Strict Frequency Reuse in Ultra Dense Networks

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***Abstract*—In this paper, the Ultra Dense Network (UDN) using Strict Frequency Reuse (FR) is modelled according to the 3GPP recommendations. The recent developed stretch path loss model is utilised to replace the traditional one. The user performance expressions in terms of user classification probability and average coverage probability are derived in Rayleigh fading environment. Throughout the analytical results, which are verified by Monte Carlo simulation, the new findings are presented such as in deep fading environment, the received signal strength at the user keeps constant when the transmit power of Base Station (BS) varies; when the density of BSs increases, the Cell-Center User (CCU) performance reduces continuously while Cell-Edge User (CEU) performance fluctuates.**

Keyword: Strict Frequency Reuse, Poisson, Ultra Dense Network, Coverage Probability.

I. INTRODUCTION

In recent years, there has been rapid development of mobile cellular networks. According to Cisco report [1], the number of mobile users has a 5-fold growth over the past 15 years. In 2015 more than a half of a million devices have joined the cellular networks. It is predicted that the number of mobile users will reach 5.5 billion by 2020 which represents 70% of the global population. This will make mobile data traffic experience eight-fold over the next five years. To adapt with the changes of mobile users and data traffic, a new cellular network needs to be investigated. Therefore, Ultra Dense Network (UDN) [2] in which the Base Stations (BSs) are distributed with a high density was introduced as a new paradigm candidate of the future 5G network system [3]. By using a high number of BSs, the UDN can provide an enormous number of connections; reduce the distances from BSs to UEs and consequently provide high user data rate.

In the literature, performance of UDN has attracted of many research works [2]. In Reference [4], Signal-to-Interference-plus-Noise Ratio (SINR) and throughput were investigated in the dense urban network. In [5] and [6], an interference management and user association approaches were investigated to improve the network performance. Although the works discussed above provided basis knowledge about performance analysis of UDN, the path loss was assumed to be homogeneous in which all distances had the same path loss exponents. Meanwhile in a practical network, the rate of path loss increases with distances. A dual-slope path loss model [7] in which the signal travels from the source to the destination experiences path loss with different path loss exponents. Authors in [8], [9] studied the impacts of Light-of-Sight (LoS) and non Light-of-Sight (nLoS) on the performance of UDN. In recent work [10], stretched path loss model was proposed. The empirical measurements proved that the stretched path loss is a suitable model for short-range communications in UDN.

In next generation of cellular networks, Strict Frequency Reuse (FR) [11] is an basic approach of InterCell Interference Coordination (ICIC), which was introduced in the literature to minimize the InterCell Interference (ICI). In our recent work [12], the operation of Strict FR was modelled according to the recommendations of 3GPP [13]. The operation of Strict FR with two phases, called *establishment phase* and *communication phase*, has recently modelled according to recommendations of 3GPP. During establishment phase, SINR on the control channel is measured and compared with the pre-defined SINR threshold for Cell-Center User (CCU) and Cell-Edge User (CEU) classification purpose. After that, communication link is established between the user and it's associated BS, then data is exchanged. In this work, the regular cellular network with low densities of BSs and regular path loss model with a constant path loss model were investigated.

In the literature, research work on UDN [4], [7]–[9], [14] considered frequency reuse. However, there were limitations such as: *(i)* two-phase operation has not been investigated, only communication phase was considered; *(ii)* there was no difference between CCU and CEU. All users were served with the same power.

In this paper, we use stretched path loss model to analyse performance of UDN using Strict FR. This work can be considered as the development of our work in [12] for UDN. The work is distinguished from other works by the following aspects: *(i)* compared to our previous work in [12], the ultra dense network with the stretched path loss model [10] is considered; *(ii)* compared to the related work in the literature, this work strictly follows the two-phase operation of Strict FR, particularly 3GPP recommendations; *(iii)* compared to the work in [10], besides the differences listed in *(ii)*, instead of considering the interference-limited network (Gaussian noise $\sigma^2 \approx 0$, then Signal-to-Noise $SNR \to \infty$), this work analyses the effects of SNR on the network performance; *(iv)* in stead of analysing the overall network performance as in [10], this work investigates on user performance and thus some interesting performance trends are found.

Throughout the paper,the following interesting findings are presented for UDN using Strict FR: *(i)* in deep fading environment, increasing the transmit power of BSs cannot improve the received signal strength at the user; *(ii)* for CCUs, increasing the density of BSs will reduces the performance ; *(iii)* for CEU the user performance fluctuates when the density of BSs increases.

II. NETWORK MODEL

We consider a model of an ultra-dense single-tier network in which the BSs' distribution follows a spatial Point Poisson Process (PPP) with a density of λ. The nearest BS association in which each user is served by the nearest BS at a distance r is studied. Thus, the Probability Density Function (PDF) of the distance is given by [15]:

$$f_R(r) = 2\pi\lambda r e^{-\lambda\pi r^2} \tag{1}$$

The downlink signal experiences Rayleigh fading with instantaneous power g. Thus, the PDF of the channel power gain is $\exp(-g)$. The stretch path loss model is modelled as follows [10]:

$$PL(r) = \exp\left(-\alpha r^{\beta}\right) \tag{2}$$

in which α and β are tunable parameters. In [10], by comparing with the existing path loss model, it is claimed that the stretch path loss model is the best path loss model to model large-scale attenuation.

It is assumed that the number of users in each cell is greater that that of the number of RBs. Thus, all RBs are employed to serve the active users. Consequently, each user is affected from ICI originating from all adjacent BSs.

A. Fractional Frequency Reuse

Conventionally, the CEU is served with a higher transmit power than CCU, the transmit power of user z is denoted by $P^{(z)} = \phi^{(z)} P$ where $z = (c, e)$ corresponds to the CCU and CEU, $\phi^{(e)} = \phi$ $(\phi > 1)$ is a transmit ratio between the CE and CC powers, and $\phi^{(c)} = 1$. The set of BSs that create interference, which is due to the use of the same RB at the same time, to user z is denoted by $\theta_{FR}^{(z)}$ and $I_{FR}^{(z)}$ is the corresponding interference power, in which $FR = (Str, Sof)$ correspond to Strict FR and Soft FR. Denote θ as the set of all BSs in the networks, hence $\theta = \theta_{FR}^{(c)} \bigcup \theta_{FR}^{(e)}$.

Establishment phase: During the establishment phase, the users under both Strict FR measure and report the received SINRs on the downlink control channels [13], [16] for user classification purpose. Every BS is continuously transmitting downlink control information, and subsequently each control channel experiences the ICI from all adjacent BSs. Furthermore, since all BSs are assumed to transmit on the control channels at the CC power, the interference of the measured SINR during this phase is given by

$$I = \sum_{j \in \theta} P g_{jz} \exp\left(-\alpha r_{jz}^{\beta}\right) \tag{3}$$

where g_{jz} and r_{jz} are the power gain and distance from interfering BS j to user z.

Communication phase: Under Strict FR, $I_{Str}^{(z)}$ originates from BSs in either $\theta_{Str}^{(c)}$ or $\theta_{Str}^{(e)}$ whose densities are given by λ and $\frac{\lambda}{\Delta}$ [12]. The power of interference $I_{Str}^{(z)}$ of user z is

$$I_{Str}^{(z)} = \sum_{j \in \theta_{Str}^{(z)}} P_j^{(z)} g_{jz} \exp\left(-\alpha r_{jz}^{\beta}\right) \tag{4}$$

B. User Classification Probability

In the downlink 3GPP cellular networks, the SINR on the control channel during the establishment phase, denoted by $SINR^{(o)}(1, r)$, is reported to the BS by the user, in which 1 in $SINR^{(o)}(1, r)$ means the transmit power on the control channel is P. If the reported SINR is greater than the SINR threshold T, the user will be classified as a CCU; otherwise it will be classified as a CEU.

The probability, that a user at a distance r from its serving BS is classified as a CCU, is denoted by $A^{(c)}(T, \lambda|r)$. Since in the PPP cellular network model, $SINR^{(o)}(1, r)$, and consequently the probability $P(SINR^{(o)}(1, r) > T)$ are functions of RVs such as the distances from the user to the BSs, to calculate $A^{(c)}(T, \lambda|r)$, the expected value of $P(SINR^{(o)}(1, r) > T)$ has to be evaluated. Hence, $\mathbb{P}$ is used to denote the probability, instead of P in this thesis. Consequently, $A^{(c)}(T, \lambda|r) = \mathbb{P}(SINR^{(o)}(1, r) > T)$.

The CCU classification probability is obtained by integrating $A^{(c)}(T, \lambda|r)$ over the networks, and then it is denoted by $A^{(c)}(T, \lambda)$. It is clear that the *CEU classification probability* can be obtained by $A_{FR}^{(e)}(T, \lambda) = 1 - A_{FR}^{(c)}(T, \lambda)$.

Remark 1: (CCU Classification Probability) The CCU Classification can be obtained from [10]

$$\begin{aligned} A_{FR}^{(c)}(T, \lambda) \\ &= 2\pi\lambda \int_0^{\infty} e^{-\pi\lambda r^2 - \frac{T}{SNR\zeta(r)}} \\ &\quad \exp\left[-\frac{2\pi\lambda}{\beta\alpha^{2/\beta}} \int_0^{\zeta(r)} \frac{T\left[-ln(y)\right]^{\frac{2-\beta}{\beta}}}{\zeta(r) + Ty} dy\right] r dr \end{aligned} \tag{5}$$

in which $SNR = \frac{P}{\sigma}$; and $\zeta(r) = \exp\left(-\alpha r^{\beta}\right)$.
In a comparison to the coverage probability expression in [10], since the SNR is considered in this paper, there is an appearance of SNR in the equation.

III. AVERAGE COVERAGE PROBABILITY

A. Average Coverage Probability Definition

In case of the CCU, the CCU is covered by the network when it's downlink SINRs during the establishment phase and communication phase, denoted by $SINR^{(o)}(1, r)$ and $SINR(1, r)$, are greater than the SINR threshold T and the coverage threshold $\hat{T}$ respectively. Hence, the average coverage probability is defined as the following conditional probability:

$$\mathcal{P}^{(c)}(T,\lambda) = \mathbb{P}\left(SINR(1,r) > \hat{T} | SINR^{(o)}(1,r) > T\right)$$

Similarity, in the case of CEU, the average coverage probability is defined by the following equation:

$$\mathcal{P}^{(e)}(T,\lambda) = \mathbb{P}\left(SINR(\phi,r) > \hat{T} | SINR^{(o)}(1,r) < T\right)$$

B. CCU and CEU Average Coverage Probabilities

Theorem 3.1: The CCU average coverage probability is obtained from

$$\mathcal{P}^{(c)}(T,\lambda) = \frac{\int_0^\infty re^{-\pi\lambda r^2} e^{-\frac{(T+\hat{T})}{SNR\zeta(r)}} \mathscr{L}(T,\hat{T},\lambda) dr}{2\pi\lambda \int_0^\infty e^{-\pi\lambda r^2 - \frac{T}{SNR\zeta(r)}} \mathscr{L}\left(T,0,\lambda\right) rdr} \tag{6}$$

in which $\mathscr{L}(T,\hat{T},\lambda)$ is defined in Equation 10.

The average coverage probability expression of the CCU in Equation 6 is similar to the result in the previous work in [12], except the path loss model is defined by $\zeta(r)$. The following proof highlights the main steps to obtain the desired result.

Proof 1: The CCU average coverage probability can be computed as follows

$$\mathcal{P}^{(c)}(T,\lambda) = \frac{\mathbb{P}\left(SINR(1,r) > \hat{T}, SINR^{(o)}(1,r) > T\right)}{\mathbb{P}\left(SINR^{(o)}(1,r) > T\right)} \tag{7}$$

In Equation 7, the denominator is given in Remark 1. By letting $\zeta(r) = \exp\left(-\alpha r^\beta\right)$, the numerator is given by

$$\begin{aligned} &\mathbb{P}\left(SINR(1,r) > \hat{T}, SINR^{(o)}(1,r) > T\right) \\ &= \mathbb{P}\left(\frac{Pg\zeta(r)}{I_{Str}^{(z)} + \sigma^2} > \hat{T}, \frac{Pg^{(o)}\zeta(r)}{I + \sigma^2} > T\right) \\ &= \int_0^\infty re^{-\pi\lambda r^2} e^{-\frac{(T+\hat{T})}{SNR\zeta(r)}} \mathbb{E}\left[e^{-\frac{\hat{T} I_{Str}^{(c)}}{P^{(c)}\zeta(r)} - \frac{T I_{Str}^{(oc)}}{P^{(c)}\zeta(r)}}\right] dr \end{aligned} \tag{8}$$

The expectation in Equation 8 is denoted by $\mathscr{L}(T,\hat{T},\lambda)$, and since Rayleigh fading is considered.

$$\begin{aligned} &\mathbb{E}\left[e^{-T\sum_{j\in\theta} g_{jz}\zeta(r_{jz})/\zeta(r) - \hat{T}\sum_{j\in\theta_{Str}^{(c)}} g_{jz}^{(o)}\zeta(r_{jz})/\zeta(r)}\right] \\ &= \prod_{j\in\theta} \mathbb{E}\left[\frac{1}{1+T\zeta(r_{jz})/\zeta(r)} \frac{1}{1+\hat{T}\zeta(r_{jz})/\zeta(r)}\right] \end{aligned}$$

Employing the properties of PGF, thus $\mathscr{L}(T,\hat{T},\lambda) =$

$$e^{-2\pi\lambda \int_r^\infty \left[1 - \frac{1}{1+T\zeta(r_{jz})/\zeta(r)} \frac{1}{1+\hat{T}\zeta(r_{jz})/\zeta(r)}\right] r_{jz} dr_{jz}} \tag{9}$$

Employing a change of variable $y = \exp\left(-\alpha r_{jz}^\beta\right)$, $\mathscr{L}(T,\hat{T},\lambda) =$

$$e^{-\frac{2\pi\lambda}{\beta\alpha^{2/\beta}} \int_0^{\zeta(r)} \frac{[-ln(y)]^{\frac{2-\beta}{\beta}}}{y} \left[1 - \frac{\zeta(r)}{\zeta(r)+Ty} \frac{\zeta(r)}{\zeta(r)+\hat{T}y}\right] dy} \tag{10}$$

Theorem 3.2: The CEU average coverage probability is obtained from

$$\mathcal{P}_{Str}^{(e)}(T,\lambda) = \frac{2\pi\lambda \int_0^\infty re^{-\pi\lambda r^2} \begin{bmatrix} e^{-\frac{T}{\phi SNR\zeta(r)}} \mathscr{L}(\hat{T},\frac{\lambda}{\Delta}) & \\ & -e^{-\left(\frac{\hat{T}}{\phi}+T\right)\frac{1}{SNR\zeta}} \mathscr{L}(T,\hat{T},\lambda) \\ 0 & 0 \qquad \times\mathscr{L}\left(T,\frac{(\Delta-1)\lambda}{\Delta}\right) \end{bmatrix} dr}{1 - 2\pi\lambda \int_0^\infty re^{-\pi\lambda r^2 - \frac{Tr^\alpha}{SNR}} \mathscr{L}(T,0,\lambda) dr} \tag{11}$$

Proof 2: The CCU average coverage probability is derived based on the approach in [17]. It is noted that the density of interfering BSs is λ during establishment phase and λ/Δ during communication phase. Hence,

$$\begin{aligned} &\mathcal{P}_{Str}^{(e)}(T,\epsilon) \\ &= \frac{\mathbb{P}\left(\frac{P^{(e)}g\zeta(r)}{\sigma^2+I_{Str}^{(e)}} > \hat{T}, \frac{P^{(c)}g^{(o)}\zeta(r)}{\sigma^2+I_{Str}^{(oc)}} < T\right)}{\mathbb{P}\left(\frac{Pg\zeta(r)}{\sigma^2+I_{Str}^{(oc)}} < T\right)} \\ &= 2\pi\lambda \times \\ &\frac{\int_0^\infty re^{-\pi\lambda r^2} \mathbb{E}\left[e^{-\frac{\hat{T}\left(\sigma^2+I_{Str}^{(e)}\right)}{P^{(e)}\zeta(r)}} \left(1 - e^{-\frac{T\left(\sigma^2+I_{Str}^{(oc)}\right)}{P^{(c)}\zeta(r)}}\right)\right] dr}{1 - \int_0^\infty 2\pi\lambda re^{-\pi\lambda r^2} e^{-\frac{T\sigma^2}{P^{(c)}\zeta(r)}} \mathbb{E}\left[-\frac{TI_{Str}^{(oc)}}{P^{(c)}\zeta(r)} dr\right] dr} \end{aligned} \tag{12}$$

The expected value of the numerator can be separated into two expectations in which the first one can be obtained from Remark 1; the second one, i.e. $E_2 = \mathbb{E}\left[e^{-\frac{\hat{T} I_{Str}^{(e)}}{P^{(e)}\zeta(r)}} e^{-\frac{T I_{Str}^{(oc)}}{P^{(c)}\zeta(r)}}\right]$ can be computed based on the following steps

$$\begin{aligned} E_2 &= \mathbb{E}\left[e^{-\hat{T}\sum_{j\in\theta_{Str}^{(e)}} \zeta(r_{je})/\zeta(r) g_{je}} e^{-T\sum_{j\in\theta_{Str}} \zeta(r_{jc})/\zeta(r) g_{jc}^{(o)}}\right] \\ &= \mathbb{E}\left[\prod_{j\in\theta_{Str}^{(e)}} \frac{1}{1+T\zeta(r_{je})/\zeta(r)} \frac{1}{1+\hat{T}\zeta(r_{je})/\zeta(r)}\right. \\ &\qquad \left.\prod_{j\in\theta_{Str}\backslash\theta_{Str}^{(e)}} \frac{1}{1+T\zeta(r_{jc})/\zeta(r)}\right] \end{aligned}$$

Since $\theta_{Str}^{(e)}$ and $\theta_{Str}^{(c)}$ are independent Poisson Process, and employing the properties of Probability Generating Function (PGF) with notes that the densities of BSs in $\theta_{Str}^{(e)}$ and $\theta_{Str}\backslash\theta_{Str}^{(c)}$ are $\frac{\lambda}{\Delta}$ and $\frac{\Delta-1}{\Delta}\lambda$ respectively, the expectation equals

$$= e^{-\frac{2\pi\lambda}{\Delta}\int_r^\infty \left[1-\frac{1}{1+T\zeta(r_{je})/\zeta(r)}\frac{1}{1+\hat{T}\zeta(r_{je})/\zeta(r)}\right] r_{je}dr_{je}}$$

$$e^{-\frac{2\pi\lambda(\Delta-1)}{\Delta}\int_r^\infty \left[1-\frac{1}{1+T\zeta(r_{jc})/\zeta(r)}\right] r_{jc}dr_{jc}}$$

$$= \mathscr{L}(T,\hat{T},\frac{1}{\lambda})\mathscr{L}(T,\frac{\Delta-1}{\Delta}\lambda) \tag{13}$$

IV. SIMULATION AND DISCUSSION

The Monte Carlo simulation is utilized throughout this section to verify the analytical results. The values of β are selected at 2, 1 and 0.3 which based on the empirical result in Reference [10]:

- $\beta = 2, \alpha = 3 \times 10^{-5}$: The signal experiences deep fading since the number of obstacles is assumed to be proportional to the square of the distance between the user and BSs (r^2).
- $\beta = 1, \alpha = 3 \times 10^{-2}$: The obstacles is assumed to be uniformly distributed in the area and thus the number of BSs is proportional to r.
- $\beta = 2/3, \alpha = 0.3$: The BSs are regularly distributed on the propagation line of the signal. Compared to the scenarios with $\beta = 1$ and $\beta = 3$, the signal in the scenario with $\beta = 2/3$ can experience the lowest fading.

A. Validation of the Analytical Results

In this section, the CCU Classification Probability is analysed for different values of SNR as in Figure 1. After that, the average coverage probability is analysed with different values of coverage threshold $\hat{T}$ as in Figures 2 and 3. It is reminded that the CCU Classification Probability $A_{FR}^{(c)}(T,\lambda)$ also represents the signal strength on the control channel of the user.

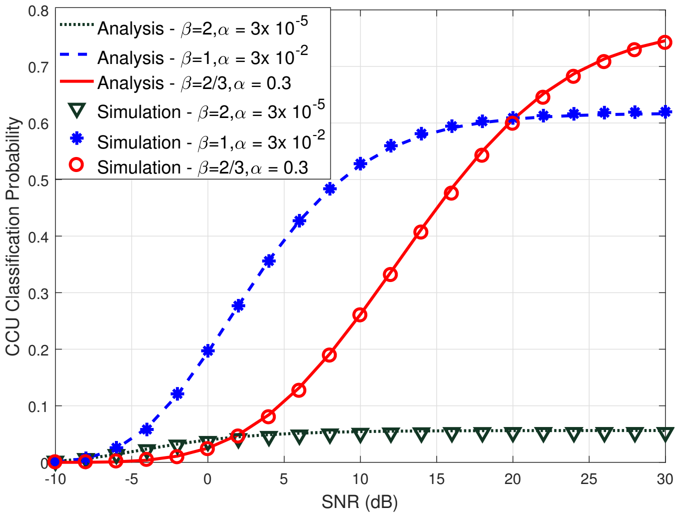

Fig. 1: CCU Classification Probability with different values of SNR

As shown in Figure 1, $A_{FR}^{(c)}(T,\lambda)$ increases significantly with SNR in the case of $\beta = 1$ and $\beta = 2/3$. For example, when SNR increases from 0 dB to 15 dB, $A_{FR}^{(c)}(T,\lambda)$ increases by 201.42% from 0.1972 to 0.5944 in the case $\beta = 1$. While $A_{FR}^{(c)}(T,\lambda)$ lightly increases until $SNR = 5$ dB and keeps constant at 0.05 when $SNR > 5$ dB. Since the environment with $\beta = 2$ is affected by the strongest fading comparing to others with $\beta = 1$ and $\beta = 2/3$, it can be concluded that increasing SNR can not improve the signal strength at the receiver in the case of deep fading.

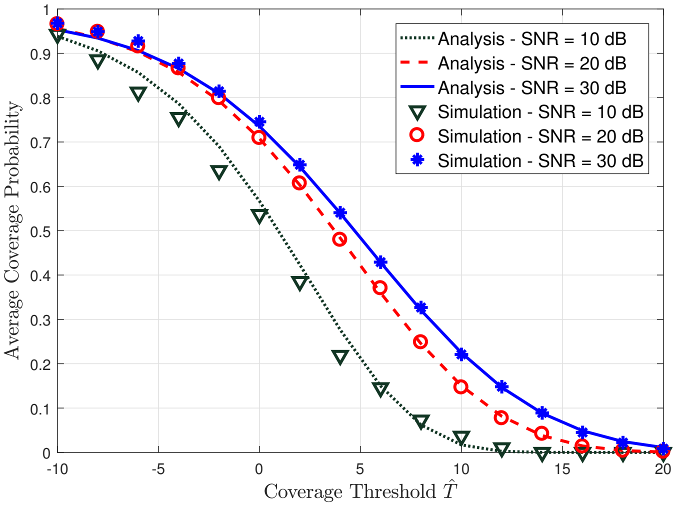

Fig. 2: *(CCU)* Average Coverage Probability with different values of Coverage Threshold $\hat{T}$

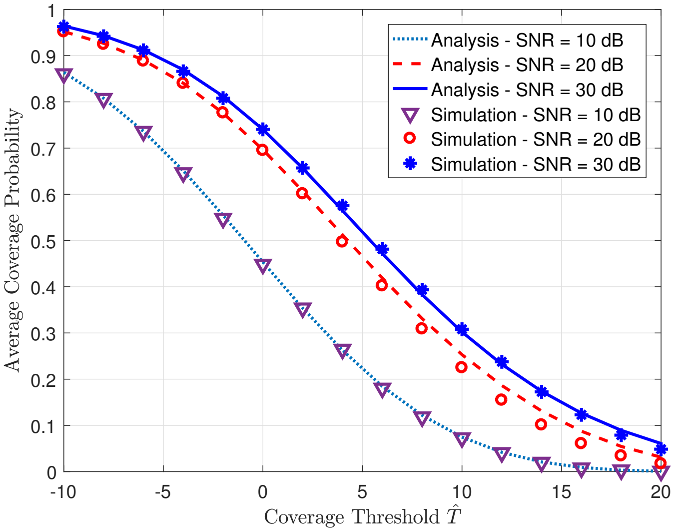

Fig. 3: *(CEU)* Average Coverage Probability with different values of Coverage Threshold $\hat{T}$

B. Performance Analysis

In this section, the performance of CCU and CEU are analysed with different values of the density of BSs.

a) CCU performance: It is observed from Figure 2 that the average coverage probability of CCU reduces when the density of BSs increases. Take $\beta = 2$ for example, the average coverage probability reduces by 79.43% from 0.4446

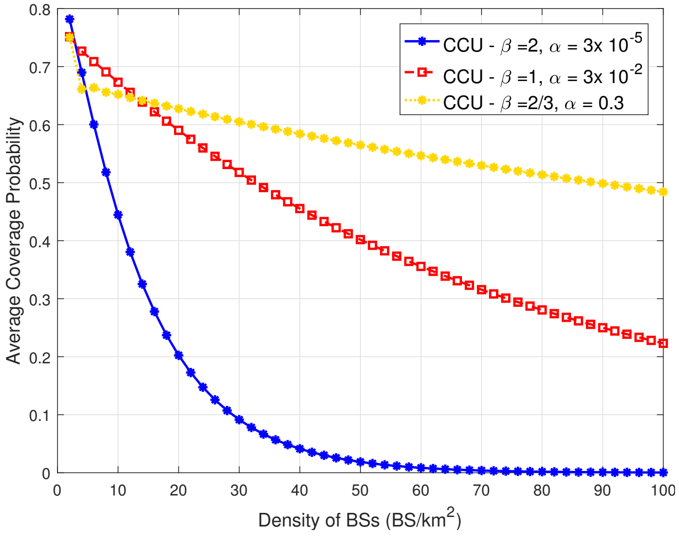

Fig. 4: *(CCU)* Average Coverage Probability with different values of Coverage Threshold $\hat{T}$

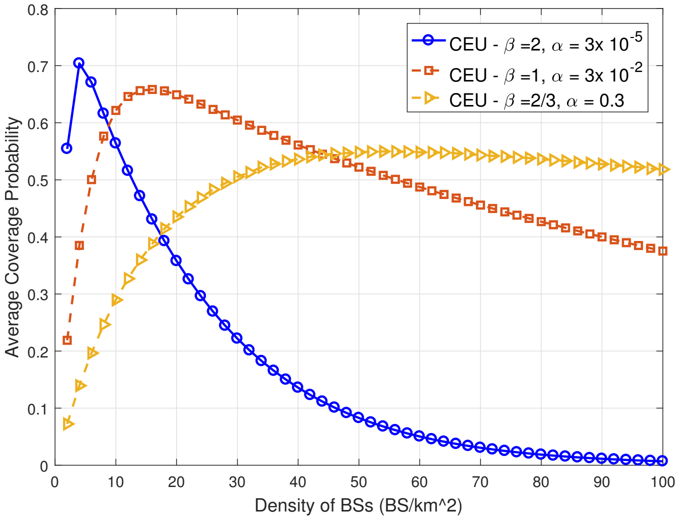

Fig. 5: *(CEU)* Average Coverage Probability with different values of Coverage Threshold $\hat{T}$

to 0.09145 when λ increases from 10 to 30 (BS/km^2). This phenomenon can be explained as follows: when λ increases, the distances between the user and BSs reduces and consequently the signal strength including the serving signal and interfering signals increases. Since the serving signal has a high power while the interfering signals have lower powers. Thus when the distance reduces, the power of the serving signal increases at a lower rate that that of the interfering signal. Consequently, the SINR and user performance reduces.

This finding is very interesting since this performance trend is opposite with the well-known fact for the regular cellular network which stated that the user performance increases to the upper bound with λ [18], [19]. It also contrasts with the conclusion for UDN in [8] which stated that the user performance increases before decreasing when λ increases.

b) CEU performance: It is observed from Figure 3 that the CEU average coverage probability increases to a peak value before going through a decline. This is due to the serving signal increases at a higher rate with small λ and lower rate with high λ thant the interfering signals. The similar trends were found in the literature such as in [8], [10].

V. CONCLUSION

In this paper, Strict FR in UDN with the stretched path loss model was modelled according to the 3GPP recommendations and analysed in term of CCU classification probability and average coverage probability. Throughout the analytical and simulation results, the following interesting findings are found for UDN: *(i)* increasing the SNR, particular transmit power of BSs, cannot improve the received signal strength at the user; *(ii)* for CCUs which have strong received signals on the control channel, increasing the density of BSs will reduce the performance; *(iii)* for CEU which have low SINR on the control channel, the user performance will reach the peak before undergoing a decline when the density of BSs increases.

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