

Eventually, the inequalities (14) and (15) are useful only when $\max_{w_k \in W_j} u_k$ is equal to 1.

All the valid inequalities of the set multi-covering polytope $conv\{y\colon\ \Sigma b_k y_k \geq u_k, y_k \in \{0,\ 1\}\}$ where b_k is the binary coefficient, are valid for *mm*-CTP-*p*. Here we extend the facets with coefficients in {0,1,2} proposed by Balas and Ng (1986) for the set covering polytope which is a particular case of the set multi-covering polytope with $u_k = 1, \forall k$. Let S be a nonempty subset of W and define for each $v_i \in V \backslash T$ the coefficient

$$\alpha_i^S = \begin{cases} 0 \text{ if } v_i \text{ does not cover any vertex in } S, \text{i.e. } \lambda_{ik} = 0 \\ \qquad \text{for all } w_k \in S \\ 2 \text{ if } v_i \text{ covers all vertices in } S, \text{i.e. } \lambda_{ik} = 1 \text{ for all } w_k \in S \\ 1 \text{ if } v_i \text{ covers some but not all vertices in } S. \end{cases}$$

Then the following inequality is valid for *mm*-CTP-*p*

$$\sum_{v_i \in V \backslash T} \alpha_i^S y_i \geq \left\lceil \frac{\sum_{w_k \in S} u_k}{|S| - \epsilon} \right\rceil. \tag{16}$$

where ϵ is a real number and $\epsilon > 0.5$.

It is easy to see that the inequality (16) is indeed obtainable from the covering constraints (2) by the following procedure:

- Add $|S|$ inequalities $\sum_{v_i \in V \backslash T} \lambda_{ik} y_i \geq u_k, w_k \in S$; we have $\sum_{v_i \in V \backslash T} (\sum_{w_k \in S} \lambda_{ik}) y_i \geq \sum_{w_k \in S} u_k$;
- Divide the resulting inequality by $|S| - \epsilon$;
- Round up all coefficients to the nearest integer. The resulting coefficient of the variable y_i will be equal to α_i^S. Indeed, the coefficient is equal to 0 if v_i does not cover any vertex in S. It is rounded up to 1 if v_i partially covers the vertices in S; and to 2 if v_i covers all the vertices in S.

Based on the formulation above, we develop a branch-and-cut procedure to solve the problem to the optimality. We solve a linear program containing the constraints (1)–(8). We then search for violated constraints of type (12)–(15) and (16), and the detected constraints are added to the current LP, which is then reoptimized. This process is repeated until all the constraints are satisfied. If there are fractional variables, we branch. If all the variables are integer, we explore another tree node. Our branch-and-cut algorithm is built around CPLEX 12.6 with the Callable Library. All parameters are set to their default values.

The separation of the constraints of type (12), (14), (15) and (13) is straightforward. For constraints (16), to reduce the computational effort we verify only the sets S that include three elements.

3. Metaheuristics for finding approximate solutions

In this section, we present a genetic algorithm for solving efficiently the *mm*-CTP. Because the problem is new, there is no available metaheuristic for the comparison. As can be seen in the experimental section, we mainly use the GRASP-ELS proposed in Hà et al. (2013) as a reference method to assess the performance of the GA. Since the GRASP-ELS was designed to solve the *m*-CTP-*p*, a special case of the *mm*-CTP, several components of the algorithm must be modified when dealing with the considering problem. We next describe these modifications.

3.1. GRASP-ELS algorithm

The GRASP-ELS is constructed in two phases. The aim of the first phase is to randomly generate a number of subsets of $V\backslash T$ such that each subset meets the covering requirements. This can be done by solving the following mixed integer programming problems:

$$\text{Minimize} \quad \sum_{v_i \in V\backslash T} b_i y_i \tag{17}$$

$$\text{subject to} \quad \sum_{v_i \in V\backslash T} \lambda_{ij} y_i \geq u_j, \qquad \forall w_j \in W \tag{18}$$

$$y_i = \{0, 1\}, \qquad \forall v_i \in V \setminus T \tag{19}$$

where b_i is a random integer varying from 1 to a given number B.

Let θ_1 be the number of mixed integer programs solved in the first phase. After each program is solved, the vertices corresponding to the variables y_i equal to 1 in the solutions of the model above combined with the vertices of T create a set of the vertices that must be visited. The problem now becomes a distance-constrained VRP with unit demands, and it is solved in the second phase by an algorithm based on the ELS method of Prins (2009). We apply the ELS due to its simplicity, speed, and good performance. In this method, a single solution is mutated to obtain several children that are then improved by local search. The next generation is the best solution among the parent and its children. As in Hà et al. (2013), four local search operators are used: swap two nodes, relocate a node, combine two routes and try a new node. The three former operators are re-applied without any modification but in the latter operator, we must take into the account the multi-covering requirements while trying to replace a node in a solution by a new one.

The value of B is selected so that it is neither too large nor too small. If it is too large, the number of vertices in the solution may be more than necessary. By necessary, we mean the number of visited vertices in the optimal solution. On the contrary, if B is too small, the number of visited vertices may be less than necessary. Both of these cases often lead to sub-optimal solutions. In Hà et al. (2013), B was set to 2 and θ_1 was set to $5(|V| - |T|)$. Our tests show that the algorithm can give slightly better solutions if we increase the values of θ_1 to $10|V|$ and B to 3. Therefore, we use these parameter values in our algorithm. For more details about the GRASP-ELS (settings, implementation and parameters), we recommend readers to Hà et al. (2013).

The aforementioned GRASP-ELS is relatively simple and has very few parameters. However, it also has several disadvantages. First, the disconnection between the two phases makes it rather random. Moreover, since a solution created from a GRASP iteration could be completely different from previous ones, the algorithm cannot utilize historical information to find better solutions. Therefore, it is not easy to design a mechanism which guides the algorithm exploring more potential search spaces. This gives us motivation to build a GA for the *mm*-CTP.

3.2. Genetic algorithm

Genetic algorithm (GA) is an adaptive approach inspired by the natural evolution of biological organisms. In GA, an initial population of individuals (chromosomes) evolves through generations until some criteria of quality are satisfied. New individuals (children) are generated from individuals forming the current generation (parents) by means of genetic operators (crossover and mutation). To date, GA and GA-based hybrid algorithms have become state-of-the-art metaheuristics for solving many variants of VRP (Vidal et al., 2012; 2014). However, to the best of our knowledge, GA has not been applied to the CTP problems. This section describes a method using GA with Variable Length Genomes called GA-VLG to solve the *mm*-CTP. Our method reuses some ideas of Unified Hybrid Genetic Search (UHGS), the current state-of-the-art algorithm for many VRP variants (Vidal et al., 2014). We now describe the general structure of the UHGS.

3.2.1. Unified Hybrid Genetic Search

The UHGS is a general framework for VRPs that hybridizes genetic algorithms with local search operators (i.e it is similar to memetic algorithms). It maintains two distinct sub-populations, one for feasible solutions, and the other for infeasible solutions. The size of each sub-population is in the range from μ_{min} to μ_{max} and determined by a survivor selection procedure that is triggered when the size of a sub-population reaches a maximal size μ_{max}. Each individual is represented as a giant tour without trip delimiters (Prins, 2004). This giant tour is then converted to an explicit feasible solution by using a *Split* algorithm (Prins et al., 2008). The computation of the individual fitness is based not only on the total cost (distance/duration) of the tours, but also on its feasibility by using penalty coefficients associated to capacity constraints as well as on its contribution to the population diversity. The penalty parameters are adaptively changed during the search to achieve a ratio of feasible solutions within a predefined interval.

The main steps of UHGS and their brief description are shown in Algorithm 1. The algorithm evolves the population by iteratively

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Algorithm 1: Main steps in UHGS

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population initialization ;
while *stopping conditions are not satisfied* **do**
  Selection: Select two individuals as parents via a binary tournament based on the biased fitness measure;
  Mating: Create an offspring from these parents by crossovers. It generates new giant tours which inherit common characteristics from both parents while introducing a significant level ofrandomness.
  Education: Educate the offspring by local searches. It is applied to any new offspring and is the main force which improves solutions.
  Repair: Repair an offspring with a probability of 0.5 if it is infeasible. This operator simply consists in running the local searches with higher penalty values with the aim of converging towards a feasible solution.
  Population management: Insert the offspring into an appropriate sub-population. The survivor operator is triggered if the sub-population size exceeds μ_{max}.
  Diversification: Keep only 1/3 best individuals in each sub-population and reinserts new random initial solutions after a given number of consecutive iterations without improvement of the best solution. This operator is to avoid a premature convergence of the method due to elitism.
return the best individual ;

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executing operators: Selection, Mating, Education, Repair, Population management and Diversification. It terminates and returns the best feasible individual after processing $Iters_{max}$ iterations without any improvement, or when the running time exceeds a time limit. For more details on UHGS, the readers are recommended to Vidal et al. (2012, 2014).

3.2.2. Genetic algorithm with variable length genomes (GA-VLG)

In the standard GAs including the UHGS, feasible solutions generally are encoded in chromosomes with a fixed length. However, *mm*-CTP solutions could have different number of visited vertices. This requires us to make a number of modifications when adapting the UHGS to mainly deal with variable length genomes. Consequently, the new features of GA-VLG compared to UHGS are:

- **Individual representation:** Our system has to use a variable length representation. That is, the length of each individual is not fixed during the evolutionary process.
- **Crossover operator:** Most of popular crossovers for VRPs, such as Order Crossover (OX), Partial Mapped Crossover (PMX), etc., cannot be directly applied for the variable length representation. Therefore, we propose two new crossover operators adapted from OX namely Shaking Order Crossover (SOX) and Modified Order Crossover (MOX). SOX creates an offspring by replacing at most nb_{SOX} successive vertices from a random position of the shorter individual with those of the longer individual according to the OX's mechanism. The experiments show that the value of nb_{SOX} equal to 3 leads to the best performance of our method. However, making relatively *small* changes in many situations might trap the search in local optima. Therefore, we combine SOX with MOX, a crossover that could make *larger* changes. This crossover is similar to OX except that two crossover points are chosen from shorter individual and an offspring has the same length as shorter individual. Algorithms 2 and 3 describe in more details SOX and MOX respectively. Fig. 2 depicts two examples of our crossovers. Gray rectangles represent the genes copied from a parent to the offspring.

Algorithm 2: Shaking Order Crossover (SOX)

```
Data: smallInd, bigInd
/* smallInd, bigInd are respectively shorter and longer
   chromosomes                                          */
Result: offspring
Let p1, p2(p1 < p2) are crossover points between 0 to len
(smallInd) ;
if p2 − p1 > 3 then
    p2 = p1 + random number in [0, 3] ;
offspring = copy of smallInd ;
offspring[i] = 0, i = p1..p2 ;
for i from p1 to p2 do
    Find a vertex v in bigInd starting from i, v ∉ offspring ;
    offspring[i] = v ;
return (offspring) ;
```

- **Local search:** The UHGS used classical local searches (Vidal et al., 2012; 2014) which did not change the length of chromosomes. This might lead to a poor performance of our algorithm if they are used without the support of other local searches dealing with an additional decision layer in the *mm*-CTP that requires to select visited vertices. Hence, we propose to add three simple but effective local search operators as follows: ***(1) Add***: a vertex that is not in a solution is added into that solution, ***(2) Remove***: a visited vertex is removed from a solution, and ***(3) Swap***: it swaps a vertex in a solution with another vertex that is not in the solution. These operators also help us to remove redundant vertices without which the solution is still feasible with regard to the covering constraints. All neighborhoods are explored in a random order with a first improvement move acceptance policy. The LS search stops when no improving move can be found in the entire neighborhood, and the resulting solution is transformed back to a giant tour, which is then inserted into the corresponding populations.

Algorithm 3: Modified Order Crossover (MOX)

```
Data: smallInd, bigInd
Result: offspring
Let p1, p2(p1 < p2) are crossover points between 0 and len
(smallInd) ;
Initialise an offspring with size equal to len (bigInd) ;
offspring[i] = 0, i = 0, .., len(offspring) ;
Copy vertices in [p1, p2] from smallInd to offspring ;
pos = p2 + 1 ;
bigSize = len(bigInd) ;
for i = 1 to len(bigInd) do
    v = bigInd[(p2 + i)%bigSize] ;
    if v ∉ offspring then
        offspring[pos%bigSize] = v ;
        pos + + ;
    if pos == p1 then
        break ;
return (offspring) ;
```

- **Fitness evaluation:** Our algorithm maintains not only feasible solutions, but also infeasible solutions. Therefore, the fitness of each individual has to take into account its feasibilities regarding not only two capacity constraints but also the covering constraints.
  For any route σ with distance $\varphi^D(\sigma)$, load $\varphi^Q(\sigma)$, and length $\varphi^L(\sigma)$, define $\phi(\sigma)$ - the cost of a route σ as in Eq. (20), where ω^Q, and ω^L are the penalty coefficients for the load and length violations.

$$\phi(\sigma) = \varphi^D(\sigma) + \omega^Q \max(0, \varphi^Q(\sigma) - Q) + \omega^L \max(0, \varphi^L(\sigma) - L) \tag{20}$$

For any solution S with a set of routes $\Re$, the covering violation $\Delta^C(S)$ and the fitness of S denoted by $F(S)$, are calculated as in Eqs. (21) and (22) respectively, where ω^C is the penalty coefficient for the covering violation.

$$\Delta^C(S) = \sum_{w_k \in W} \max(0, u_k - \sum_{v_i \in S} \lambda_{ik}) \tag{21}$$

$$F(S) = \sum_{\sigma \in \Re} \phi(\sigma) + \omega^C \Delta^C(S) \tag{22}$$

- **Adaptive penalty coefficients:** Penalty coefficients of the UHGS are adaptively changed according to the ratio of feasible solutions. In GA-VLG, we also update regularly these penalty coefficients every 100 iterations. Let $\epsilon_{\min}$ and $\epsilon_{\max}$ are the minimum and maximal ratios of new feasible individuals, ϵ_X is the ratio of feasible solutions with respect to constraint X (X can be Q, L, C for capacity, load, covering constraint respectively). These penalty coefficients are updated as follows:

$$\omega^X = \begin{cases} \omega^X \times 1.2, & if \quad \epsilon_X \le \epsilon_{\min} \\ \omega^X \times 0.8, & if \quad \epsilon_X \ge \epsilon_{\max} \end{cases} \tag{23}$$

In our experiments, $\epsilon_{\min} = 0.4^{1/|W|}$ and $\epsilon_{\max} = 0.6^{1/|W|}$

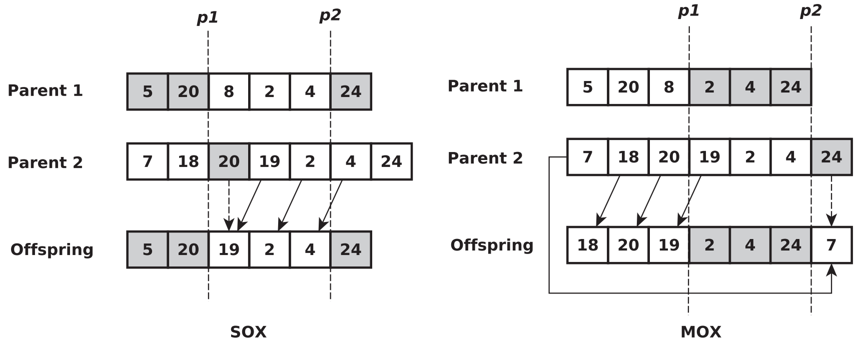

Fig. 2. Examples of SOX and MOX.

4. Computational Experiments

In this section, we describe the *mm*-CTP instances and the computational evaluation of the proposed algorithms. All algorithms are coded in C/C++ and run on a 2.4-GHz Intel Xeon. For the GA-VLG, we use two crossovers SOX and MOX with probability of 0.8 and 0.2, respectively. The termination criteria is $Iters_{max} = 30,000$ for the *mm*-CTP variant, and $Iters_{max} = 20,000$ for other variants.

4.1. Data instances

We now explain the way to generate instances for the *mm*-CTP to test the algorithms proposed in the previous sections. The instances of TSPLIB are first used to build the instances for the *m*-CTP as described in Hà et al. (2013). More precisely, the instances kroA100, kroB100, kroC100, and kroD100 are first used to create a set of nb_{total} = |*V*| + |*W*| = 100 vertices. Tests are run for $|V| = \lceil 0.25 nb_{total} \rceil$ and $\lceil 0.5 nb_{total} \rceil$ and $|T| = 1$ and $\lceil 0.20n \rceil$, and *W* is defined by taking the remaining vertices. The covering radius c_{ij} are computed as the Euclidean distances between the vertices. The value of *c* is determined so that each vertex of $V \backslash T$ covers at least one vertex of *W*, and each vertex of *W* is covered by at least two vertices of $V \backslash T$ (see Gendreau et al., 1997; Hà et al., 2013; Jozefowiez, 2014 for further information). The value of *p* is set to {4, 5, 6, 8}. As in Jozefowiez (2014), the value of maximal route length *q* is computed by the formula $q = \beta + \rho$ where $\beta = 2 \times \max_{i \in V \backslash \{0\}} c_{0,i}$ and $\rho = \{250, 500\}$. We also use instances kroA200 and kroB200 with $nb_{total} = 200$ vertices to generate larger instances for *mm*-CTP.

And finally, the number of coverages u_k for each vertex w_k of *W* is created as followed. Let nb_k be the maximal number of nodes in *V* which can cover w_k. Then the number of coverages u_k can be set to a random integer in the interval from 1 to min(3, nb_k) in order to ensure that the generated instances are feasible in term of covering constraints.

As a result, we have 192 instances for the *mm*-CTP, each is labeled as $X-|T|-|V|-|W|-|p|-|q|$, where X is the name of the TSPLIB instance and the remaining labels are self-explained. In some situations, the labels without the value of *q* implicate instances for the case in which the constraints on the route length are relaxed.

4.2. Results for the branch-and-cut algorithm

This subsection presents the results of the branch-and-cut algorithm for the problem without the length constraints *mm*-CTP-*p*. To accelerate the solving process, we integrate into the algorithm the best solutions found by the metaheuristics as the initial upper bounds on the objective function. The running time of the branch-and-cut algorithm is limited to 2 hours for each instance. During the solution process, we observe that the cuts of type (14)–(16) are very rarely activated on the tested instances. Therefore, we do not use these cuts in our branch-and-cut algorithm. Moreover, the experiment shows that our algorithm cannot solve the instances with more than 50 vertices of *V*. As a result, we report the results for only the instances with $|V| \leq 50$. We also present the best solutions provided by the metaheuristics GRASP-ELS and GA-VLG

In the tables of results, the blank entries indicate that the algorithm could not solve an instance to optimality, and the bolded entries in GAP column indicate the better solutions. The column headings are as follows:

- Data: name of instance.
- *m*: number of vehicles in solution;
- *Nv*: number of vertices of *V* visited by the route in solution;
- Search tree: number of nodes in search tree of branch-and-cut algorithm;
- Time: total running time in seconds.
- Result: objective value of solution;
- GAP: deviation between solution of metaheuristic and the best lower bound found by CPLEX. A solution is proved optimal if its GAP is equal to zero.

Table 1 shows that our branch-and-cut algorithm can solve 69 out of 80 instances with 50 vertices of *V*. Since a similar exact method could solve almost all *m*-CTP-*p* instances with up to 100 vertices (Hà et al., 2013), this indicates that the *mm*-CTP-*p* is much more difficult than the *m*-CTP-*p*. This can be explained by the fact that, in the *mm*-CTP-*p*, we need to visit more vertices of *V* to satisfy the covering constraints. Hence, the "underlying" model with regard to routing aspect is larger and CPLEX needs more computational time.

The problem difficulty increases with *n* and the instances with $p = 4$ or 5 are usually solved more readily than the instances with higher *p* values. These are similar to problems 1-CTP in Gendreau et al. (1997) and *m*-CTP-*p* in Hà et al. (2013). Another interesting observation is that the difficulty depends on the size of *W*. This is contrary to *m*-CTP-*p* when the dependency is not clear (see Hà et al., 2013). Moreover, for the *m*-CTP-*p* and the 1-CTP, the greater the value of |*T*|, the harder the problem. But the hardness of the *mm*-CTP-*p* is fairly insensitive to |*T*|. In many cases, for example A2–1–50–150–4 and B1–1–50–50–8 etc., the augmentation of |*T*| makes the instances easier to solve.

The results also confirm the high quality of solutions provided by the metaheuristics. The branch-and-cut algorithm provides the optimal solution for 69 instances and our methaheuristics can find all these solutions. In addition, for all the instances, the branch-and-cut algorithm can not give any solution better than one of the metaheuristics and we believe that the large GAP in some cases are due to the poor quality of the lower bounds. Two metaheuristic methods are here very competitive but GA-VLG is

Table 1
Computational results of branch-and-cut algorithm on instances with $|V| \leq 50$.

Data	Branch-and-Cut					GRASP-ELS			GA-VLG		
	m	*Nv*	Search tree	Time	Result	Result	GAP	Time	Result	GAP	Time
A1–1–25–75–4	5	17	4	1.0	17,774	17,774	0.00	56.8	17,774	0.00	178.3
A1–1–25–75–5	4	17	1016	4.1	15,793	15,793	0.00	61.9	15,793	0.00	177.5
A1–1–25–75–6	3	17	11,222	16.4	14,628	14,628	0.00	59.1	14,628	0.00	178.3
A1–1–25–75–8	3	17	0	0.8	12,590	12,590	0.00	60.4	12,590	0.00	179.2
A1–1–50–50–4	6	23	3004	17.6	21,473	21,473	0.00	860.5	21,473	0.00	283.1
A1–1–50–50–5	5	23	43,127	348.4	18,680	18,680	0.00	898.8	18,680	0.00	287.9
A1–1–50–50–6	4	23	930,785	5796.5	17,481	17,481	0.00	944.4	17,481	0.00	285.0
A1–1–50–50–8	3	23	52,200	355.9	14,380	14,380	0.00	965.4	14,380	0.00	278.4
A1–10–50–50–4	7	28	5472	35.0	25,340	25,340	0.00	1166.0	25,340	0.00	298.6
A1–10–50–50–5	6	28	3165	18.0	21,712	21,712	0.00	1133.2	21,712	0.00	309.5
A1–10–50–50–6	5	28	1,005,828	5326.7	20,125	20,125	0.00	1144.1	20,125	0.00	300.3
A1–10–50–50–8	–	–	1,064,416	7200.0	–	17,603	3.66	1253.4	17,603	3.66	309.0
A1–5–25–75–4	3	11	28	0.7	13,082	13,082	0.00	20.8	13,082	0.00	159.8
A1–5–25–75–5	3	11	234	1.5	11,969	11,969	0.00	21.7	11,969	0.00	159.8
A1–5–25–75–6	2	11	4274	9.8	11,746	11,746	0.00	21.1	11,746	0.00	162.5
A1–5–25–75–8	2	11	0	0.6	9081	9081	0.00	21.5	9081	0.00	155.1
A2–1–50–150–4	–	–	706,135	7200.0	–	23,601	2.07	798.4	23,601	2.07	533.4
A2–1–50–150–5	–	–	932,839	7200.0	–	20,439	1.78	835.1	20,439	1.78	617.6
A2–1–50–150–6	–	–	785,820	7200.0	–	18,410	4.41	829.2	18,410	4.41	493.0
A2–1–50–150–8	–	–	676,066	7200.0	–	15,565	3.42	768.6	15,502	**3.02**	371.1
A2–10–50–150–4	7	26	78,061	375.9	25,702	25,702	0.00	1072.5	25,702	0.00	380.9
A2–10–50–150–5	5	25	10,634	47.4	21,503	21,503	0.00	1046.2	21,503	0.00	369.1
A2–10–50–150–6	–	–	1,122,196	7200.0	–	20,250	2.18	1126.1	20,250	2.18	353.1
A2–10–50–150–8	4	25	46,422	214.7	16,676	16,676	0.00	1091.2	16,676	0.00	354.5
B1–1–25–75–4	4	16	148	3.9	17,417	17,417	0.00	71.6	17,417	0.00	194.1
B1–1–25–75–5	4	16	3436	10.6	15,891	15,891	0.00	77.5	15,891	0.00	183.7
B1–1–25–75–6	3	16	5064	13.3	14,260	14,260	0.00	70.9	14,260	0.00	186.4
B1–1–25–75–8	2	16	39	4.3	11,538	11,538	0.00	72.9	11,538	0.00	188.1
B1–1–50–50–4	5	19	52,758	381.4	19,966	19,966	0.00	555.0	19,966	0.00	280.3
B1–1–50–50–5	4	20	41,221	284.1	17,113	17,113	0.00	573.7	17,113	0.00	328.0
B1–1–50–50–6	4	20	444,025	3624.3	15,989	15,989	0.00	534.8	15,989	0.00	292.2
B1–1–50–50–8	–	–	918,165	7200.0	–	14,027	1.54	540.4	14,027	1.54	296.4
B1–10–50–50–4	6	23	1068	7.5	20,075	20,075	0.00	735.6	20,075	0.00	277.2
B1–10–50–50–5	5	23	122,766	754.6	17,986	17,986	0.00	789.6	17,986	0.00	307.1
B1–10–50–50–6	4	22	21,480	159.0	15,924	15,924	0.00	803.4	15,924	0.00	258.9
B1–10–50–50–8	3	23	61,380	413.2	13,672	13,672	0.00	703.8	13,672	0.00	267.9
B1–5–25–75–4	4	15	1228	5.7	17,079	17,079	0.00	54.8	17,079	0.00	202.0
B1–5–25–75–5	3	15	3446	10.8	15,110	15,110	0.00	59.7	15,110	0.00	190.7
B1–5–25–75–6	3	15	153,942	285.8	14,707	14,707	0.00	62.3	14,707	0.00	192.4
B1–5–25–75–8	2	16	126	6.1	11,319	11,319	0.00	60.7	11,319	0.00	194.4
B2–1–50–150–4	6	23	183,130	1060.1	23,288	23,288	0.00	882.0	23,288	0.00	339.1
B2–1–50–150–5	5	23	148,963	800.9	20,039	20,039	0.00	866.4	20,039	0.00	332.4
B2–1–50–150–6	–	–	830,854	7200.0	–	18,046	0.98	891.9	18,046	0.98	345.8
B2–1–50–150–8	–	–	806702	7200.0	–	15,668	5.27	959.2	15668	5.27	313.8
B2–10–50–150–4	7	28	300697	1457.7	25,967	25967	0.00	1452.2	25967	0.00	346.4
B2–10–50–150–5	6	28	184455	1268.3	22,359	22359	0.00	1422.0	22359	0.00	334.2
B2–10–50–150–6	5	28	175686	914.8	19,792	19792	0.00	1539.9	19792	0.00	348.4
B2–10–50–150–8	4	28	182972	1149.7	17106	17106	0.00	1386.2	17106	0.00	361.9
C1–1–25–75–4	3	10	2349	4.4	13012	13012	0.00	31.9	13012	0.00	160.6
C1–1–25–75–5	2	10	1329	4.2	11666	11666	0.00	31.4	11666	0.00	159.9
C1–1–25–75–6	2	10	0	0.9	9820	9820	0.00	30.0	9820	0.00	156.8
C1–1–25–75–8	2	10	382	1.3	9818	9818	0.00	31.9	9818	0.00	159.0
C1–1–50–50–4	5	20	18056	101.6	20294	20294	0.00	574.6	20294	0.00	259.0
C1–1–50–50–5	4	20	2066	11.5	17378	17378	0.00	619.5	17378	0.00	268.8
C1–1–50–50–6	4	20	70181	451.6	16365	16365	0.00	636.5	16365	0.00	265.5
C1–1–50–50–8	3	20	341157	2254.3	13900	13900	0.00	616.4	13900	0.00	260.3
C1–10–50–50–4	7	26	43620	208.8	26931	26931	0.00	937.8	26931	0.00	291.9
C1–10–50–50–5	6	26	48229	263.4	23544	23544	0.00	1075.8	23544	0.00	412.6
C1–10–50–50–6	5	26	51231	207.8	20818	20818	0.00	1001.7	20818	0.00	331.6
C1–10–50–50–8	4	26	123008	593.2	18154	18154	0.00	980.8	18154	0.00	292.6
C1–5–25–75–4	3	12	75	0.4	13738	13738	0.00	35.9	13738	0.00	168.4
C1–5–25–75–5	3	12	10095	10.3	13575	13575	0.00	34.9	13575	0.00	175.3
C1–5–25–75–6	2	12	1	0.3	10826	10826	0.00	37.0	10826	0.00	166.6
C1–5–25–75–8	2	13	366	2.7	10556	10556	0.00	34.4	10556	0.00	169.0
D1–1–25–75–4	4	15	546	3.4	18127	18127	0.00	35.4	18127	0.00	175.3
D1–1–25–75–5	3	15	408	3.6	15972	15972	0.00	36.8	15972	0.00	175.9
D1–1–25–75–6	3	15	1166	5.0	14532	14532	0.00	39.3	14532	0.00	175.7
D1–1–25–75–8	2	15	1681	5.0	12700	12700	0.00	36.7	12700	0.00	174.5
D1–1–50–50–4	6	22	661253	5406.0	23275	23275	0.00	716.3	23275	0.00	271.1
D1–1–50–50–5	5	22	115345	1238.4	20402	20402	0.00	719.3	20402	0.00	275.1
D1–1–50–50–6	4	22	644239	6685.7	18072	18072	0.00	741.8	18072	0.00	257.4
D1–1–50–50–8	3	22	73029	625.8	14930	14930	0.00	685.0	14930	0.00	249.7
D1–10–50–50–4	7	28	12408	68.4	30390	30390	0.00	1407.2	30390	0.00	309.0
D1–10–50–50–5	6	28	927581	4808.5	26284	26284	0.00	1509.5	26284	0.00	331.6
D1–10–50–50–6	–	–	982448	7200.0	–	23646	2.49	1433.9	23646	2.49	304.1
D1–10–50–50–8	–	–	789953	7200.0	–	19986	4.59	1404.4	19986	4.59	323.8
D1–5–25–75–4	4	15	69	2.8	18464	18464	0.00	22.0	18464	0.00	177.6
D1–5–25–75–5	3	15	27	1.1	15767	15767	0.00	21.9	15767	0.00	176.2
D1–5–25–75–6	3	15	1334	4.4	14851	14851	0.00	21.9	14851	0.00	180.3
D1–5–25–75–8	2	15	660	3.4	12705	12705	0.00	20.6	12705	0.00	183.8

Table 2
Impacts of generated cuts on the branch-and-cut algorithm.

Data	Do1	Do2	Do3	Cov	Flow	*LB*0	*LB*1
A1–1–50–50–4	387	5	74	0	541	20.25	12.18
A1–1–50–50–5	390	7	72	0	635	24.43	14.97
A1–1–50–50–6	386	16	216	0	700	25.78	17.53
A1–1–50–50–8	404	16	224	0	738	32.96	23.70
B1–1–50–50–4	191	0	2	0	300	21.16	7.81
B1–1–50–50–5	357	0	10	0	629	27.04	13.93
B1–1–50–50–6	551	6	79	0	969	30.74	18.66
B1–1–50–50–8	606	7	92	0	1147	35.69	24.64
C1–1–50–50–4	323	13	90	0	506	23.63	15.55
C1–1–50–50–5	372	19	131	0	587	26.41	17.41
C1–1–50–50–6	255	11	71	0	409	24.14	15.37
C1–1–50–50–8	325	16	131	0	486	30.55	21.86
D1–1–50–50–4	864	32	415	18	1366	26.24	17.01
D1–1–50–50–5	747	31	327	10	1339	29.57	19.69
D1–1–50–50–6	760	33	429	12	1419	33.50	24.15
D1–1–50–50–8	873	33	375	10	1473	38.01	29.51

slightly better than GRASP-ELS when it gives a better solution for the instance A2–1–50–150–8 with smaller running time.

We now analyse the impacts of valid cuts proposed in the previous section. During the solution process, the flow constraints (13) and the domination constraints (12) are the most frequent. However, as mentioned above, the cuts of type (14)–(16) are very rarely activated on the tested instances. One of reasons could be that, for instance, the inequalities (14) and (15), are useful only when $\max_{w_k \in W_j} u_k$ is equal to 1 as mentioned in Section 2. But in our current instances, very few vertices in V satisfy this condition due to a majority of vertices in W requiring to be covered more than once. Therefore, to investigate the usefulness of all the proposed cuts we test our branch-and-cut algorithm on new instances generated in such a way that they include a large number of vertices in V (70% in our experiment) with unit covering demands, i.e. those need to be covered only once. $X-|1|-|50|-|50|-$ settings are chosen to create these new instances.

Table 2 presents the number of constraints generated in the branch-and-cut algorithm. We also compute the linear relaxations before and after adding the valid cuts to analyse the ability of these cuts in improving lower bounds. Note that all automatic CPLEX's cuts are turned off to avoid their unintended impacts on the linear relaxations. In the table, the column headings are as follows:

Do1: number of constraints of type (12);

Do2: number of constraints of type (14);

Do3: number of constraints of type (15)

Cov: number of constraints of type (16);

Flow: number of constraints of type (13);

Gap_0: deviation between the value of linear relaxation (LB_0) before adding any cut and the best solution found so far;

Gap_1: deviation between the value of linear relaxation (LB_1) after adding the proposed cuts and the best solution found so far;

Let UB be the value of the final solution found by branch-and-cut algorithm or the value of the solution of the metaheuristic if the branch-and-cut algorithm fails to find a solution), Gap_0 and Gap_1 in Table 2 are computed as:

$$Gap_i = \frac{100.(UB - LB_i)}{UB} \qquad i = 1, 2 \tag{24}$$

Table 2 clearly shows the performance of valid inequalities in improving the linear relaxation of *mm*-CTP-*p*. All the cuts are activated during the solving process. Among them, the flow constraints (13) are the most frequent while the cover constraints (18) are the least. This is similar to problems 1-CTP in (Gendreau et al., 1997) and *m*-CTP-*p* in (Hà et al., 2013).

4.3. Results for metaheuristics

We now investigate the performance of our metaheuristic. Six variants of the *mm*-CTP are selected for the experiments:

- *m*-CTP-*p*: multi-vehicle covering tour problem with only constraints on route length;
- *m*-CTP: general multi-vehicle covering problem;
- *mm*-CTP-*p*: multi-vehicle multi-covering tour problem with only constraints on route length;
- *mm*-CTP: general multi-vehicle multi-covering tour problem;
- *mm*-CTP-o: general multi-vehicle multi-covering tour problem with overnight;
- *mm*-CTP-wo: general multi-vehicle multi-covering tour problem without overnight;

To solve the *mm*-CTP-o and the *mm*-CTP-wo, we use the graph transformation to convert the instances to binary *mm*-CTP problems and then apply directly the methods. Because the size of generated instances is too large, we only run the metaheuristics on 32 instances with $|V| = 50, |T| = 1$ and $|W| = 50$.

In the following, we compare our GA with current best metaheuristics (if available) for each variant, that is with the modified GRASP-ELS on all six variants and with the VNS (Kammoun et al., 2017) on the *m*-CTP-*p*. Further, the GRASP-ELS and GA are run 10 times for each instance to better observe their variance. It is worthy to mention that the GRASP-ELS (Hà et al., 2013) and VNS were run only once for each instance and only the single-run results for the *m*-CTP-*p* were reported. To avoid long and tedious tables, we summarize the results by reporting the average values for each variant. The detailed results for the separate instances are presented in Appendix A.

In Table 3, the criteria used for the comparison are the total running time in seconds of 10 runs (column "Time") and the average gaps of average solution (column "Avg."), and best solution (column "Best") over 10 runs to the current best known solution found by existing and considering methods. In addition, column "Better" represents the number of problem instances on which an algorithm (GRASP-ELS or GA-VLG) finds a better solution than the other does. For each criterion we indicate the better results in bold. The results obtained show that the GA-VLG performs better in all criteria on all variants.

Another interesting observation is that by allowing to revisit a vertex, we can significantly reduce the transportation cost. Specifically, we can save on average 2.33% with GRASP-ELS and 2.60% with GA-VLG of the cost in the case without overnight. The savings in the case with overnight is even better up to 13.64% with GRASP-ELS and 14.01% with GA-VLG of the cost on average (see column 'Save' in Tables A.10 and A.11 for more detail). Therefore, allowing the revisiting (if possible) is an effective way to reduce the cost.

The computational time of our metaheuristic is acceptable. The running time for each run in general can be measured in minutes. Between two methods, on average, the GA-VLG is usually faster than the GRASP-ELS, even more than 40 times in some cases such as instance B2–20–* of *mm*–CTP–*p* and *mm*-CTP. More precisely, GA-VLG is often faster on large instances and slower on small instances (see more details in Appendix A). This can be explained by the fact that the parameters of GA-VLG are set to the fixed values for every instance while those of GRASP-ELS depend on the instances' size.

We also compare our methods with VNS proposed by Kammoun et al. (2017), the current best metaheuristic for the *m*-CTP-*p* (see Table A.6 for detailed results). Note that only single-run results provided by the VNS were reported in Kammoun et al. (2017). Since the platform which was used to run the VNS was not presented, we could not compare the running times of three algorithms. In terms of solution quality, our methods have found

Table 3
Performance analysis of GRASP-ELS and GA-VLG on variants of *mm*-CTP problems.

Variants	Methods and criteria					
	Methods	Avg.(%)	Better	Best(%))	Better	Time(secs)
m–CTP–*p*	GRASP-ELS	0.011	0	0.000	0	530.28
	GA-VLG	**0.002**	**9**	0.000	0	**229.80**
m–CTP	GRASP-ELS	0.092	7	0.001	0	624.85
	GA-VLG	**0.006**	**29**	**0.000**	**7**	**232.17**
mm–CTP–*p*	GRASP-ELS	0.042	6	0.010	1	3936.32
	GA-VLG	**0.015**	**19**	**0.001**	**7**	**347.01**
mm–CTP	GRASP-ELS	0.156	9	0.033	3	4729.70
	GA-VLG	**0.021**	**59**	**0.001**	**17**	**487.92**
mm–CTP–o	GRASP-ELS	0.889	4	0.428	0	1840.55
	GA-VLG	**0.369**	**25**	**0.000**	**18**	**612.90**
mm–CTP–wo	GRASP-ELS	0.523	8	0.312	2	2346.35
	GA-VLG	**0.141**	**20**	**0.031**	**10**	**559.16**

Table 4
Stability of GRASP-ELS and GA-VLG on variants of *mm*-CTP problems.

Variants	Methods and criteria			
	Methods	Same Cost	GA-VLG Better	GRASP-ELS Better
m–CTP–*p*	GRASP-ELS	2	0	0
	GA-VLG	**8**	0	0
m–CTP	GRASP-ELS	5	**3**	0
	GA-VLG	**24**	1	0
mm–CTP–*p*	GRASP-ELS	4	**4**	0
	GA-VLG	**13**	3	**1**
mm–CTP	GRASP-ELS	9	6	**2**
	GA-VLG	**39**	**11**	1
mm–CTP–o	GRASP-ELS	5	9	0
	GA-VLG	**6**	9	0
mm–CTP–wo	GRASP-ELS	6	**6**	0
	GA-VLG	**10**	4	**2**

solutions at least as good as ones of the VNS. More interestingly, we found a new best known solution for the instance B2–20–100–100–5. It is noted that our methods are more general than the VNS when they can deal with not only the *m*-CTP-*p* but also many other variants of the *mm*-CTP.

Moreover, we examine the performance of our metaheuristic on 4 *m*-CTP instances where the constraints on the number of visited vertices on each route are relaxed, i.e. $p = +\infty$. The instances were named A2–1–50–150–250/500 and B2–1–50–150–250/500 and their exact solutions were found in Jozefowiez (2014) by a branch-and-price algorithm. The obtained results show that, while GRASP-ELS finds only 3 optimal solutions, GA-VLG finds all 4 optimal solutions. In particular, all ten runs of GA-VLG on each instance reach the optimal solutions. This again confirms the performance of our GA-VLG.

Finally, we investigate the stability of our metaheuristics by calculating the variance of solution costs over 10 runs. Variance values of each problem instance are detailed in Appendix A. Table 4 summarizes this result, where "Same Cost" column presents the number of problem instances that an algorithm (GRASP-ELS or GA-VLG) has better (smaller) variance than the other while they provide the same best cost; "GA-VLG Better" and "GRASP-ELS Better" columns are the same as "Same Cost" column, but when GA-VLG provides better solutions than GRASP-ELS does or vice versa, respectively. It is clear from this table that, when GRASP-ELS and GA-VLG achieve the same best cost, GA-VLG is much more stable (reliable) than GRASP-ELS. In other cases, when an algorithm (GRASP-ELS or GA-VLG) finds a better result, it tends to have worse variance, especially in the case of GA-VLG. This can be explained as an algorithm, that finds worse solution, gets the same bad results for each run, it has smaller variance. In this case, comparing variances between the two methods is less meaningful.

4.4. Sensitivity analysis of the main resolution strategies

This section investigates the sensitivity of two new important components of GA-VLG: crossovers and adaptive covering penalty coefficients. Overall, we tested 4 configurations:

- Standard (Conf-Stand): this configuration was described in Section 3.2.2.
- SOX Crossover (Conf-SOX): this is similar to the standard configuration except that only SOX crossover is used.
- MOX Crossover (Conf-MOX): this is similar to Conf-SOX, but SOX crossover is now replaced by MOX crossover.
- No Adaptive Covering Penalty Coefficient (Conf-NoACPC): this is the same as in Conf-Stand, but the penalty coefficients of the covering constraints are set to very large values and do not change during the evolving process. Our goal here is to observe the impact of the adaptive covering penalty coefficient on the performance of the method.

Table 5 presents sensitivity analysis of 4 configurations. We have executed 10 runs for each instance and reported the total

Table 5
Sensitivity analysis of the new features of GA-VLG.

Variants		Conf-SOX	Conf-MOX	Conf-NoACPC	Conf-Stand
m-CTP-*p*	Best (%)	**0.00**	5.98	11.61	**0.00**
	Avg. (%)	1.17	40.85	53.91	**0.23**
m-CTP	Best (%)	0.02	13.04	25.45	**0.00**
	Avg. (%)	3.30	50.30	98.42	**1.17**
mm-CTP-*p*	Best (%)	**0.00**	5.19	16.08	0.06
	Avg. (%)	3.79	31.81	67.15	**1.43**
mm-CTP	Best (%)	**0.06**	13.98	42.20	0.22
	Avg. (%)	8.09	69.90	137.63	**4.17**
mm-CTP-o	Best (%)	2.03	8.87	36.62	**0.00**
	Avg. (%)	19.99	56.87	142.74	**11.80**
mm-CTP-wo	Best (%)	**0.00**	17.04	53.51	0.82
	Avg. (%)	4.41	67.24	133.09	**4.33**

gaps of average solutions (row 'Avg. (%)'), and best solutions (row 'Best (%)'). The gap here is calculated as the distance to the best known solution. As can be seen in the table, all of our new features contribute to the overall performance of the method. Particularly, the adaptive covering penalty mechanism significantly improves the solution quality. Using MOX as the only crossover operator leads to a poor performance of the approach because it creates too much randomness. On the other hand, the Conf-SOX configuration does not provide enough randomness to guide the searching process escaping from local optima. As a consequence, although the crossover SOX seems to work well on finding the best solution, its results related to the average solution are not really good. The SOX-MOX combination provides a better trade-off between the best and average solutions. The standard configuration performs in the most stable and reliable way. Its 'Best' criterion is worst than one of the Conf-SOX configuration on several cases but better on *m*-CTP and *mm*-CTP-o variants. More importantly, it outperforms all other configurations on the 'Average' criterion.

5. Conclusions

In this paper, we have generalized several existing variants of the CTP problem to introduce a new problem called *mm*-CTP. The new characteristic of this problem is that vertices must be covered multiple times. We discussed three variants of the *mm*-CTP: the binary *mm*-CTP where a vertex is visited at most once, the *mm*-CTP with overnight where revisiting a vertex is freely permitted and the *mm*-CTP without overnight where revisiting a vertex is allowed only after passing through another vertex.

An exact method and a metaheuristic have been proposed to deal with the first version. For remaining variants, we proposed the graph transformations to convert them into the binary variant. The exact method based on the branch-and-cut principle could solve to optimality instances in which the tour contains up to 50 vertices of a special case with the relaxed length constraints. Its solutions were used to analyse the problem complexity as well as the performance of our metaheuristic. Our metaheuristic was adapted from the genetic algorithm proposed by Vidal et al. (2014) with several new features exploring the problem characteristics. The extensive experiments on different *mm*-CTP variants have confirmed its performance. The metaheuristic retrieved all known optimal solutions and was much more reliable. It outperformed the GRASP-ELS proposed by Hà et al. (2013) especially on the large instances. When tested on the special existing case of the problem, it provided and improved best known solutions. All in all, our GA-VLG has become the state-of-the-art metaheuristic for a wide class of the CTP problems.

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Appendix A. Detailed experimental results

Tables A.6–A.11 present computational results of GRASP-ELS and GA-VLG on instances of the problems: *m*-CTP-*p*, *m*-CTP, *mm*-CTP-*p*, *mm*-CTP, *mm*-CTP-o and *mm*-CTP-wo.

Table A.6
Computational results of experiments on *m*-CTP-*p* problem

Data instances	GRASP-ELS				GA-VLG				VNS	
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time	Best	Time
A1–1–25–75–4–250	8479.00	8479.00	0.00	6.93	8479.00	8479.00	0.00	132.09	8479	0.016
A1–1–25–75–5–250	8479.00	8479.00	0.00	7.59	8479.00	8479.00	0.00	132.92	8479	0.016
A1–1–25–75–6–250	8479.00	8479.00	0.00	7.55	8479.00	8479.00	0.00	133.44	8479	0.013
A1–1–25–75–8–250	7985.00	7985.00	0.00	7.19	7985.00	7985.00	0.00	130.80	7985	0.014
A1–1–50–50–4–250	10271.00	10271.00	0.00	77.26	10271.00	10271.00	0.00	198.18	10271	0.022
A1–1–50–50–5–250	9220.00	9220.00	0.00	75.54	9220.00	9220.00	0.00	199.82	9220	0.017
A1–1–50–50–6–250	9130.00	9130.00	0.00	77.64	9130.00	9130.00	0.00	202.69	9130	0.023
A1–1–50–50–8–250	9130.00	9130.00	0.00	79.14	9130.00	9130.00	0.00	196.39	9130	0.018
A1–10–50–50–4–250	17953.00	17954.30	15.21	214.47	17953.00	**17953.00**	**0.00**	262.73	17953	1.041
A1–10–50–50–5–250	15440.00	15440.00	0.00	236.19	15440.00	15440.00	0.00	261.06	15440	0.020
A1–10–50–50–6–250	14064.00	14064.00	0.00	231.80	14064.00	14064.00	0.00	258.19	14064	0.041
A1–10–50–50–8–250	13369.00	13369.00	0.00	254.16	13369.00	13369.00	0.00	250.93	13369	0.078
A1–5–25–75–4–250	10827.00	10827.00	0.00	15.64	10827.00	10827.00	0.00	138.20	10827	0.015
A1–5–25–75–5–250	8659.00	8659.00	0.00	17.00	8659.00	8659.00	0.00	135.20	8659	0.014
A1–5–25–75–6–250	8659.00	8659.00	0.00	16.00	8659.00	8659.00	0.00	135.19	8659	0.015
A1–5–25–75–8–250	8265.00	8265.00	0.00	16.46	8265.00	8265.00	0.00	131.59	8265	0.017
A2–1–100–100–4–250	11885.00	11885.00	0.00	279.63	11885.00	11885.00	0.00	314.09	11885	0.154
A2–1–100–100–5–250	10234.00	10234.00	0.00	275.83	10234.00	10234.00	0.00	302.21	10234	0.058
A2–1–100–100–6–250	10020.00	10020.00	0.00	290.37	10020.00	10020.00	0.00	339.10	10020	0.026
A2–1–100–100–8–250	9093.00	9093.00	0.00	280.16	9093.00	9093.00	0.00	315.57	9093	0.270
A2–1–50–150–4–250	11550.00	11550.00	0.00	77.53	11550.00	11550.00	0.00	221.31	11550	0.024
A2–1–50–150–5–250	10407.00	10407.00	0.00	76.50	10407.00	10407.00	0.00	249.38	10407	0.025
A2–1–50–150–6–250	10068.00	10068.00	0.00	73.51	10068.00	10068.00	0.00	258.61	10068	0.023
A2–1–50–150–8–250	8896.00	8896.00	0.00	77.53	8896.00	8896.00	0.00	225.47	8896	0.063
A2–10–50–150–4–250	17083.00	17083.00	0.00	132.12	17083.00	17083.00	0.00	277.74	17083	0.474
A2–10–50–150–5–250	14977.00	14977.00	0.00	141.36	14977.00	14977.00	0.00	269.84	14977	0.120
A2–10–50–150–6–250	13894.00	13894.00	0.00	139.97	13894.00	13894.00	0.00	264.63	13894	0.190
A2–10–50–150–8–250	11942.00	11942.00	0.00	133.42	11942.00	11942.00	0.00	254.10	11942	0.068
A2–20–100–100–4–250	26594.00	26597.60	8.64	2478.58	26594.00	**26594.00**	**0.00**	458.28	26594	0.891
A2–20–100–100–5–250	23419.00	23419.00	0.00	2385.05	23419.00	23419.00	0.00	407.73	23419	5.201
A2–20–100–100–6–250	20966.00	20966.00	0.00	2583.07	20966.00	20966.00	0.00	481.03	20966	5.813
A2–20–100–100–8–250	18415.00	18443.70	**241.21**	2590.73	18415.00	**18435.80**	416.76	638.16	18415	123.884

(*continued on next page*)

Table A.6 (*continued*)

Data instances	GRASP-ELS				GA-VLG				VNS	
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time	Best	Time
B1–1–25–75–4–250	7146.00	7146.00	0.00	28.22	7146.00	7146.00	0.00	130.81	7146	0.004
B1–1–25–75–5–250	6901.00	6901.00	0.00	26.66	6901.00	6901.00	0.00	128.96	6901	0.005
B1–1–25–75–6–250	6450.00	6450.00	0.00	21.43	6450.00	6450.00	0.00	128.60	6450	0.004
B1–1–25–75–8–250	6450.00	6450.00	0.00	25.87	6450.00	6450.00	0.00	128.10	6450	0.004
B1–1–50–50–4–250	10107.00	10107.00	0.00	66.38	10107.00	10107.00	0.00	189.40	10107	0.012
B1–1–50–50–5–250	9723.00	9723.00	0.00	68.95	9723.00	9723.00	0.00	188.37	9723	0.009
B1–1–50–50–6–250	9382.00	9382.00	0.00	67.18	9382.00	9382.00	0.00	193.56	9382	0.016
B1–1–50–50–8–250	8348.00	8348.00	0.00	69.66	8348.00	8348.00	0.00	183.72	8348	0.016
B1–10–50–50–4–250	15209.00	15209.00	0.00	156.94	15209.00	15209.00	0.00	224.26	15209	0.004
B1–10–50–50–5–250	13535.00	13535.00	0.00	162.78	13535.00	13535.00	0.00	212.58	13535	0.052
B1–10–50–50–6–250	12067.00	12067.00	0.00	148.02	12067.00	12067.00	0.00	212.11	12067	0.012
B1–10–50–50–8–250	10344.00	10344.00	0.00	135.57	10344.00	10344.00	0.00	212.55	10344	0.016
B1–5–25–75–4–250	9465.00	9465.00	0.00	17.98	9465.00	9465.00	0.00	139.84	9465	0.004
B1–5–25–75–5–250	9460.00	9460.00	0.00	21.41	9460.00	9460.00	0.00	139.59	9460	0.004
B1–5–25–75–6–250	9148.00	9148.00	0.00	21.24	9148.00	9148.00	0.00	139.83	9148	0.004
B1–5–25–75–8–250	8306.00	8306.00	0.00	20.62	8306.00	8306.00	0.00	136.00	8306	0.004
B2–1–100–100–4–250	18370.00	18370.00	0.00	1083.06	18370.00	18370.00	0.00	339.71	18370	0.057
B2–1–100–100–5–250	15876.00	15876.00	0.00	1156.48	15876.00	15876.00	0.00	401.00	15876	0.076
B2–1–100–100–6–250	14867.00	14878.60	577.44	1069.07	14867.00	**14867.00**	**0.00**	340.53	14867	0.05
B2–1–100–100–8–250	13137.00	13137.00	0.00	1106.34	13137.00	13137.00	0.00	340.25	13137	0.026
B2–1–50–150–4–250	11175.00	11175.00	0.00	89.00	11175.00	11175.00	0.00	231.35	11175	0.177
B2–1–50–150–5–250	10502.00	10502.00	0.00	84.56	10502.00	10502.00	0.00	241.88	10502	0.020
B2–1–50–150–6–250	9799.00	9799.00	0.00	84.19	9799.00	9799.00	0.00	260.78	9799	0.018
B2–1–50–150–8–250	8846.00	8846.00	0.00	85.20	8846.00	8846.00	0.00	223.76	8846	0.072
B2–10–50–150–4–250	16667.00	16667.00	0.00	219.46	16667.00	16667.00	0.00	255.75	16667	0.019
B2–10–50–150–5–250	14188.00	14188.00	0.00	209.76	14188.00	14188.00	0.00	237.36	14188	0.010
B2–10–50–150–6–250	12954.00	12954.00	0.00	194.40	12954.00	12954.00	0.00	231.03	12954	0.026
B2–10–50–150–8–250	11495.00	11495.00	0.00	191.97	11495.00	11495.00	0.00	226.99	11495	0.016
B2–20–100–100–4–250	34062.00	34095.20	1370.56	7282.60	34062.00	**34062.00**	**0.00**	585.17	34062	0.033
B2–20–100–100–5–250	29405.00	29413.40	242.04	7200.69	29405.00	**29410.30**	**28.81**	732.45	29412	78.155
B2–20–100–100–6–250	25960.00	25960.10	0.09	7120.46	25960.00	25960.10	**0.09**	495.59	25960	0.06
B2–20–100–100–8–250	22082.00	22141.10	**873.49**	7054.38	22082.00	**22104.10**	1139.69	577.58	22082	168.606
C1–1–25–75–4–250	6161.00	6161.00	0.00	20.14	6161.00	6161.00	0.00	125.19	6161	0.004
C1–1–25–75–5–250	6161.00	6161.00	0.00	20.66	6161.00	6161.00	0.00	124.07	6161	0.004
C1–1–25–75–6–250	6161.00	6161.00	0.00	20.31	6161.00	6161.00	0.00	125.08	6161	0.004
C1–1–25–75–8–250	6161.00	6161.00	0.00	20.60	6161.00	6161.00	0.00	124.63	6161	0.004
C1–1–50–50–4–250	11372.00	11372.00	0.00	62.26	11372.00	11372.00	0.00	191.35	11372	0.028
C1–1–50–50–5–250	9900.00	9900.00	0.00	61.32	9900.00	9900.00	0.00	195.98	9900	0.013
C1–1–50–50–6–250	9895.00	9895.00	0.00	63.67	9895.00	9895.00	0.00	193.51	9895	0.017
C1–1–50–50–8–250	8699.00	8699.00	0.00	62.41	8699.00	8699.00	0.00	188.30	8699	0.007
C1–10–50–50–4–250	18212.00	18212.00	0.00	142.03	18212.00	18212.00	0.00	244.49	18212	0.025
C1–10–50–50–5–250	16362.00	16362.00	0.00	154.07	16362.00	16362.00	0.00	249.33	16362	0.043
C1–10–50–50–6–250	14749.00	14749.00	0.00	154.93	14749.00	14749.00	0.00	229.26	14749	0.017
C1–10–50–50–8–250	12394.00	12396.00	36.00	146.19	12394.00	**12394.00**	**0.00**	229.63	12394	0.043
C1–5–25–75–4–250	9898.00	9898.00	0.00	17.84	9898.00	9898.00	0.00	137.43	9898	0.011
C1–5–25–75–5–250	9707.00	9707.00	0.00	18.58	9707.00	9707.00	0.00	138.98	9707	0.004
C1–5–25–75–6–250	9321.00	9321.00	0.00	19.18	9321.00	9321.00	0.00	139.40	9324	0.004
C1–5–25–75–8–250	7474.00	7474.00	0.00	18.92	7474.00	7474.00	0.00	133.53	7474	0.004
D1–1–25–75–4–250	7671.00	7671.00	0.00	12.08	7671.00	7671.00	0.00	131.65	7671	0.020
D1–1–25–75–5–250	7465.00	7465.00	0.00	12.08	7465.00	7465.00	0.00	130.17	7465	0.022
D1–1–25–75–6–250	6651.00	6651.00	0.00	11.95	6651.00	6651.00	0.00	130.27	6651	0.015
D1–1–25–75–8–250	6651.00	6651.00	0.00	11.62	6651.00	6651.00	0.00	128.79	6651	0.014
D1–1–50–50–4–250	11606.00	11606.00	0.00	83.26	11606.00	11606.00	0.00	185.36	11606	0.021
D1–1–50–50–5–250	10770.00	10770.00	0.00	71.20	10770.00	10770.00	0.00	185.08	10770	0.263
D1–1–50–50–6–250	10525.00	10571.50	5045.25	73.46	10525.00	**10525.00**	**0.00**	189.46	10525	0.026
D1–1–50–50–8–250	9361.00	9361.00	0.00	78.67	9361.00	9361.00	0.00	185.13	9361	0.028
D1–10–50–50–4–250	20982.00	20982.00	0.00	229.02	20982.00	20982.00	0.00	225.78	20982	0.038
D1–10–50–50–5–250	18576.00	18576.00	0.00	195.16	18576.00	18576.00	0.00	221.79	18576	0.164
D1–10–50–50–6–250	16330.00	16330.00	0.00	184.65	16330.00	16330.00	0.00	207.83	16330	0.011
D1–10–50–50–8–250	14204.00	14204.00	0.00	222.76	14204.00	14204.00	0.00	206.66	14204	0.008
D1–5–25–75–4–250	11820.00	11820.00	0.00	17.88	11820.00	11820.00	0.00	145.57	11820	0.013
D1–5–25–75–5–250	10982.00	10982.00	0.00	18.07	10982.00	10982.00	0.00	143.53	10982	0.016
D1–5–25–75–6–250	9669.00	9669.00	0.00	17.09	9669.00	9669.00	0.00	146.51	9669	0.018
D1–5–25–75–8–250	8200.00	8200.00	0.00	18.58	8200.00	8200.00	0.00	141.31	8200	0.020

Table A.7
Computational results of experiments on general m-CTP problem

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
A1–1–25–75–4–250	12182.00	12182.00	0.00	6.58	12182.00	12182.00	0.00	134.14
A1–1–25–75–4–500	12182.00	12182.00	0.00	6.96	12182.00	12182.00	0.00	135.02
A1–1–25–75–5–250	12182.00	12182.00	0.00	6.64	12182.00	12182.00	0.00	134.21
A1–1–25–75–5–500	12182.00	12182.00	0.00	7.07	12182.00	12182.00	0.00	134.94
A1–1–25–75–6–250	12182.00	12182.00	0.00	6.68	12182.00	12182.00	0.00	132.93
A1–1–25–75–6–500	12182.00	12182.00	0.00	6.97	12182.00	12182.00	0.00	133.71
A1–1–25–75–8–250	12182.00	12182.00	0.00	6.54	12182.00	12182.00	0.00	133.07
A1–1–25–75–8–500	12182.00	12182.00	0.00	6.98	12182.00	12182.00	0.00	134.16
A1–1–50–50–4–250	12685.00	12685.00	0.00	76.49	12685.00	12685.00	0.00	205.99
A1–1–50–50–4–500	10271.00	10271.00	0.00	82.85	10271.00	10271.00	0.00	196.79
A1–1–50–50–5–250	12685.00	12685.00	0.00	79.87	12685.00	12685.00	0.00	211.29
A1–1–50–50–5–500	9220.00	9220.00	0.00	82.90	9220.00	9220.00	0.00	199.59
A1–1–50–50–6–250	12685.00	12685.00	0.00	82.36	12685.00	12685.00	0.00	210.81
A1–1–50–50–6–500	9220.00	9220.00	0.00	88.54	9220.00	9220.00	0.00	200.18
A1–1–50–50–8–250	12685.00	12685.00	0.00	78.89	12685.00	12685.00	0.00	211.14
A1–1–50–50–8–500	9220.00	9220.00	0.00	86.94	9220.00	9220.00	0.00	201.19
A1–10–50–50–4–250	18241.00	18241.00	0.00	237.75	18241.00	18241.00	0.00	237.91
A1–10–50–50–4–500	18241.00	18241.00	0.00	245.59	18241.00	18241.00	0.00	246.62
A1–10–50–50–5–250	15440.00	15440.00	0.00	231.24	15440.00	15440.00	0.00	241.16
A1–10–50–50–5–500	15440.00	15440.00	0.00	232.61	15440.00	15440.00	0.00	248.56
A1–10–50–50–6–250	14916.00	14916.00	0.00	259.38	14916.00	14916.00	0.00	240.56
A1–10–50–50–6–500	14550.00	14550.00	0.00	253.00	14550.00	14550.00	0.00	238.27
A1–10–50–50–8–250	14206.00	14206.00	0.00	247.15	14206.00	14206.00	0.00	230.36
A1–10–50–50–8–500	14206.00	14206.00	0.00	244.55	14206.00	14206.00	0.00	232.57
A1–5–25–75–4–250	15194.00	15194.00	0.00	17.04	15194.00	15194.00	0.00	131.26
A1–5–25–75–4–500	12558.00	12558.00	0.00	16.26	12558.00	12558.00	0.00	132.40
A1–5–25–75–5–250	15194.00	15194.00	0.00	16.14	15194.00	15194.00	0.00	131.71
A1–5–25–75–5–500	12558.00	12558.00	0.00	16.54	12558.00	12558.00	0.00	132.33
A1–5–25–75–6–250	15194.00	15194.00	0.00	16.60	15194.00	15194.00	0.00	131.29
A1–5–25–75–6–500	12558.00	12558.00	0.00	16.10	12558.00	12558.00	0.00	131.82
A1–5–25–75–8–250	15194.00	15194.00	0.00	15.59	15194.00	15194.00	0.00	131.31
A1–5–25–75–8–500	12558.00	12558.00	0.00	16.32	12558.00	12558.00	0.00	132.29
A2–1–100–100–4–250	12701.00	12821.70	43636.01	284.86	12701.00	**12701.00**	**0.00**	418.57
A2–1–100–100–4–500	11885.00	11885.00	0.00	285.28	11885.00	11885.00	0.00	357.83
A2–1–100–100–5–250	10618.00	10877.40	79299.24	302.70	10618.00	**10618.00**	**0.00**	299.43
A2–1–100–100–5–500	10234.00	10332.90	3353.69	285.80	10234.00	**10234.00**	**0.00**	300.45
A2–1–100–100–6–250	10186.00	10356.50	96114.45	291.33	10186.00	**10186.00**	**0.00**	275.04
A2–1–100–100–6–500	10020.00	10020.00	0.00	291.31	10020.00	10020.00	0.00	326.75
A2–1–100–100–8–250	9931.00	10820.70	243389.21	280.15	**9924.00**	**9924.00**	**0.00**	268.52
A2–1–100–100–8–500	9924.00	9924.10	0.09	283.13	9924.00	**9924.00**	**0.00**	278.32
A2–1–50–150–4–250	12039.00	12039.00	0.00	89.66	12039.00	12039.00	0.00	221.65
A2–1–50–150–4–500	11612.00	11612.00	0.00	92.97	11612.00	11612.00	0.00	219.94
A2–1–50–150–5–250	11024.00	11024.00	0.00	86.55	11024.00	11024.00	0.00	218.28
A2–1–50–150–5–500	11024.00	11024.00	0.00	86.67	11024.00	11024.00	0.00	221.18
A2–1–50–150–6–250	11022.00	11022.00	0.00	92.62	11022.00	11022.00	0.00	226.92
A2–1–50–150–6–500	11022.00	11022.00	0.00	87.38	11022.00	11022.00	0.00	238.22
A2–1–50–150–8–250	11022.00	11022.00	0.00	89.32	11022.00	11022.00	0.00	246.18
A2–1–50–150–8–500	11022.00	11022.00	0.00	86.30	11022.00	11022.00	0.00	241.44
A2–10–50–150–4–250	17083.00	17083.00	0.00	144.45	17083.00	17083.00	0.00	274.00
A2–10–50–150–4–500	17083.00	17083.00	0.00	143.50	17083.00	17083.00	0.00	272.82
A2–10–50–150–5–250	14977.00	14977.00	0.00	161.69	14977.00	14977.00	0.00	245.67
A2–10–50–150–5–500	14977.00	14977.00	0.00	152.06	14977.00	14977.00	0.00	264.62
A2–10–50–150–6–250	14370.00	14370.00	0.00	163.17	14370.00	14370.00	0.00	258.32
A2–10–50–150–6–500	13894.00	13894.00	0.00	158.60	13894.00	13894.00	0.00	256.96
A2–10–50–150–8–250	14370.00	14370.00	0.00	166.00	14370.00	14370.00	0.00	251.24
A2–10–50–150–8–500	12179.00	12179.00	0.00	175.52	12179.00	12179.00	0.00	255.50
A2–20–100–100–4–250	26663.00	**26687.30**	**72.81**	2807.51	**26649.00**	26688.60	1760.24	563.47
A2–20–100–100–4–500	26594.00	**26597.00**	**9.00**	2696.12	26594.00	26598.90	216.09	439.30
A2–20–100–100–5–250	23521.00	**23533.50**	156.25	2998.41	23521.00	23536.00	**150.00**	550.10
A2–20–100–100–5–500	23419.00	**23419.00**	**0.00**	2696.58	23419.00	23419.40	1.44	402.30
A2–20–100–100–6–250	21636.00	**21646.80**	**986.16**	3172.32	**21623.00**	21727.40	1211.04	536.90
A2–20–100–100–6–500	20966.00	20966.00	0.00	3031.94	20966.00	20966.00	0.00	448.47
A2–20–100–100–8–250	19346.00	19362.60	61.44	3381.31	19346.00	**19347.50**	**7.65**	553.09
A2–20–100–100–8–500	18458.00	18458.40	1.44	3234.43	18458.00	**18458.00**	**0.00**	463.11
B1–1–25–75–4–250	7146.00	7146.00	0.00	26.13	7146.00	7146.00	0.00	130.50
B1–1–25–75–4–500	7146.00	7146.00	0.00	30.18	7146.00	7146.00	0.00	130.11
B1–1–25–75–5–250	7114.00	7114.00	0.00	30.44	7114.00	7114.00	0.00	130.78
B1–1–25–75–5–500	6901.00	6901.00	0.00	30.28	6901.00	6901.00	0.00	129.33
B1–1–25–75–6–250	7114.00	7114.00	0.00	27.89	7114.00	7114.00	0.00	131.15
B1–1–25–75–6–500	6450.00	6450.00	0.00	25.43	6450.00	6450.00	0.00	128.52
B1–1–25–75–8–250	7114.00	7114.00	0.00	29.47	7114.00	7114.00	0.00	131.27
B1–1–25–75–8–500	6450.00	6450.00	0.00	22.98	6450.00	6450.00	0.00	128.71
B1–1–50–50–4–250	10107.00	10107.00	0.00	61.92	10107.00	10107.00	0.00	191.45

(*continued on next page*)

Table A.7 (*continued*)

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
B1–1–50–50–4–500	10107.00	10107.00	0.00	62.30	10107.00	10107.00	0.00	190.11
B1–1–50–50–5–250	9723.00	9723.00	0.00	68.05	9723.00	9723.00	0.00	189.47
B1–1–50–50–5–500	9723.00	9723.00	0.00	63.02	9723.00	9723.00	0.00	189.60
B1–1–50–50–6–250	9382.00	9382.00	0.00	65.74	9382.00	9382.00	0.00	188.43
B1–1–50–50–6–500	9382.00	9382.00	0.00	61.81	9382.00	9382.00	0.00	190.09
B1–1–50–50–8–250	9234.00	9246.50	681.25	62.08	9234.00	**9238.00**	**4.00**	214.72
B1–1–50–50–8–500	9234.00	9238.00	9.00	62.45	9234.00	**9237.50**	**5.25**	208.23
B1–10–50–50–4–250	15209.00	15209.00	0.00	150.12	15209.00	15209.00	0.00	230.55
B1–10–50–50–4–500	15209.00	15209.00	0.00	152.21	15209.00	15209.00	0.00	226.57
B1–10–50–50–5–250	13535.00	13535.00	0.00	170.19	13535.00	13535.00	0.00	217.53
B1–10–50–50–5–500	13535.00	13535.00	0.00	162.57	13535.00	13535.00	0.00	217.91
B1–10–50–50–6–250	12067.00	12067.00	0.00	164.59	12067.00	12067.00	0.00	209.35
B1–10–50–50–6–500	12067.00	12067.00	0.00	161.64	12067.00	12067.00	0.00	208.17
B1–10–50–50–8–250	10344.00	10344.00	0.00	158.95	10344.00	10344.00	0.00	210.21
B1–10–50–50–8–500	10344.00	10344.00	0.00	156.39	10344.00	10344.00	0.00	206.45
B1–5–25–75–4–250	9465.00	9465.00	0.00	19.12	9465.00	9465.00	0.00	139.19
B1–5–25–75–4–500	9465.00	9465.00	0.00	17.78	9465.00	9465.00	0.00	139.34
B1–5–25–75–5–250	9460.00	9460.00	0.00	18.86	9460.00	9460.00	0.00	139.44
B1–5–25–75–5–500	9460.00	9460.00	0.00	19.23	9460.00	9460.00	0.00	139.86
B1–5–25–75–6–250	9460.00	9460.00	0.00	19.16	9460.00	9460.00	0.00	138.51
B1–5–25–75–6–500	9460.00	9460.00	0.00	17.91	9460.00	9460.00	0.00	139.41
B1–5–25–75–8–250	9460.00	9460.00	0.00	18.06	9460.00	9460.00	0.00	138.30
B1–5–25–75–8–500	9460.00	9460.00	0.00	18.74	9460.00	9460.00	0.00	138.66
B2–1–100–100–4–250	18650.00	18716.20	489.16	1182.89	18650.00	**18650.00**	**0.00**	338.53
B2–1–100–100–4–500	18650.00	18664.60	852.64	1174.24	18650.00	**18650.00**	**0.00**	334.75
B2–1–100–100–5–250	16572.00	16572.00	0.00	1258.55	16572.00	16572.00	0.00	428.94
B2–1–100–100–5–500	16325.00	16455.70	13940.41	1255.83	16325.00	**16325.00**	**0.00**	476.51
B2–1–100–100–6–250	15452.00	15452.00	0.00	1194.33	15452.00	15452.00	0.00	405.14
B2–1–100–100–6–500	15010.00	15066.80	2586.76	1189.59	15010.00	**15010.00**	**0.00**	397.56
B2–1–100–100–8–250	15312.00	15312.00	0.00	1366.38	15312.00	15312.00	0.00	421.01
B2–1–100–100–8–500	13292.00	13341.80	3764.16	1385.83	13292.00	**13292.00**	**0.00**	344.94
B2–1–50–150–4–250	11175.00	11175.00	0.00	88.47	11175.00	11175.00	0.00	249.81
B2–1–50–150–4–500	11175.00	11175.00	0.00	89.54	11175.00	11175.00	0.00	228.28
B2–1–50–150–5–250	10585.00	10585.00	0.00	89.71	10585.00	10585.00	0.00	226.42
B2–1–50–150–5–500	10585.00	10585.00	0.00	91.42	10585.00	10585.00	0.00	227.81
B2–1–50–150–6–250	9799.00	9799.00	0.00	93.01	9799.00	9799.00	0.00	231.43
B2–1–50–150–6–500	9799.00	9799.00	0.00	96.80	9799.00	9799.00	0.00	243.82
B2–1–50–150–8–250	9362.00	9373.50	1190.25	94.68	9362.00	**9362.00**	**0.00**	239.02
B2–1–50–150–8–500	9362.00	9362.00	0.00	97.00	9362.00	9362.00	0.00	241.34
B2–10–50–150–4–250	16667.00	16667.00	0.00	229.51	16667.00	16667.00	0.00	253.07
B2–10–50–150–4–500	16667.00	16667.00	0.00	230.63	16667.00	16667.00	0.00	245.05
B2–10–50–150–5–250	14188.00	14188.00	0.00	250.89	14188.00	14188.00	0.00	237.95
B2–10–50–150–5–500	14188.00	14188.00	0.00	234.70	14188.00	14188.00	0.00	235.58
B2–10–50–150–6–250	12954.00	12954.00	0.00	238.72	12954.00	12954.00	0.00	235.55
B2–10–50–150–6–500	12954.00	12954.00	0.00	221.54	12954.00	12954.00	0.00	234.99
B2–10–50–150–8–250	11495.00	11495.00	0.00	231.97	11495.00	11495.00	0.00	233.41
B2–10–50–150–8–500	11495.00	11495.00	0.00	240.50	11495.00	11495.00	0.00	232.57
B2–20–100–100–4–250	34062.00	34084.10	791.49	8015.92	34062.00	**34062.00**	**0.00**	537.93
B2–20–100–100–4–500	34062.00	34069.70	25.41	7814.01	34062.00	**34062.00**	**0.00**	539.23
B2–20–100–100–5–250	29405.00	29413.40	179.04	8502.01	29405.00	**29412.20**	**71.76**	716.95
B2–20–100–100–5–500	29405.00	29426.20	270.96	8390.54	29405.00	**29409.10**	**11.29**	688.94
B2–20–100–100–6–250	25960.00	25960.20	0.16	8983.15	25960.00	**25960.10**	**0.09**	531.65
B2–20–100–100–6–500	25960.00	**25960.00**	**0.00**	8355.50	25960.00	25960.30	0.21	519.32
B2–20–100–100–8–250	22086.00	22143.80	**490.56**	9206.32	**22082.00**	**22118.50**	1332.25	632.76
B2–20–100–100–8–500	22082.00	22140.90	**877.69**	9720.07	22082.00	**22111.30**	1287.81	570.66
C1–1–25–75–4–250	7420.00	7420.00	0.00	20.96	7420.00	7420.00	0.00	126.37
C1–1–25–75–4–500	7420.00	7420.00	0.00	20.88	7420.00	7420.00	0.00	126.53
C1–1–25–75–5–250	7420.00	7420.00	0.00	21.29	7420.00	7420.00	0.00	126.86
C1–1–25–75–5–500	7420.00	7420.00	0.00	20.82	7420.00	7420.00	0.00	126.68
C1–1–25–75–6–250	7420.00	7420.00	0.00	18.65	7420.00	7420.00	0.00	127.18
C1–1–25–75–6–500	7420.00	7420.00	0.00	20.92	7420.00	7420.00	0.00	127.71
C1–1–25–75–8–250	7420.00	7420.00	0.00	20.82	7420.00	7420.00	0.00	127.36
C1–1–25–75–8–500	7420.00	7420.00	0.00	20.37	7420.00	7420.00	0.00	128.50
C1–1–50–50–4–250	11372.00	11372.00	0.00	65.07	11372.00	11372.00	0.00	193.31
C1–1–50–50–4–500	11372.00	11372.00	0.00	64.30	11372.00	11372.00	0.00	194.10
C1–1–50–50–5–250	9900.00	9900.00	0.00	66.49	9900.00	9900.00	0.00	195.96
C1–1–50–50–5–500	9900.00	9900.00	0.00	66.27	9900.00	9900.00	0.00	193.65
C1–1–50–50–6–250	9895.00	9895.00	0.00	67.76	9895.00	9895.00	0.00	194.36
C1–1–50–50–6–500	9895.00	9895.00	0.00	65.24	9895.00	9895.00	0.00	193.51
C1–1–50–50–8–250	9895.00	9895.00	0.00	66.27	9895.00	9895.00	0.00	193.64
C1–1–50–50–8–500	9895.00	9895.00	0.00	66.32	9895.00	9895.00	0.00	195.86
C1–10–50–50–4–250	18212.00	18212.00	0.00	149.66	18212.00	18212.00	0.00	247.74
C1–10–50–50–4–500	18212.00	18212.00	0.00	149.94	18212.00	18212.00	0.00	241.64
C1–10–50–50–5–250	16362.00	16362.00	0.00	161.62	16362.00	16362.00	0.00	251.96

(*continued on next page*)

Table A.7 (*continued*)

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
C1–10–50–50–5–500	16362.00	16362.00	0.00	152.77	16362.00	16362.00	0.00	238.97
C1–10–50–50–6–250	15164.00	15164.00	0.00	156.56	15164.00	15164.00	0.00	223.85
C1–10–50–50–6–500	14749.00	14749.00	0.00	164.05	14749.00	14749.00	0.00	218.58
C1–10–50–50–8–250	15164.00	15164.00	0.00	164.92	15164.00	15164.00	0.00	219.00
C1–10–50–50–8–500	14683.00	14683.00	0.00	184.57	14683.00	14683.00	0.00	258.00
C1–5–25–75–4–250	11934.00	11934.00	0.00	19.09	11934.00	11934.00	0.00	140.84
C1–5–25–75–4–500	9898.00	9898.00	0.00	18.47	9898.00	9898.00	0.00	136.65
C1–5–25–75–5–250	10894.00	10894.00	0.00	18.94	10894.00	10894.00	0.00	140.13
C1–5–25–75–5–500	9898.00	9898.00	0.00	18.71	9898.00	9898.00	0.00	139.03
C1–5–25–75–6–250	10779.00	10779.00	0.00	22.59	10779.00	10779.00	0.00	137.15
C1–5–25–75–6–500	9898.00	9898.00	0.00	17.74	9898.00	9898.00	0.00	139.30
C1–5–25–75–8–250	10779.00	10779.00	0.00	18.76	10779.00	10779.00	0.00	136.97
C1–5–25–75–8–500	9898.00	9898.00	0.00	18.99	9898.00	9898.00	0.00	138.08
D1–1–25–75–4–250	7671.00	7671.00	0.00	14.18	7671.00	7671.00	0.00	130.61
D1–1–25–75–4–500	7671.00	7671.00	0.00	14.38	7671.00	7671.00	0.00	130.23
D1–1–25–75–5–250	7671.00	7671.00	0.00	14.09	7671.00	7671.00	0.00	131.24
D1–1–25–75–5–500	7671.00	7671.00	0.00	14.47	7671.00	7671.00	0.00	130.79
D1–1–25–75–6–250	7671.00	7671.00	0.00	14.39	7671.00	7671.00	0.00	132.31
D1–1–25–75–6–500	7671.00	7671.00	0.00	14.19	7671.00	7671.00	0.00	131.01
D1–1–25–75–8–250	7671.00	7671.00	0.00	14.12	7671.00	7671.00	0.00	132.23
D1–1–25–75–8–500	7671.00	7671.00	0.00	14.57	7671.00	7671.00	0.00	130.94
D1–1–50–50–4–250	11606.00	11606.00	0.00	93.21	11606.00	11606.00	0.00	183.60
D1–1–50–50–4–500	11606.00	11606.00	0.00	90.10	11606.00	11606.00	0.00	181.22
D1–1–50–50–5–250	11090.00	11090.00	0.00	91.57	11090.00	11090.00	0.00	188.42
D1–1–50–50–5–500	11090.00	11090.00	0.00	85.93	11090.00	11090.00	0.00	188.02
D1–1–50–50–6–250	11036.00	11036.60	0.24	87.62	11036.00	**11036.10**	**0.09**	242.54
D1–1–50–50–6–500	11037.00	11037.00	0.00	86.50	**11036.00**	**11036.00**	0.00	247.92
D1–1–50–50–8–250	11037.00	11037.00	0.00	86.98	**11036.00**	**11036.00**	0.00	219.19
D1–1–50–50–8–500	11037.00	11037.00	0.00	85.33	**11036.00**	**11036.00**	0.00	226.07
D1–10–50–50–4–250	21112.00	21112.00	0.00	284.89	21112.00	21112.00	0.00	216.44
D1–10–50–50–4–500	20982.00	20982.00	0.00	268.39	20982.00	20982.00	0.00	221.48
D1–10–50–50–5–250	18696.00	18696.00	0.00	261.87	18696.00	18696.00	0.00	232.73
D1–10–50–50–5–500	18696.00	18696.00	0.00	250.56	18696.00	18696.00	0.00	234.68
D1–10–50–50–6–250	17059.00	17063.60	27.44	227.90	17059.00	**17059.00**	**0.00**	231.87
D1–10–50–50–6–500	16711.00	**16711.00**	**0.00**	219.54	16711.00	16715.50	182.25	315.28
D1–10–50–50–8–250	16989.00	16990.40	7.84	235.89	16989.00	**16989.00**	**0.00**	218.92
D1–10–50–50–8–500	16341.00	16341.00	0.00	232.51	16341.00	16341.00	0.00	234.01
D1–5–25–75–4–250	12411.00	12411.00	0.00	20.66	12411.00	12411.00	0.00	152.69
D1–5–25–75–4–500	12411.00	12411.00	0.00	20.51	12411.00	12411.00	0.00	147.94
D1–5–25–75–5–250	11432.00	11432.00	0.00	22.18	11432.00	11432.00	0.00	144.95
D1–5–25–75–5–500	11432.00	11432.00	0.00	22.94	11432.00	11432.00	0.00	145.24
D1–5–25–75–6–250	11432.00	11432.00	0.00	20.80	11432.00	11432.00	0.00	146.18
D1–5–25–75–6–500	9669.00	9669.00	0.00	22.16	9669.00	9669.00	0.00	144.58
D1–5–25–75–8–250	11432.00	11432.00	0.00	20.98	11432.00	11432.00	0.00	144.76
D1–5–25–75–8–500	9312.00	9312.00	0.00	22.76	9312.00	9312.00	0.00	142.26

Table A.8
Computational results of experiments on *mm*-CTP-*p*

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
A1–1–25–75–4–250	17774.00	17774.00	0.00	56.77	17774.00	17774.00	0.00	178.34
A1–1–25–75–5–250	15793.00	15793.00	0.00	61.86	15793.00	15793.00	0.00	177.52
A1–1–25–75–6–250	14628.00	14628.00	0.00	59.14	14628.00	14628.00	0.00	178.32
A1–1–25–75–8–250	12590.00	12590.00	0.00	60.39	12590.00	12590.00	0.00	179.16
A1–1–50–50–4–250	21473.00	21473.00	0.00	860.54	21473.00	21473.00	0.00	283.15
A1–1–50–50–5–250	18680.00	18680.00	0.00	898.80	18680.00	18680.00	0.00	287.94
A1–1–50–50–6–250	17481.00	17481.00	0.00	944.39	17481.00	17481.00	0.00	284.99
A1–1–50–50–8–250	14380.00	14380.00	0.00	965.42	14380.00	14380.00	0.00	278.43
A1–10–50–50–4–250	25340.00	25340.00	0.00	1165.99	25340.00	25340.00	0.00	298.60
A1–10–50–50–5–250	21712.00	21712.00	0.00	1133.15	21712.00	21712.00	0.00	309.51
A1–10–50–50–6–250	20125.00	20125.00	0.00	1144.12	20125.00	20125.00	0.00	300.33
A1–10–50–50–8–250	17603.00	17603.00	0.00	1253.44	17603.00	17603.00	0.00	308.99
A1–5–25–75–4–250	13082.00	13082.00	0.00	20.83	13082.00	13082.00	0.00	159.75
A1–5–25–75–5–250	11969.00	11969.00	0.00	21.74	11969.00	11969.00	0.00	159.81
A1–5–25–75–6–250	11746.00	11746.00	0.00	21.10	11746.00	11746.00	0.00	162.48
A1–5–25–75–8–250	9081.00	9081.00	0.00	21.46	9081.00	9081.00	0.00	155.13
A2–1–100–100–4–250	25051.00	25058.20	**60.96**	3656.09	**25026.00**	**25033.60**	134.84	538.48
A2–1–100–100–5–250	21626.00	21677.30	**292.41**	4140.76	21626.00	**21669.10**	629.89	717.98
A2–1–100–100–6–250	19119.00	19180.20	7823.96	4026.62	**19108.00**	**19108.00**	**0.00**	565.29
A2–1–100–100–8–250	16226.00	**16241.00**	**235.80**	4051.89	**16209.00**	16266.40	3803.84	564.42
A2–1–50–150–4–250	23601.00	23613.60	635.04	798.37	23601.00	**23601.00**	**0.00**	533.38

(*continued on next page*)

Table A.8 (*continued*)

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
A2–1–50–150–5–250	20439.00	**20443.20**	**158.76**	835.08	20439.00	20483.40	3591.84	617.65
A2–1–50–150–6–250	18410.00	18410.00	0.00	829.21	18410.00	18410.00	0.00	493.03
A2–1–50–150–8–250	15565.00	15593.30	145.41	768.58	**15502.00**	**15502.00**	**0.00**	371.09
A2–10–50–150–4–250	25702.00	25712.40	432.64	1072.54	25702.00	**25702.00**	**0.00**	380.91
A2–10–50–150–5–250	21503.00	21503.00	0.00	1046.22	21503.00	21503.00	0.00	369.09
A2–10–50–150–6–250	20250.00	20250.00	0.00	1126.13	20250.00	20250.00	0.00	353.10
A2–10–50–150–8–250	16676.00	16676.00	0.00	1091.21	16676.00	16676.00	0.00	354.46
A2–20–100–100–4–250	38074.00	38104.40	316.04	16115.14	38074.00	**38078.60**	**84.64**	704.69
A2–20–100–100–5–250	32646.00	32680.90	**872.09**	16736.41	**32583.00**	**32634.20**	5179.16	825.72
A2–20–100–100–6–250	28490.00	28576.00	3811.60	18798.71	28490.00	**28490.00**	**0.00**	683.06
A2–20–100–100–8–250	24615.00	24652.90	555.49	16746.77	**24593.00**	**24605.10**	**351.09**	901.37
B1–1–25–75–4–250	17417.00	17417.00	0.00	71.63	17417.00	17417.00	0.00	194.08
B1–1–25–75–5–250	15891.00	15891.00	0.00	77.48	15891.00	15891.00	0.00	183.65
B1–1–25–75–6–250	14260.00	14260.00	0.00	70.93	14260.00	14260.00	0.00	186.37
B1–1–25–75–8–250	11538.00	11538.00	0.00	72.92	11538.00	11538.00	0.00	188.12
B1–1–50–50–4–250	19966.00	19966.00	0.00	555.05	19966.00	19966.00	0.00	280.26
B1–1–50–50–5–250	17113.00	17179.10	10915.89	573.71	17113.00	**17113.00**	**0.00**	328.04
B1–1–50–50–6–250	15989.00	15999.50	785.45	534.80	15989.00	**15989.00**	**0.00**	292.18
B1–1–50–50–8–250	14027.00	14027.00	0.00	540.37	14027.00	14027.00	0.00	296.37
B1–10–50–50–4–250	20075.00	20075.00	0.00	735.56	20075.00	20075.00	0.00	277.25
B1–10–50–50–5–250	17986.00	17986.00	0.00	789.64	17986.00	17986.00	0.00	307.10
B1–10–50–50–6–250	15924.00	15924.00	0.00	803.43	15924.00	15924.00	0.00	258.94
B1–10–50–50–8–250	13672.00	13705.60	4515.84	703.80	13672.00	**13672.00**	**0.00**	267.91
B1–5–25–75–4–250	17079.00	17079.00	0.00	54.82	17079.00	17079.00	0.00	201.98
B1–5–25–75–5–250	15110.00	15110.00	0.00	59.68	15110.00	15110.00	0.00	190.72
B1–5–25–75–6–250	14707.00	14707.00	0.00	62.32	14707.00	14707.00	0.00	192.43
B1–5–25–75–8–250	11319.00	11319.00	0.00	60.69	11319.00	11319.00	0.00	194.38
B2–1–100–100–4–250	40974.00	**40993.10**	**741.69**	20287.35	40974.00	41001.50	3025.05	821.72
B2–1–100–100–5–250	34848.00	34856.30	34.61	21132.51	34848.00	**34848.00**	**0.00**	883.70
B2–1–100–100–6–250	**30829.00**	**30880.10**	1715.69	21999.00	30849.00	30894.30	**996.81**	856.52
B2–1–100–100–8–250	25804.00	25914.10	3048.29	20826.46	25804.00	**25820.00**	**256.00**	993.06
B2–1–50–150–4–250	23288.00	23288.00	0.00	881.96	23288.00	23288.00	0.00	339.12
B2–1–50–150–5–250	20039.00	20039.00	0.00	866.44	20039.00	20039.00	0.00	332.39
B2–1–50–150–6–250	18046.00	18046.00	0.00	891.85	18046.00	18046.00	0.00	345.81
B2–1–50–150–8–250	15668.00	15668.00	0.00	959.18	15668.00	15668.00	0.00	313.84
B2–10–50–150–4–250	25967.00	25967.00	0.00	1452.23	25967.00	25967.00	0.00	346.35
B2–10–50–150–5–250	22359.00	22359.00	0.00	1421.98	22359.00	22359.00	0.00	334.17
B2–10–50–150–6–250	19792.00	19792.00	0.00	1539.92	19792.00	19792.00	0.00	348.38
B2–10–50–150–8–250	17106.00	17106.10	0.09	1386.24	17106.00	**17106.00**	**0.00**	361.92
B2–20–100–100–4–250	53590.00	53591.90	32.49	38501.00	53590.00	**53590.00**	**0.00**	763.91
B2–20–100–100–5–250	45209.00	**45209.00**	**0.00**	42990.69	45209.00	45213.40	174.24	743.29
B2–20–100–100–6–250	39184.00	39194.20	560.16	41914.83	39184.00	**39184.00**	**0.00**	712.49
B2–20–100–100–8–250	32513.00	**32524.20**	**1128.96**	38976.69	**32512.00**	32531.00	1444.00	861.18
C1–1–25–75–4–250	13012.00	13012.00	0.00	31.86	13012.00	13012.00	0.00	160.63
C1–1–25–75–5–250	11666.00	11666.00	0.00	31.39	11666.00	11666.00	0.00	159.93
C1–1–25–75–6–250	9820.00	9820.00	0.00	30.00	9820.00	9820.00	0.00	156.82
C1–1–25–75–8–250	9818.00	9818.00	0.00	31.94	9818.00	9818.00	0.00	159.01
C1–1–50–50–4–250	20294.00	20294.00	0.00	574.60	20294.00	20294.00	0.00	258.98
C1–1–50–50–5–250	17378.00	17378.00	0.00	619.47	17378.00	17378.00	0.00	268.75
C1–1–50–50–6–250	16365.00	16365.00	0.00	636.53	16365.00	16365.00	0.00	265.50
C1–1–50–50–8–250	13900.00	13900.00	0.00	616.37	13900.00	13900.00	0.00	260.33
C1–10–50–50–4–250	26931.00	26931.00	0.00	937.78	26931.00	26931.00	0.00	291.93
C1–10–50–50–5–250	23544.00	23544.00	0.00	1075.82	23544.00	23544.00	0.00	412.64
C1–10–50–50–6–250	20818.00	20818.00	0.00	1001.74	20818.00	20818.00	0.00	331.56
C1–10–50–50–8–250	18154.00	18158.80	34.56	980.82	18154.00	**18154.00**	**0.00**	292.64
C1–5–25–75–4–250	13738.00	13738.00	0.00	35.89	13738.00	13738.00	0.00	168.41
C1–5–25–75–5–250	13575.00	13575.00	0.00	34.92	13575.00	13575.00	0.00	175.35
C1–5–25–75–6–250	10826.00	10826.00	0.00	37.02	10826.00	10826.00	0.00	166.63
C1–5–25–75–8–250	10556.00	10556.00	0.00	34.40	10556.00	10556.00	0.00	168.97
D1–1–25–75–4–250	18127.00	18127.00	0.00	35.35	18127.00	18127.00	0.00	175.32
D1–1–25–75–5–250	15972.00	15972.00	0.00	36.79	15972.00	15972.00	0.00	175.92
D1–1–25–75–6–250	14532.00	14532.00	0.00	39.30	14532.00	14532.00	0.00	175.72
D1–1–25–75–8–250	12700.00	12700.00	0.00	36.71	12700.00	12700.00	0.00	174.48
D1–1–50–50–4–250	23275.00	23275.00	0.00	716.26	23275.00	23275.00	0.00	271.06
D1–1–50–50–5–250	20402.00	20402.00	0.00	719.32	20402.00	20402.00	0.00	275.12
D1–1–50–50–6–250	18072.00	18072.00	0.00	741.83	18072.00	18072.00	0.00	257.36
D1–1–50–50–8–250	14930.00	14930.00	0.00	684.95	14930.00	14930.00	0.00	249.68
D1–10–50–50–4–250	30390.00	30390.00	0.00	1407.15	30390.00	30390.00	0.00	308.98
D1–10–50–50–5–250	26284.00	26284.00	0.00	1509.47	26284.00	26284.00	0.00	331.55
D1–10–50–50–6–250	23646.00	23646.00	0.00	1433.92	23646.00	23646.00	0.00	304.10
D1–10–50–50–8–250	19986.00	19986.00	0.00	1404.40	19986.00	19986.00	0.00	323.79
D1–5–25–75–4–250	18464.00	18464.00	0.00	21.99	18464.00	18464.00	0.00	177.63
D1–5–25–75–5–250	15767.00	15767.00	0.00	21.86	15767.00	15767.00	0.00	176.24
D1–5–25–75–6–250	14851.00	14851.00	0.00	21.89	14851.00	14851.00	0.00	180.31
D1–5–25–75–8–250	12705.00	12705.00	0.00	20.65	12705.00	12705.00	0.00	183.84

Table A.9
Computational results of experiments on *mm*-CTP

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
A1–1–25–75–4–250	21806.00	21806.00	0.00	56.75	21806.00	21806.00	0.00	275.73
A1–1–25–75–4–500	20553.00	20553.00	0.00	55.82	20553.00	20553.00	0.00	261.70
A1–1–25–75–5–250	21282.00	21282.00	0.00	54.08	21282.00	21282.00	0.00	258.66
A1–1–25–75–5–500	19561.00	19561.00	0.00	59.80	19561.00	19561.00	0.00	273.83
A1–1–25–75–6–250	20808.00	20808.00	0.00	65.62	20808.00	20808.00	0.00	265.90
A1–1–25–75–6–500	19012.00	19012.00	0.00	59.87	19012.00	19012.00	0.00	267.62
A1–1–25–75–8–250	20003.00	20003.00	0.00	58.43	20003.00	20003.00	0.00	256.28
A1–1–25–75–8–500	18563.00	18563.00	0.00	59.57	18563.00	18563.00	0.00	263.53
A1–1–50–50–4–250	21529.00	21529.00	0.00	836.00	21529.00	21529.00	0.00	416.83
A1–1–50–50–4–500	21529.00	21529.00	0.00	867.47	21529.00	21529.00	0.00	411.23
A1–1–50–50–5–250	19762.00	19762.00	0.00	854.81	19762.00	19762.00	0.00	426.87
A1–1–50–50–5–500	19581.00	19581.00	0.00	950.36	19581.00	19581.00	0.00	425.64
A1–1–50–50–6–250	18208.00	18216.80	696.96	1023.57	18208.00	**18208.00**	**0.00**	435.65
A1–1–50–50–6–500	17976.00	17976.00	0.00	952.88	17976.00	17976.00	0.00	422.75
A1–1–50–50–8–250	16941.00	16941.00	0.00	965.86	16941.00	16941.00	0.00	428.45
A1–1–50–50–8–500	15399.00	15399.00	0.00	985.77	15399.00	15399.00	0.00	414.47
A1–10–50–50–4–250	25340.00	25340.00	0.00	1137.83	25340.00	25340.00	0.00	432.17
A1–10–50–50–4–500	25340.00	25340.00	0.00	1146.13	25340.00	25340.00	0.00	439.03
A1–10–50–50–5–250	22626.00	22786.60	6976.44	1198.89	22626.00	**22626.00**	**0.00**	544.12
A1–10–50–50–5–500	22650.00	22680.60	104.04	1200.45	**22626.00**	**22626.00**	**0.00**	572.30
A1–10–50–50–6–250	20841.00	20848.90	561.69	1181.82	20841.00	**20841.00**	**0.00**	489.82
A1–10–50–50–6–500	20841.00	20878.50	3598.65	1181.43	20841.00	**20841.00**	**0.00**	505.40
A1–10–50–50–8–250	19420.00	19425.90	313.29	1502.19	19420.00	**19420.00**	**0.00**	448.59
A1–10–50–50–8–500	18136.00	18136.00	0.00	1442.02	18136.00	18136.00	0.00	480.46
A1–5–25–75–4–250	17657.00	17657.00	0.00	20.72	17657.00	17657.00	0.00	229.80
A1–5–25–75–4–500	16359.00	16359.00	0.00	20.31	16359.00	16359.00	0.00	239.82
A1–5–25–75–5–250	17657.00	17657.00	0.00	20.53	17657.00	17657.00	0.00	227.58
A1–5–25–75–5–500	15861.00	15861.00	0.00	21.09	15861.00	15861.00	0.00	231.91
A1–5–25–75–6–250	17657.00	17657.00	0.00	21.15	17657.00	17657.00	0.00	228.47
A1–5–25–75–6–500	15861.00	15861.00	0.00	21.27	15861.00	15861.00	0.00	233.05
A1–5–25–75–8–250	17657.00	17657.00	0.00	20.42	17657.00	17657.00	0.00	228.97
A1–5–25–75–8–500	15861.00	15861.00	0.00	20.67	15861.00	15861.00	0.00	233.18
A2–1–100–100–4–250	25042.00	25065.20	**59.96**	4064.08	25042.00	**25057.00**	150.00	836.24
A2–1–100–100–4–500	25051.00	25058.20	**64.16**	4132.39	**25026.00**	**25033.60**	134.84	767.31
A2–1–100–100–5–250	22247.00	22539.60	**19154.64**	4310.79	**22225.00**	**22401.20**	41774.56	1059.88
A2–1–100–100–5–500	21900.00	**21937.20**	1811.36	4523.04	21900.00	21969.60	**700.04**	985.26
A2–1–100–100–6–250	20311.00	20818.90	70188.89	4311.42	**19867.00**	**19907.30**	**3260.01**	784.42
A2–1–100–100–6–500	**19372.00**	19441.80	**1606.76**	4609.52	19374.00	**19436.20**	3977.76	1276.15
A2–1–100–100–8–250	**16720.00**	17579.70	335674.01	4593.41	16724.00	**16750.70**	**6416.01**	915.19
A2–1–100–100–8–500	17095.00	17308.40	16817.04	4718.67	**16724.00**	**16724.00**	**0.00**	734.34
A2–1–50–150–4–250	23601.00	**23619.90**	**833.49**	833.83	23601.00	23626.20	952.56	744.91
A2–1–50–150–4–500	23601.00	23632.50	992.25	855.30	23601.00	**23626.20**	**952.56**	713.13
A2–1–50–150–5–250	20573.00	20576.10	86.49	849.69	20573.00	**20573.00**	**0.00**	488.85
A2–1–50–150–5–500	20573.00	20573.00	0.00	896.83	20573.00	20573.00	0.00	525.43
A2–1–50–150–6–250	18971.00	19023.20	1787.56	898.65	18971.00	**18971.00**	**0.00**	482.62
A2–1–50–150–6–500	18791.00	18791.00	**0.00**	925.24	**18779.00**	**18789.80**	12.96	553.90
A2–1–50–150–8–250	16632.00	17347.30	127749.41	863.75	**16614.00**	**16614.00**	**0.00**	485.87
A2–1–50–150–8–500	15624.00	15838.60	8868.04	995.61	**15502.00**	**15502.00**	**0.00**	466.23
A2–10–50–150–4–250	25702.00	25712.40	432.64	1129.50	25702.00	**25702.00**	**0.00**	537.85
A2–10–50–150–4–500	25702.00	25728.00	1216.80	1144.89	25702.00	**25702.00**	**0.00**	551.62
A2–10–50–150–5–250	21503.00	21503.00	0.00	1076.68	21503.00	21503.00	0.00	497.11
A2–10–50–150–5–500	21503.00	21503.00	0.00	1049.79	21503.00	21503.00	0.00	582.18
A2–10–50–150–6–250	20250.00	20250.00	0.00	1245.77	20250.00	20250.00	0.00	511.71
A2–10–50–150–6–500	20250.00	20250.00	0.00	1226.91	20250.00	20250.00	0.00	561.45
A2–10–50–150–8–250	17469.00	17469.00	0.00	1229.00	17469.00	17469.00	0.00	476.54
A2–10–50–150–8–500	16676.00	16676.00	0.00	1257.36	16676.00	16676.00	0.00	482.76
A2–20–100–100–4–250	38074.00	38099.20	442.36	18389.65	38074.00	**38074.00**	**0.00**	1040.52
A2–20–100–100–4–500	38074.00	38098.10	296.89	19409.64	38074.00	**38078.60**	**84.64**	1003.44
A2–20–100–100–5–250	32965.00	32994.80	509.76	21065.60	**32902.00**	**32909.80**	**323.36**	1188.38
A2–20–100–100–5–500	32642.00	32672.90	771.69	19980.46	**32583.00**	**32583.00**	**0.00**	1222.22
A2–20–100–100–6–250	29204.00	29282.30	**1873.21**	20956.43	**29195.00**	**29270.60**	23231.24	1022.63
A2–20–100–100–6–500	28490.00	28609.40	3748.04	21706.62	28490.00	**28490.00**	**0.00**	1027.93
A2–20–100–100–8–250	25558.00	25571.70	148.01	21350.77	**25547.00**	**25547.90**	**7.29**	1181.82
A2–20–100–100–8–500	24629.00	24674.80	1948.36	21310.66	**24618.00**	**24619.80**	**12.96**	1180.46
B1–1–25–75–4–250	17498.00	17498.00	0.00	93.66	17498.00	17498.00	0.00	284.15
B1–1–25–75–4–500	17498.00	17498.00	0.00	77.29	17498.00	17498.00	0.00	289.14
B1–1–25–75–5–250	16016.00	16016.00	0.00	92.93	16016.00	16016.00	0.00	299.77
B1–1–25–75–5–500	15891.00	15891.00	0.00	89.74	15891.00	15891.00	0.00	270.91
B1–1–25–75–6–250	15447.00	15450.40	104.04	94.09	15447.00	**15447.00**	**0.00**	279.80
B1–1–25–75–6–500	14260.00	14260.00	0.00	89.22	14260.00	14260.00	0.00	270.71
B1–1–25–75–8–250	15414.00	15414.00	0.00	93.15	15414.00	15414.00	0.00	288.20
B1–1–25–75–8–500	13176.00	13176.00	0.00	88.48	13176.00	13176.00	0.00	270.20
B1–1–50–50–4–250	19966.00	19966.00	0.00	560.83	19966.00	19966.00	0.00	376.11
B1–1–50–50–4–500	19966.00	19966.00	0.00	566.48	19966.00	19966.00	0.00	393.02

(continued on next page)

Table A.9 (*continued*)

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
B1–1–50–50–5–250	17113.00	17315.70	7108.81	641.37	17113.00	**17113.00**	**0.00**	466.63
B1–1–50–50–5–500	17113.00	17240.50	16256.25	625.33	17113.00	**17113.00**	**0.00**	436.24
B1–1–50–50–6–250	16000.00	16030.50	169.05	627.36	**15999.00**	**15999.10**	**0.09**	539.78
B1–1–50–50–6–500	15989.00	15996.10	221.69	615.54	15989.00	**15989.00**	**0.00**	398.64
B1–1–50–50–8–250	14027.00	14058.60	8987.04	626.09	14027.00	**14027.00**	**0.00**	388.30
B1–1–50–50–8–500	14027.00	14027.00	0.00	693.50	14027.00	14027.00	0.00	399.09
B1–10–50–50–4–250	20075.00	20075.00	0.00	786.08	20075.00	20075.00	0.00	391.87
B1–10–50–50–4–500	20075.00	20077.00	36.00	797.22	20075.00	**20075.00**	**0.00**	388.82
B1–10–50–50–5–250	17986.00	17986.40	1.44	830.37	17986.00	**17986.00**	**0.00**	416.16
B1–10–50–50–5–500	17986.00	17986.00	0.00	854.08	17986.00	17986.00	0.00	432.46
B1–10–50–50–6–250	15924.00	15924.00	0.00	848.42	15924.00	15924.00	0.00	369.12
B1–10–50–50–6–500	15924.00	15924.00	0.00	886.10	15924.00	15924.00	0.00	372.89
B1–10–50–50–8–250	13672.00	13672.00	0.00	865.69	13672.00	13672.00	0.00	389.59
B1–10–50–50–8–500	13672.00	13672.00	0.00	881.17	13672.00	13672.00	0.00	391.43
B1–5–25–75–4–250	17079.00	17079.00	0.00	60.05	17079.00	17079.00	0.00	289.34
B1–5–25–75–4–500	17079.00	17079.00	0.00	60.27	17079.00	17079.00	0.00	290.14
B1–5–25–75–5–250	15110.00	15110.00	0.00	70.42	15110.00	15110.00	0.00	279.87
B1–5–25–75–5–500	15110.00	15110.00	0.00	61.98	15110.00	15110.00	0.00	278.43
B1–5–25–75–6–250	14921.00	14933.80	342.96	69.80	14921.00	**14921.00**	**0.00**	308.33
B1–5–25–75–6–500	14707.00	14707.00	0.00	66.18	14707.00	14707.00	0.00	278.53
B1–5–25–75–8–250	14837.00	14887.40	282.24	68.23	14837.00	**14837.00**	**0.00**	316.12
B1–5–25–75–8–500	14395.00	14395.00	0.00	65.94	14395.00	14395.00	0.00	271.27
B2–1–100–100–4–250	40974.00	**40990.00**	**448.00**	23193.36	40974.00	41013.40	3648.84	1016.86
B2–1–100–100–4–500	40974.00	**40985.00**	**160.80**	23136.23	40974.00	40999.60	2641.44	976.59
B2–1–100–100–5–250	34848.00	**34860.80**	**120.76**	24930.12	34848.00	34862.80	1971.36	945.10
B2–1–100–100–5–500	34848.00	34862.30	137.01	24963.08	34848.00	**34848.00**	**0.00**	1092.92
B2–1–100–100–6–250	**30829.00**	**30878.50**	**597.25**	25740.75	30849.00	30896.50	2767.45	973.81
B2–1–100–100–6–500	30829.00	**30891.20**	**1259.36**	25583.81	30829.00	30927.20	1668.36	1101.55
B2–1–100–100–8–250	25871.00	25918.80	**703.36**	26606.30	**25804.00**	**25899.00**	9184.20	1291.55
B2–1–100–100–8–500	25804.00	25920.10	**2829.29**	26533.78	25804.00	**25852.50**	4394.25	1272.65
B2–1–50–150–4–250	23288.00	23288.00	0.00	985.47	23288.00	23288.00	0.00	478.89
B2–1–50–150–4–500	23288.00	23288.00	0.00	961.00	23288.00	23288.00	0.00	482.41
B2–1–50–150–5–250	20039.00	20039.00	0.00	995.30	20039.00	20039.00	0.00	478.53
B2–1–50–150–5–500	20039.00	20039.00	0.00	966.98	20039.00	20039.00	0.00	471.43
B2–1–50–150–6–250	18046.00	18046.00	0.00	1012.47	18046.00	18046.00	0.00	460.21
B2–1–50–150–6–500	18046.00	18046.00	0.00	988.27	18046.00	18046.00	0.00	485.07
B2–1–50–150–8–250	15668.00	15670.20	43.56	1069.97	15668.00	**15668.00**	**0.00**	461.46
B2–1–50–150–8–500	15668.00	15668.00	0.00	1027.69	15668.00	15668.00	0.00	458.48
B2–10–50–150–4–250	25967.00	25967.00	0.00	1540.06	25967.00	25967.00	0.00	477.09
B2–10–50–150–4–500	25967.00	25967.00	0.00	1562.53	25967.00	25967.00	0.00	477.24
B2–10–50–150–5–250	22359.00	22359.00	0.00	1609.42	22359.00	22359.00	0.00	471.19
B2–10–50–150–5–500	22359.00	22359.00	0.00	1596.65	22359.00	22359.00	0.00	472.53
B2–10–50–150–6–250	19792.00	19792.00	0.00	1777.58	19792.00	19792.00	0.00	503.46
B2–10–50–150–6–500	19792.00	19792.00	0.00	1786.16	19792.00	19792.00	0.00	480.19
B2–10–50–150–8–250	17106.00	17115.80	134.56	1668.27	17106.00	**17106.00**	**0.00**	486.99
B2–10–50–150–8–500	17106.00	17110.80	92.16	1632.95	17106.00	**17106.00**	**0.00**	482.47
B2–20–100–100–4–250	53590.00	53591.90	32.49	44095.38	53590.00	**53590.00**	**0.00**	1015.18
B2–20–100–100–4–500	53590.00	53591.90	32.49	43115.38	53590.00	**53590.00**	**0.00**	1012.73
B2–20–100–100–5–250	45209.00	45222.80	179.96	49082.43	45209.00	**45213.40**	**174.24**	1073.34
B2–20–100–100–5–500	45209.00	45212.60	51.84	50914.97	45209.00	**45209.00**	**0.00**	1049.80
B2–20–100–100–6–250	39184.00	39230.60	1981.04	51925.36	39184.00	**39193.10**	**745.29**	882.57
B2–20–100–100–6–500	39184.00	39194.60	**562.24**	52363.99	39184.00	**39193.10**	745.29	1094.54
B2–20–100–100–8–250	32610.00	32642.20	970.16	54596.88	**32607.00**	**32616.40**	**652.84**	1363.54
B2–20–100–100–8–500	32513.00	**32526.70**	**1146.81**	55181.70	**32512.00**	32569.00	2166.00	1220.32
C1–1–25–75–4–250	13574.00	13574.00	0.00	29.57	13574.00	13574.00	0.00	231.58
C1–1–25–75–4–500	13012.00	13012.00	0.00	29.39	13012.00	13012.00	0.00	235.05
C1–1–25–75–5–250	13574.00	13574.00	0.00	28.56	13574.00	13574.00	0.00	233.98
C1–1–25–75–5–500	13010.00	13010.00	0.00	30.07	13010.00	13010.00	0.00	236.55
C1–1–25–75–6–250	13574.00	13574.00	0.00	29.25	13574.00	13574.00	0.00	234.54
C1–1–25–75–6–500	13010.00	13010.00	0.00	29.90	13010.00	13010.00	0.00	233.99
C1–1–25–75–8–250	13574.00	13574.00	0.00	29.29	13574.00	13574.00	0.00	234.56
C1–1–25–75–8–500	13010.00	13010.00	0.00	29.38	13010.00	13010.00	0.00	234.15
C1–1–50–50–4–250	20294.00	20294.00	0.00	603.98	20294.00	20294.00	0.00	379.73
C1–1–50–50–4–500	20294.00	20294.00	0.00	587.14	20294.00	20294.00	0.00	386.19
C1–1–50–50–5–250	17378.00	17378.00	0.00	690.86	17378.00	17378.00	0.00	397.29
C1–1–50–50–5–500	17378.00	17378.00	0.00	661.52	17378.00	17378.00	0.00	399.57
C1–1–50–50–6–250	16365.00	16365.00	0.00	739.05	16365.00	16365.00	0.00	382.08
C1–1–50–50–6–500	16365.00	16365.00	0.00	710.37	16365.00	16365.00	0.00	384.23
C1–1–50–50–8–250	14334.00	14347.60	46.24	774.89	14334.00	**14334.00**	**0.00**	379.25
C1–1–50–50–8–500	14334.00	14339.10	60.69	750.78	14334.00	**14334.00**	**0.00**	381.49
C1–10–50–50–4–250	26931.00	26931.00	0.00	923.63	26931.00	26931.00	0.00	417.08
C1–10–50–50–4–500	26931.00	26931.00	0.00	937.78	26931.00	26931.00	0.00	414.35
C1–10–50–50–5–250	23544.00	23544.00	0.00	1039.98	23544.00	23544.00	0.00	519.04
C1–10–50–50–5–500	23544.00	**23544.00**	**0.00**	1080.10	23544.00	23548.90	216.09	482.43

(*continued on next page*)

Table A.9 (*continued*)

Data instances	GRASP-ELS				GA-VLG			
	Best	Avg.	σ^2	Time	Best	Avg.	σ^2	Time
C1–10–50–50–6–250	20818.00	20818.00	0.00	999.85	20818.00	20818.00	0.00	471.09
C1–10–50–50–6–500	20818.00	20818.00	0.00	1031.01	20818.00	20818.00	0.00	480.31
C1–10–50–50–8–250	18748.00	18750.50	43.05	1037.88	18748.00	**18748.00**	**0.00**	467.98
C1–10–50–50–8–500	18154.00	18170.00	133.60	1069.45	18154.00	**18154.00**	**0.00**	431.25
C1–5–25–75–4–250	15028.00	15028.00	0.00	34.59	15028.00	15028.00	0.00	250.06
C1–5–25–75–4–500	13738.00	13738.00	0.00	35.12	13738.00	13738.00	0.00	243.31
C1–5–25–75–5–250	13951.00	13951.00	0.00	34.95	13951.00	13951.00	0.00	250.22
C1–5–25–75–5–500	13646.00	13646.00	0.00	35.26	13646.00	13646.00	0.00	251.17
C1–5–25–75–6–250	13934.00	13934.00	0.00	35.84	13934.00	13934.00	0.00	250.23
C1–5–25–75–6–500	13273.00	13273.00	0.00	36.08	13273.00	13273.00	0.00	249.24
C1–5–25–75–8–250	13934.00	13934.00	0.00	35.18	13934.00	13934.00	0.00	250.17
C1–5–25–75–8–500	12664.00	12664.00	0.00	37.71	12664.00	12664.00	0.00	249.68
D1–1–25–75–4–250	18127.00	18127.00	0.00	40.23	18127.00	18127.00	0.00	257.96
D1–1–25–75–4–500	18127.00	18127.00	0.00	41.06	18127.00	18127.00	0.00	259.20
D1–1–25–75–5–250	15972.00	15972.00	0.00	43.44	15972.00	15972.00	0.00	254.18
D1–1–25–75–5–500	15972.00	15972.00	0.00	44.47	15972.00	15972.00	0.00	256.75
D1–1–25–75–6–250	15811.00	15811.00	0.00	40.13	15811.00	15811.00	0.00	257.67
D1–1–25–75–6–500	15811.00	15811.00	0.00	41.21	15811.00	15811.00	0.00	262.81
D1–1–25–75–8–250	15811.00	15811.00	0.00	41.70	15811.00	15811.00	0.00	257.14
D1–1–25–75–8–500	15811.00	15811.00	0.00	43.54	15811.00	15811.00	0.00	260.23
D1–1–50–50–4–250	23275.00	23275.00	0.00	830.87	23275.00	23275.00	0.00	407.66
D1–1–50–50–4–500	23275.00	23275.00	0.00	824.11	23275.00	23275.00	0.00	398.67
D1–1–50–50–5–250	20574.00	20574.00	0.00	814.78	20574.00	20574.00	0.00	408.26
D1–1–50–50–5–500	20402.00	20402.00	0.00	813.52	20402.00	20402.00	0.00	394.40
D1–1–50–50–6–250	18854.00	18854.00	0.00	910.22	18854.00	18854.00	0.00	380.01
D1–1–50–50–6–500	18072.00	18072.00	0.00	916.18	18072.00	18072.00	0.00	373.56
D1–1–50–50–8–250	17056.00	17056.00	0.00	921.68	17056.00	17056.00	0.00	392.02
D1–1–50–50–8–500	14930.00	14936.70	404.01	1020.60	14930.00	**14930.00**	**0.00**	371.29
D1–10–50–50–4–250	30390.00	30390.00	0.00	1939.51	30390.00	30390.00	0.00	422.12
D1–10–50–50–4–500	30390.00	30390.00	0.00	1844.94	30390.00	30390.00	0.00	429.72
D1–10–50–50–5–250	26284.00	26284.00	0.00	2122.53	26284.00	26284.00	0.00	461.95
D1–10–50–50–5–500	26284.00	26284.00	0.00	2002.74	26284.00	26284.00	0.00	443.68
D1–10–50–50–6–250	23646.00	23647.50	20.25	2053.15	23646.00	**23646.00**	**0.00**	431.83
D1–10–50–50–6–500	23646.00	23646.00	0.00	1947.01	23646.00	23646.00	0.00	435.04
D1–10–50–50–8–250	19986.00	19988.60	27.04	2217.45	19986.00	**19986.00**	**0.00**	436.62
D1–10–50–50–8–500	19986.00	19986.00	0.00	2161.55	19986.00	19986.00	0.00	436.41
D1–5–25–75–4–250	18464.00	18464.00	0.00	25.13	18464.00	18464.00	0.00	256.40
D1–5–25–75–4–500	18464.00	18464.00	0.00	25.05	18464.00	18464.00	0.00	260.01
D1–5–25–75–5–250	15767.00	15767.00	0.00	25.34	15767.00	15767.00	0.00	259.66
D1–5–25–75–5–500	15767.00	15767.00	0.00	24.66	15767.00	15767.00	0.00	259.76
D1–5–25–75–6–250	15333.00	15333.00	0.00	24.10	15333.00	15333.00	0.00	258.88
D1–5–25–75–6–500	15333.00	15333.00	0.00	24.12	15333.00	15333.00	0.00	258.06
D1–5–25–75–8–250	15333.00	15333.00	0.00	25.98	15333.00	15333.00	0.00	258.00
D1–5–25–75–8–500	15333.00	15333.00	0.00	25.51	15333.00	15333.00	0.00	254.00

Table A.10
Computational results of experiments on *mm*-CTP-o

Data Instances	GRASP-ELS					GA-VLG				
	Best	Save(%)	Avg.	σ^2	Time	Best	Save(%)	Avg.	σ^2	Time
A1–1–50–50–4–250	18114.00	15.86	18212.30	**4937.01**	1979.92	**17825.00**	**17.20**	**17942.20**	20069.56	537.28
A1–1–50–50–4–500	18096.00	15.95	18250.50	**5093.85**	1971.20	**17825.00**	**17.20**	**17895.20**	12073.76	776.47
A1–1–50–50–5–250	15904.00	19.52	**15955.90**	**6285.09**	2096.74	15904.00	19.52	16143.20	15447.16	820.98
A1–1–50–50–5–500	15904.00	18.78	**16017.90**	**9519.29**	2120.54	15904.00	18.78	16049.10	23563.09	633.34
A1–1–50–50–6–250	14409.00	20.86	14434.20	1274.76	2230.50	**14389.00**	**20.97**	**14409.40**	**101.04**	557.82
A1–1–50–50–6–500	14409.00	19.84	14429.80	974.56	2175.45	**14389.00**	**19.95**	**14403.40**	**113.64**	672.24
A1–1–50–50–8–250	12789.00	24.51	12821.10	1175.29	2111.33	**12690.00**	**25.09**	**12690.90**	**7.29**	643.65
A1–1–50–50–8–500	12161.00	21.03	12195.00	**1245.60**	2129.41	**12072.00**	**21.61**	**12086.30**	1674.21	682.76
B1–1–50–50–4–250	17819.00	10.75	18072.10	7124.89	1386.44	17819.00	10.75	**17819.00**	**0.00**	566.62
B1–1–50–50–4–500	17819.00	10.75	17909.60	15747.24	1385.73	17819.00	10.75	**17825.60**	**392.04**	505.28
B1–1–50–50–5–250	14573.00	14.84	**14580.40**	**394.84**	1372.15	14573.00	14.84	14625.00	4056.00	558.62
B1–1–50–50–5–500	14573.00	14.84	**14573.90**	**0.09**	1360.15	14573.00	14.84	14625.00	4056.00	539.55
B1–1–50–50–6–250	14140.00	11.62	14145.10	**60.69**	1414.08	**13938.00**	**12.88**	**14013.30**	4819.81	792.10
B1–1–50–50–6–500	14114.00	11.73	14140.80	**124.76**	1404.24	**13938.00**	**12.83**	**14018.40**	1651.64	809.27
B1–1–50–50–8–250	12509.00	10.82	12562.00	**1129.40**	1421.78	**12432.00**	**11.37**	**12515.80**	13161.36	506.65
B1–1–50–50–8–500	12546.00	10.56	12574.20	**1205.96**	1414.26	**12432.00**	**11.37**	**12528.90**	2631.69	589.29

(*continued on next page*)

Table A.10 (*continued*)

Data Instances	GRASP-ELS					GA-VLG				
	Best	Save(%)	Avg.	σ^2	Time	Best	Save(%)	Avg.	σ^2	Time
C1–1–50–50–4–250	18390.00	9.38	18397.20	466.56	1572.46	18390.00	9.38	**18390.00**	**0.00**	424.86
C1–1–50–50–4–500	18390.00	9.38	18390.00	0.00	1581.92	18390.00	9.38	18390.00	0.00	418.84
C1–1–50–50–5–250	15296.00	11.98	15296.00	0.00	1627.47	15296.00	11.98	15296.00	0.00	384.26
C1–1–50–50–5–500	15296.00	11.98	15296.00	0.00	1637.14	15296.00	11.98	15296.00	0.00	389.38
C1–1–50–50–6–250	14736.00	9.95	14769.70	1147.41	1708.93	**14735.00**	**9.96**	**14740.10**	**135.29**	499.95
C1–1–50–50–6–500	14735.00	9.96	14774.90	949.89	1697.49	14735.00	9.96	**14743.30**	**438.01**	521.45
C1–1–50–50–8–250	12157.00	15.19	12176.40	**879.84**	1668.19	12157.00	15.19	**12171.60**	1489.64	451.22
C1–1–50–50–8–500	12157.00	15.19	12177.80	672.76	1681.43	12157.00	15.19	**12157.00**	**0.00**	562.75
D1–1–50–50–4–250	21436.00	7.90	21617.10	**6332.49**	2143.03	**21133.00**	**9.20**	**21517.70**	19687.61	786.96
D1–1–50–50–4–500	21349.00	8.27	21614.10	**17792.29**	2158.21	**21133.00**	**9.20**	**21452.60**	26659.84	833.13
D1–1–50–50–5–250	17742.00	13.76	18098.80	70949.16	2118.07	17742.00	13.76	**17742.00**	**0.00**	679.78
D1–1–50–50–5–500	17861.00	12.45	18311.50	23071.05	2113.70	**17742.00**	**13.04**	**17742.00**	**0.00**	610.21
D1–1–50–50–6–250	16618.00	11.86	16626.30	54.21	2262.53	**16601.00**	**11.95**	**16601.00**	**0.00**	729.16
D1–1–50–50–6–500	16618.00	8.05	16623.10	48.29	2302.55	**16601.00**	**8.14**	**16601.10**	**0.09**	690.45
D1–1–50–50–8–250	13619.00	20.15	13713.10	7767.29	2311.42	**13516.00**	**20.76**	**13600.30**	**2911.41**	682.25
D1–1–50–50–8–500	13592.00	8.96	13672.40	4533.04	2339.11	**13516.00**	**9.47**	**13596.00**	**1085.40**	756.27

Table A.11
Computational results of experiments on *mm*-CTP-wo

Data instances	GRASP-ELS					GA-VLG				
	Best	Save(%)	Avg.	σ^2	Time	Best	Save(%)	Avg.	σ^2	Time
A1–1–50–50–4–250	**20441.00**	**5.05**	**20604.90**	4040.89	2411.67	20611.00	4.26	20611.00	**0.00**	444.01
A1–1–50–50–4–500	20580.00	4.41	20607.90	**86.49**	2473.61	**20441.00**	**5.05**	**20577.00**	4624.00	446.47
A1–1–50–50–5–250	19044.00	3.63	19052.10	**7.29**	2564.91	**19001.00**	**3.85**	**19033.30**	331.81	742.91
A1–1–50–50–5–500	19044.00	2.74	19053.70	301.21	2671.02	**19001.00**	**2.96**	**19035.40**	**295.84**	692.74
A1–1–50–50–6–250	17163.00	5.74	17463.70	17037.81	2697.42	**17134.00**	**5.90**	**17134.00**	**0.00**	587.89
A1–1–50–50–6–500	17154.00	4.57	17205.50	**350.85**	2795.39	**17028.00**	**5.27**	**17088.90**	5702.89	584.20
A1–1–50–50–8–250	16121.00	4.84	16253.10	5452.89	2698.12	**14937.00**	**11.83**	**14943.30**	**357.21**	566.15
A1–1–50–50–8–500	14762.00	4.14	14797.90	797.29	2787.96	14762.00	4.14	**14762.00**	**0.00**	495.33
B1–1–50–50–4–250	**19658.00**	**1.54**	19718.60	1439.24	1607.45	19694.00	1.36	**19696.50**	**6.25**	730.03
B1–1–50–50–4–500	19658.00	1.54	19701.70	1339.61	1573.40	19658.00	1.54	**19691.40**	**127.84**	681.74
B1–1–50–50–5–250	16617.00	2.90	**16621.20**	**11.76**	1738.44	16617.00	2.90	16643.20	618.96	538.63
B1–1–50–50–5–500	16617.00	2.90	**16619.10**	**10.29**	1684.00	16617.00	2.90	16632.30	546.21	571.97
B1–1–50–50–6–250	15477.00	3.27	15635.30	**8058.41**	1723.57	**15452.00**	**3.42**	**15547.50**	11168.25	789.09
B1–1–50–50–6–500	15511.00	2.99	15609.90	**3290.89**	1663.88	**15452.00**	**3.36**	**15570.50**	13755.05	804.42
B1–1–50–50–8–250	13960.00	0.48	13964.00	28.00	1738.70	**13955.00**	**0.51**	**13956.70**	**26.01**	714.70
B1–1–50–50–8–500	13960.00	0.48	**13960.40**	**0.64**	1806.76	**13955.00**	**0.51**	13967.60	1428.84	660.94
C1–1–50–50–4–250	19935.00	1.77	19939.60	84.64	1883.51	19935.00	1.77	**19935.00**	**0.00**	385.64
C1–1–50–50–4–500	19935.00	1.77	19939.60	84.64	1835.55	19935.00	1.77	**19935.00**	**0.00**	378.30
C1–1–50–50–5–250	17087.00	1.67	17101.30	1179.21	2136.68	17087.00	1.67	**17087.00**	**0.00**	403.92
C1–1–50–50–5–500	17087.00	1.67	17101.30	1179.21	2029.36	17087.00	1.67	**17087.00**	**0.00**	431.07
C1–1–50–50–6–250	15991.00	2.29	15991.00	0.00	2160.49	15991.00	2.29	15991.00	0.00	362.69
C1–1–50–50–6–500	15991.00	2.29	15991.00	0.00	2126.22	15991.00	2.29	15991.00	0.00	362.94
C1–1–50–50–8–250	14189.00	1.01	14209.80	1730.56	2097.25	14189.00	1.01	**14189.00**	**0.00**	405.16
C1–1–50–50–8–500	14189.00	1.01	14189.00	0.00	2109.52	14189.00	1.01	14189.00	0.00	464.11
D1–1–50–50–4–250	23142.00	0.57	**23142.00**	**0.00**	2845.07	23142.00	0.57	23147.30	252.81	628.00
D1–1–50–50–4–500	23142.00	0.57	**23142.00**	**0.00**	2826.01	23142.00	0.57	23150.20	297.76	668.49
D1–1–50–50–5–250	20004.00	2.77	20009.30	252.81	2947.65	20004.00	2.77	**20004.00**	**0.00**	527.10
D1–1–50–50–5–500	19880.00	2.56	**19891.90**	**1274.49**	2882.18	19880.00	2.56	19904.10	5227.29	668.42
D1–1–50–50–6–250	18536.00	1.69	18537.10	10.89	3065.27	18536.00	1.69	**18536.00**	**0.00**	639.42
D1–1–50–50–6–500	17990.00	0.45	**17990.20**	**0.36**	3109.70	17990.00	0.45	17990.80	0.96	525.99
D1–1–50–50–8–250	16976.00	0.47	16976.00	0.00	3054.69	16976.00	0.47	16976.00	0.00	477.85
D1–1–50–50–8–500	14785.00	0.97	14786.50	20.25	3337.60	14785.00	0.97	**14785.00**	**0.00**	512.86

Each table shows the name of instance (column ‘Data Instances’), the best cost (column ‘Best’), average cost (column ‘Avg.’), variance of cost over 10 runs (σ^2), and total run time in seconds of 10 runs (column ‘Time’) for each instance of three methods GRASP-ELS, GA-VLG, and VNS (if available).

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.cor.2017.07.009.

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