A Frequency Dependent Investigation of Complex Shear Modulus Estimation

Quang Hai Luong[1(✉)], Manh Cuong Nguyen[1], and Tran Duc Tan[2]

[1] Biomedical Engineering Department, Le Quy Don Technical University, 236 Hoang Quoc Viet Street, Cau Giay District, Hanoi, Vietnam
luonghai@mta.edu.vn
[2] VNU University of Engineering and Technology, 144 Xuan Thuy Street, Cau Giay District, Hanoi, Vietnam
http://www.mta.edu.vn

Abstract. Mechanical properties of tissues in terms of elasticity and viscosity provide us useful information which may be used in detecting tumors. Shear wave imaging (SWI) is a new method to quantify tissue elasticity by estimating the parameters of the complex shear modulus (CSM). The shear wave is generated by a vibrating needle at a certain frequency. In fact, CSM is a function of the vibrating frequency. Therefore, in this paper, a frequency dependent investigation of CSM will be carried in order to evaluate the estimation performance. The Extended Kalman Filter (EKF) is designed to estimate the CSM for both homogeneous and heterogeneous mediums. The root mean square (rms) error is used to evaluate the quality of the CSM estimation. Several tests were implemented to determine the range of vibrating frequency should be used for the good estimation.

Keywords: Shear wave elasticity imaging · Complex shear modulus · Extended Kalman Filter · Vibration

1 Introduction

Mechanical properties of tissues in terms of elasticity and viscosity provide us useful information which may be used in medical diagnosis, especially in detecting tumors [1]. Among various elasticity imaging modalities, ultrasonic shear wave elasticity imaging (SWEI), introduced in 1998 by Sarvazyan et al. [2], is used for estimating the complex shear modulus (CSM) of biphasic hydro polymers including soft biological tissues. As a consequence, SWEI can be coupled with traditional (e.g., structural) ultrasound imaging to provide additional information in the diagnosis. In a recent survey on different state-of-art techniques of ultrasound elastography [3], Gennission et al. have confirmed that SWEI has significant advantages over the other techniques in terms of reproducibility, quantification, elasticity contrast, and automatic shear wave generation. These advantages lead to new applications of SWEI, not only for diagnosis but also for treatment.

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With respect to CSM estimation, various methods have been developed as briefly surveyed next. In 2004, by using the fact that propagation speed of shear waves is related to the frequency of vibration, the elasticity and viscosity of the medium Chen et al. proposed a method to estimate the shear elasticity and viscosity of a homogeneous medium by measuring the shear wave speed dispersion and, in turns, the CSM [4]. In 2007, Zheng et al. applied a linear Kalman filter for the reconstruction of the harmonic motion of particle velocities at distinct spatial locations. Their approach is to model displacement at the spatial points of interest as a sinusoidal function of time. From estimated quantities, absolute phase at a distinct spatial location can be found. By repeating the same procedure for another location a phase difference is found. Shear wave speed and shear wave dispersion curves are estimated over a frequency bandwidth and material properties are obtained. The stochastic filtering approach helped the authors to obtain optimal estimates of the temporal phase at the given spatial location. A drawback of this method is that, the CSM reconstruction is not optimal and is a post-processing procedure requiring several shear wave frequency measurements [5]. In 2010, Orescanin et al. have conducted an experiment whereby they modeled the nonlinear relationship between wave dynamics and material parameters. They represented the CSM parameters of the present by a nonlinear function of the CSM parameters in the past. So, they applied the Maximum Likelihood Ensemble Filter (MLEF), which is a stochastic filter capable of handling nonlinear dynamical models and nonlinear observation operators, to estimate the CSM of a homogeneous medium based on the Kelvin–Voigt model [6]. In this study, they investigated the change of the wave number and attenuation of the wave propagation of shear waves when the frequency and amplitude of vibration were changed. This approach has been extented to a heterogeneous medium [8].

Currently, for the problem of the CSM estimation, there are two key tasks. First, estimating accurately both the elasticity and the viscosity. Second, the CSM estimation can be performed in an online manner during data acquisition. In this paper, we applied the EKF to estimate the CSM. Moreover, we investigated the impact of the frequency of vibration on the quality of the CSM estimation for different type of soft tissues.

2 The Methods

2.1 Shear Wave Propagation and Generation

Shear wave is generated and measured as according to Fig. 1. A mechanical actuator was adapted to hold a stainless-steel needle. The needle is 1.5 mm in diameter and 13 cm long. It is controlled to vibrate along the z axis with frequency from 100 Hz to 500 Hz. So the shear wave is propagated in tissue. The Doppler ultrasound system was used to measure the particle velocity [6]. The needle vibrates along the vertical (z) axis. Under an assumption of cylindrical shear wave propagation along the radial axis, the particle velocity of shear wave

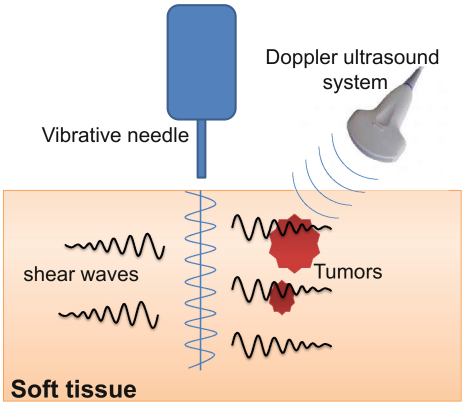

Fig. 1. Generation and measurement shear wave.

$v(r,t)$ is a spatial-temporal function of the radial distance r and time t, and is given by

$$v(r,t) = \frac{1}{\sqrt{r-r_0}} A e^{-\alpha(r-r_0)} \cos[\omega t - k_s(r-r_0) - \phi], \tag{1}$$

where A is the amplitude of source excitation, r_0 is the needle position, and ϕ is the initial temporal phase, α and k_s are attenuation coefficient and wave number at surveyed point. The particle velocities at every spatial location is measured by using the Doppler acquisition.

2.2 The Impact of Frequency of the Vibration on the Shear Wave Propagation

According to Eq. (1), the attenuation coefficient α has effect significantly on the shear wave propagation. The more α is great, the more the particle velocity of shear wave is attenuated. Essentially, α and k_s are imaginary and real components of complex wave number k'_s.

$$k'_s = k_s + i\alpha. \tag{2}$$

On the other hand, follow Kelvin–Voigt model, we have

$$c_s = \sqrt{\frac{\mu}{\rho}}, \tag{3}$$

$$\mu = \mu_1 - i\omega\eta, \tag{4}$$

where c_s is shear wave speed, ρ is mass density of medium (tissue) at the surveyed point, μ is the viscoelasticity, μ_1 and η are the elasticity and viscosity of medium at the surveyed point. Complex wave number k'_s is defined as:

$$k'_s = \frac{2\pi f}{c_s}. \tag{5}$$

From Eqs. (2), (3), (4) and (5), and replace $\omega = 2\pi f$, we have

$$k'_s = k_s + i\alpha = 2\pi f \sqrt{\frac{\rho}{\mu_1 - i(2\pi f)\eta}}. \tag{6}$$

According to Eq. (6), for every tissues (this means that the valued ranges of the elasticity μ_1 and viscosity η of medium are determined), the change of the frequency of vibration lead to the change of the attenuation coefficient α, in turns, the shear wave propagation.

2.3 Estimating the CSM Using the EKF

The CSM estimation is synonymous with estimating the elasticity μ_1 and the viscosity η. From Eq. (6), μ_1 and η at each point are calculated follow formulas

$$\mu_1 = \frac{\rho\omega^2(k_s^2 - \alpha^2)}{(k_s^2 + \alpha^2)^2}, \tag{7}$$

$$\eta = \frac{2\rho\omega k_s \alpha}{(k_s^2 + \alpha^2)^2}. \tag{8}$$

So the problem of the CSM estimation becomes one of the wave number and attenuation coefficient estimation. In this paper, we applied the EKF to estimate the wave number and attenuation coefficient. At each point in space, we built a system model which is used in the Extended Kalman problem. In discrete form, Eq. (1) becomes

$$v_n(r) = \frac{1}{\sqrt{r - r_0}} A e^{-\alpha(r-r_0)} \cos[\omega n \Delta t - k_s(r - r_0) - \phi], \tag{9}$$

where n is the discrete time index and Δt is sampling cycle. Transforming trigonometric Eq. (9), we receive

$$\begin{aligned} v_n(r) = v_{n-1}(r)\cos(\omega\Delta t) - \tfrac{1}{\sqrt{r-r_0}} A e^{-\alpha(r-r_0)} \\ \sin[\omega(n-1)\Delta t - k_s(r - r_0) - \phi] \sin(\omega \Delta t). \end{aligned} \tag{10}$$

To estimate the attenuation coefficient α and wave number k_s using the EKF, Eq. (10) is written in state equation form

$$x_n = f(x_{n-1}, p_{n-1}). \tag{11}$$

Equation (11) is equivalent to

$$\begin{bmatrix} v_n \\ \alpha_n \\ k_{s(n)} \end{bmatrix} = \begin{bmatrix} F(v_{n-1}) \\ \alpha_{n-1} \\ k_{s(n-1)} \end{bmatrix}, \tag{12}$$

where $x_n = \begin{bmatrix} v_n \\ \alpha_n \\ k_{s(n)} \end{bmatrix}$ is state vector at each point, the random variable p_n is process noise, F is a non-linear function, F describes the relation between v_{n-1}

and v_n as shown in Eq. (10), $\alpha_{n-1} = \alpha_n$ and $k_{s(n-1)} = k_{s(n)}$ because we assume that α and k_s would not be changed during the time of the experiment. By using the Doppler acquisition, the measured particle velocities at every spatial location are impacted by Gaussian noise $w_n(r)$. So the measured particle velocity is

$$\widehat{v}_n = v_n + w_n. \tag{13}$$

To use the EKF, Eq. (13) is written in measurement equation form of Kalman problem

$$y_n = h(x_n, w_n). \tag{14}$$

Equation (14) is equivalent to

$$\hat{v}_n = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_n \\ \alpha_n \\ k_{s(n)} \end{bmatrix} + w_n, \tag{15}$$

where $y_n = \widehat{v}_n$ is measurement vector at each point. From Eqs. (11) and (14), x_n is estimated by using the EKF according to the algorithm in [7]. The result is that the shear wave attenuation coefficient α and the wave number k_s at each point are estimated.

3 Results and Discussions

We built some simulation scenarios to test the proposed methods. First, we created three type of soft tissues T1, T2 and T3 (like liver, breast, prostate). Their CSM (μ_1 and η) are (2000 Pa and 0.2 Pa.s), (18000 Pa and 0.5 Pa.s) and (36000 Pa and 1 Pa.s), respectively. The above values, we refer to data table which was shown in [1]. For every type of tissues, we investigated the impact of the frequency of vibration on the attenuation coefficient follow Eq. (6). The results are shown in Fig. 2, which indicates that the attenuation coefficient for liver tissue, breast tissue and prostate tissue are nearly equal (approximates 20) at frequencies 200 Hz, 400 Hz and 450 Hz, respectively. Next, we created three heterogeneous mediums which simulate three above tissues. The details of the mediums and tumors are shown in Table 1. The surveyed number of points are 43 (they are on a line, the distance between 2 points is 0.3 mm), the coordinate of the vibration needle is 0 mm, $r_0 = 0.3$ mm, the amplitude at zero location $A = 2$ mm, the mass density of medium $\rho = 1000\,\text{kg/m}^3$, $\Delta t = 0.06$ ms, time step's number $n = 500$. the vibration frequency of the needle is changed from 100 Hz to 500 Hz (step 50 Hz).

We estimated the CSM using the EKF for all three above medium. For T1 tissue, Fig. 3 shows the estimated particle velocity and the noised particle velocity in time. The estimated particle velocity in time was filtered effectively. It is as a sinusoidal function of time.

The estimated particle velocity in space is indicated in Fig. 4. It is an attenuated-sinusoidal function and similar to the ideal particle velocity,

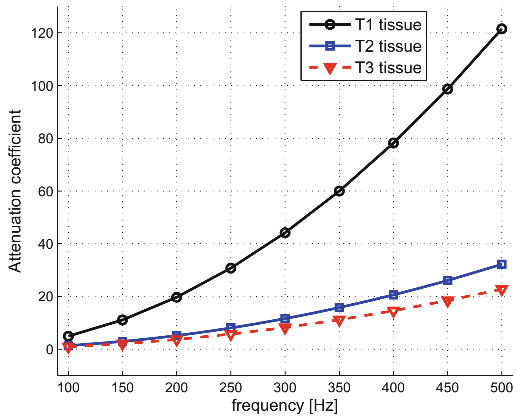

Fig. 2. For some type of tissues, the relation between the attenuation coefficient α and the frequency of vibration f.

Table 1. Parameters of tumors

Type of tissue	μ_1 (Pa)	η (Pa.s)	Location of tumor	μ_1 of tumor (Pa)	η of tumor (Pa.s)
T1	4000	0.2	(10 to 20)	5000	0.4
T2	18000	0.5	(10 to 20)	20000	0.6
T3	36000	1	(10 to 20)	42000	1.1

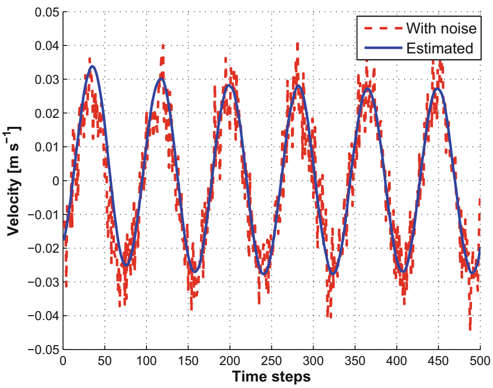

Fig. 3. Estimated particle velocity in time, with SNR = 20 dB, for T1 tissue.

while the noised particle velocity goes up and down in every short segment. This can confirm that the particle velocity in space was estimated effectively.

Fig. 5 indicates the estimated wave number in space, while Fig. 6 shows the estimated attenuation coefficient in space. The results are very good. Visually, the wave number is estimated better than the attenuation coefficient.

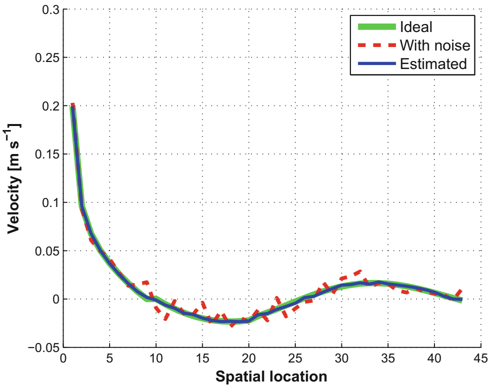

Fig. 4. Estimated particle velocity in space, with SNR = 20 dB, for T1 tissue.

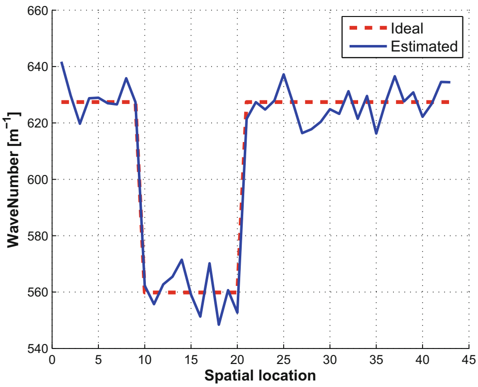

Fig. 5. Estimated wave number in space, for T1 tissue.

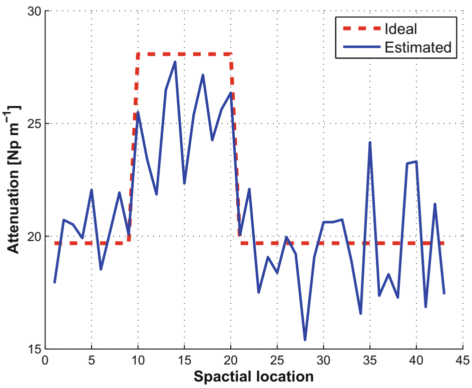

Fig. 6. Estimated attenuation coefficient in space, for T1 tissue.

To evaluate the quality of the CSM estimation when the frequency of vibration is changed, we used the *rms* error. The *rms* error is defined as in Eq. (16).

$$rms = \sqrt{\frac{\sum_{i=1}^{N} (\widehat{x}_i - x_i)^2}{N}}, \tag{16}$$

where x is ideal input, $\widehat{x}$ is the estimated value, N is the number of samples. For T1 tissue, Fig. 7 shows the change of the rms error for the estimated wave number and attenuation coefficient following the frequency of vibration. At frequency 200 Hz, the values of the rms error for both the wave number and attenuation coefficient are least. This means that the quality of the CSM estimation is best.

Of course, we tried to build the parameters of the EKF, with which, the quality of the CSM estimation is best at 200 Hz for T1 tissue. However, we applied this EKF to estimate the CSM for T2 and T3 tissues, the results illustrated that

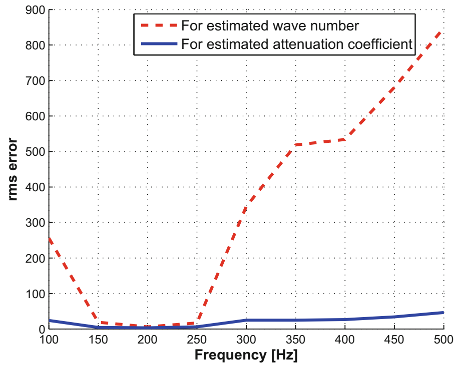

Fig. 7. The *rms* error for estimated wave number and attenuation coefficient, for T1 tissue.

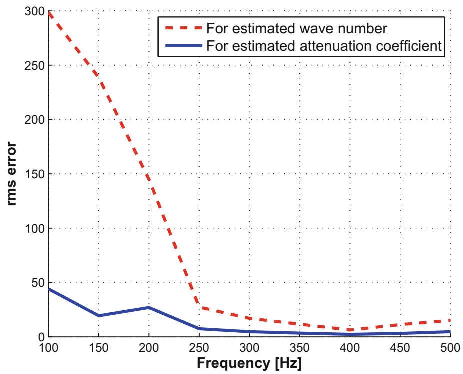

Fig. 8. The *rms* error for estimated wave number and attenuation coefficient, for T2 tissue.

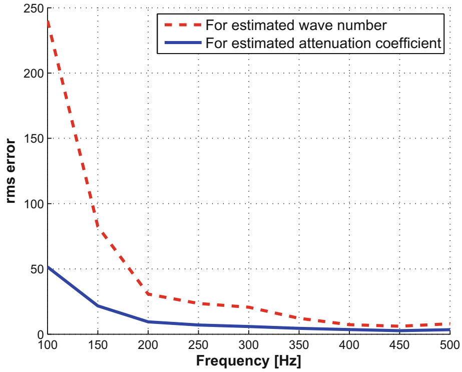

Fig. 9. The *rms* error for estimated wave number and attenuation coefficient, for T3 tissue.

the quality of the CSM estimation is best at 400 Hz (Fig. 8) and 450 Hz (Fig. 9) for T2 and T3 tissues, respectively.

4 Conclusions

This paper was successful in investigating the frequency dependent on estimating CSM for a heterogeneous medium. An effective EKF is designed to estimate the CSM at each point of a line in tissues. By extending tens of points in a line, we could estimate the CSM for a one-dimensional heterogeneous medium. Based on simulated results, we can determine the range of vibrating frequency should be used for the good estimation. In the future work, we will expand to a two-dimensional heterogeneous medium (i.e. CSM imaging). The accuracy of the CSM estimation would also be concerned, especially the viscosity of tissues.

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