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Nonlinear buckling and post-buckling of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells

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ABSTRACT

This paper aims to investigate the nonlinear buckling and post-buckling of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells resting on elastic foundations in thermal environment. The shells are reinforced by functionally graded eccentrically oblique stiffeners with deviation angles. Two types of sandwich functionally graded double curved shallow shells with the differences of distribution of functionally graded face sheets and homogeneous core are considered. Material properties of the sandwich shells and stiffeners are assumed to vary continuously and smoothly in the thickness direction according to Sigmoid power law. The formula of force and moment resultants and the nonlinear equilibrium equations are established based on the improved Donnell theory and Lekhnitskii's smeared stiffeners technique. The analytical displacement solutions are chosen based on the trigonometric forms satisfying the boundary conditions. The value of critical buckling loads and the load – deflection curves of the shells are obtained by using the Bubnov – Galerkin method. In numerical results; effect of geometrical parameters, elastic foundations, temperature increment, compressive load and oblique stiffeners on the critical buckling loads and post-buckling load – deflection curves of the shells are studied specifically. The obtained results are also compared with others from literature to validate the accuracy of the present method and approach.

1. Introduction

Functionally graded sandwich structures are multi-layered structures which made up of two face sheets and a core. Functionally graded sandwich structures are widely used in aerospace industry due to its excellent load carrying capacity, good heat resistance and effective sound insulation. In order to avoid disaggregate between layers as well as stress- focusing phenomenon, it has been proposed that the two face sheet of functionally graded sandwich structures are made of functionally graded materials with ceramic and metal constituents and the core is made of homogeneous materials. Recently, many investigations have been implemented on the mechanical behaviors of functionally graded sandwich structures. Fazzolari and Carrera [1] carried out the free vibration analysis of doubly curved shallow and deep functionally graded material shells based on the Ritz minimum energy method and the principle of virtual displacements. Mohammadzadeh and Noh [2] presented an analytical approach to investigate nonlinear dynamic responses of sandwich plates with functionally graded faces resting on elastic foundation considering blast loads. Alipour and Shariyat [3] introduced an analytical stress analysis of annular functionally graded sandwich plates with non-uniform shear and normal tractions using a zigzag-elasticity plate theory; Sobhy [4] presented a new accurate four-variable shear deformation plate theory to illustrate the hygrothermal vibration and buckling of functionally graded sandwich plates resting on Winkler-Pasternak elastic foundations. Further, Sofiyev [5] studied the stability of freely supported functionally graded sandwich conical shells subjected to the axial load within the first order shear deformation theory. Based on a layerwise higher-order theory, Pandey and Pradyumna [6] investigated thermal stresses analysis of functionally graded material sandwich beam subjected to thermal shock. In 2011, Wang and Shen [7] presented nonlinear vibration, nonlinear bending and post-buckling analyses for a sandwich plate with functionally graded face sheets resting on an elastic foundation in thermal environments. Recently, Tomar and Talha [8] focused on the influence of material uncertainties on vibration and bending behavior of skewed sandwich functionally graded plates based on Reddy's higher order shear deformation theory.

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In order to reduce the weight but increasing the load carrying capacity, structures are often reinforced by eccentrically stiffeners. In most of studies, stiffeners are supposed to be positioned orthogonal to each other. Hong et al. [9] suggested an improved wave finite element method that can be employed to predict the band-gap characteristics of stiffened shell structures efficiently. Kövesdi et al. [10] dealt with minimum requirements for transverse stiffeners on orthotropic plates subjected to compression; Ahmadi and Rahimi [11] investigated the behavior of grid stiffened composite panel subjected to transverse loading by analytical and experimental approach. A theoretical model to study the dynamic stability and nonlinear vibrations of the stiffened functionally graded cylindrical shell in thermal environments is developed in work of Sheng and Wang [12]. Besides, Zhu et al. [13] established some simple formulae based on the rigid-perfectly plastic method to examine the dynamic response of stiffened rectangular plates repeatedly impacted by a rigid knife-edged striker at any location. In 2014, Duc and Quan [14] studied the nonlinear response of eccentrically stiffened functionally graded cylindrical panels on elastic foundation subjected to mechanical loads. Tao et al. [15,15] proposed a novel foam-core sandwich cylinder to obtain a strong and weight-efficient cylindrical shell with glass fibre-reinforced plastic stiffeners inserted between the faces and the foam core. Up to date, there have been very few researches on the mechanical behaviors of structures reinforced by oblique stiffeners with any deviation angle because of the complexity in calculating the effect of stiffeners on forces and moments of the structures. Won [16] carried out an analysis for a stiffened plate with arbitrarily oblique and equally spaced eccentric stiffeners and Yang et al. [17] researched optimization design of unitized panels with stiffeners in different formats using the evolutionary strategy with covariance matrix adaptation.

Buckling and post-buckling behaviors are the basic problems of structural mechanics. The solutions from these problems provide scientists the critical buckling load and the load carrying capacity of the structures for ensuring the suitability in manufacturing and the safety in using. Gulizzi et al. [18] presented a multi-domain eXtended Ritz formulation, called X-Ritz, for the analysis of buckling and post-buckling of stiffened panels with cracks. Sobhani et al. [19] introduced a comprehensive set of designed and tested glass/epoxy composites to investigate the effect of multiple delaminations on buckling and post-buckling behaviors of laminated composites. An analytical investigation on the nonlinear post-buckling of imperfect eccentrically stiffened thin FGM plates under temperature and resting on elastic foundation using a simple power-law distribution is implemented in work of Duc and Cong [20]. Su et al. [21] considered buckling and post-buckling behavior of titanium alloy stiffened panels under shear load by experiments and numerical analysis. Tanzadeh and Amoushahi [22] developed a finite strip method for buckling and free vibration analysis of piezoelectric laminated composite plates based on various plate theories such as Zigzag, Refined plate and other higher order shear deformation theory by variation of transverse shear strains through plate thickness in the form of parabolic, sine and exponential. Recently, Milazzo et al. [23] established an extended Ritz formulation for the analysis of buckling and post-buckling behavior of cracked composite multilayered plates.

Double curved shell is special structure which is composed of two curved sheets. Due to its curvatures, a double curved shell is able to transfer applied loads by both of in-plane and out-of-plane actions. Double curved shell is widely used in civil, mechanical, architectural, and aerospace engineering. As a result, the researches on static and dynamic stability of double curved shell have been received great attention from scientists around the world. Guo et al. [24] studied dynamic analysis of composite laminated doubly-curved shells with various boundary conditions by a domain decomposition method; Tornabene and Brischetto [25] proposed a comparative study between different analytical and numerical three-dimensional and two-dimensional shell models for the bending analysis of composite and sandwich plates, spherical and doubly-curved shells subjected to a transverse normal load applied at the top surface. Further, Chen et al. [26] investigated free vibration of the functionally graded material sandwich doubly-curved shallow shells under simply supported conditions. Duc et al. [27] introduced analytical solutions for the nonlinear vibration of imperfect functionally graded nanocomposite double curved shallow shells on elastic foundations subjected to mechanical load in thermal environments; Zhai et al. [28] used the first-order shear deformation shell theory to consider the damping properties analysis of composite sandwich doubly-curved shells.

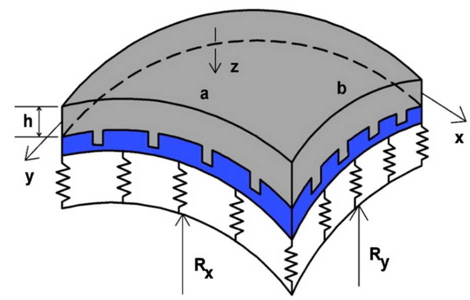

Fig. 1. Geometry and coordinate system of an eccentrically oblique stiffened functionally graded sandwich double curved shallow shells.

Up to date, there is no publication on the mechanical behaviors of oblique stiffened functionally graded sandwich double curved shallow shells. The most difficult part in this type of problem is to determine the mechanism of oblique stiffeners with any deviation angle to the coordinate axis. The calculations of oblique stiffeners are much more complicated than ones of orthogonal stiffeners. The new contribution of our paper is that we established the general formulation of the force and moment resultants from the stress components of the sandwich shell, which is reinforced by oblique stiffeners with any inclinations. The basic equations are established based on the improved Donnell shell theory. The expressions of buckling critical load and the post-buckling curves are determined by using the Galerkin method. In the numerical results, the effect of the oblique stiffeners, type of distribution, elastic foundations, temperature increment and geometrical parameters on the buckling and post-buckling of the sandwich shell are analyzed in details.

2. Eccentrically oblique stiffened functionally graded sandwich double curved shallow shells

Consider an eccentrically oblique stiffened sandwich functionally graded double curved shallow shell of radii of curvature R_x, R_y, length of edges a, b and uniform thickness h resting on elastic foundations. A coordinate system (x, y, z) is established in which (x, y) plane on the middle surface of the shell and z on thickness direction $(-h/2 \le z \le h/2)$ as shown in Fig. 1.

The functionally graded sandwich double curved shallow shell consists of a homogeneous core of thickness h_c and two functionally graded face sheets of thickness h_t and h_b that are perfectly bonded on its top and bottom surfaces. The functionally graded face sheet is made from a mixture of ceramic and metal in which the effective properties are assumed to vary continuously and smoothly in the thickness direction. Two material types of functionally graded sandwich double curved shallow shell are considered as Fig. 2. For type 1A, the homogeneous core is made of metal and for type 1B, the homogeneous core is made of ceramic.

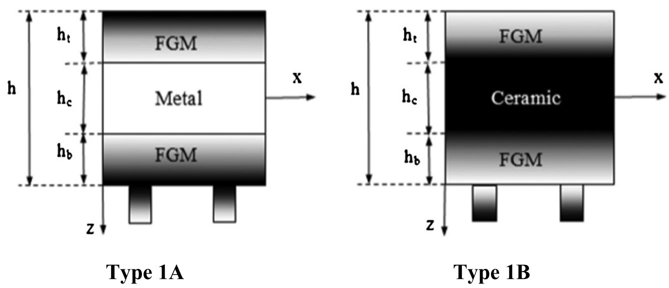

Fig. 2. Two material types of the functionally graded sandwich double curved shallow shells.

In order to ensure the continuity of materials, the effective properties such as the elastic modulus E, the thermal expansion coefficient α and the mass density ρ of the sandwich double curved shallow shells are assumed to vary in the thickness direction according to the Sigmoid law distribution as

$$[E_{sh},\alpha_{sh},\rho_{sh}]=\begin{cases}[E_i,\alpha_i,\rho_i]+[E_{kl},\alpha_{kl},\rho_{kl}](\frac{2z+h}{2h_i})^{k_t},\\ \quad -h/2\le z\le -h/2+h_t,\\ [E_j,\alpha_j,\rho_j],\\ \quad -h/2+h_t\le z\le h/2-h_b,\\ [E_i,\alpha_i,\rho_i]+[E_{kl},\alpha_{kl},\rho_{kl}](\frac{-2z+h}{2h_b})^{k_b},\\ \quad h/2-h_b\le z\le h/2,\end{cases} \tag{1}$$

where k_t, k_b are volume fraction indexes of functionally graded face sheets on the top and bottom surfaces, respectively ($0\le k_t$, $k_b<\infty$); subscripts kl, i and j stand for the constituents of the sandwich shells. For type A, $i=c$, $j=m$, $kl=mc$ and $i=m$, $j=c$, $kl=cm$ for type B with subscripts m and c denote the metal and ceramic constituents of the functionally graded face sheets, respectively and

$$\begin{aligned}&E_{mc}(T)=E_m(T)-E_c(T), &&\alpha_{mc}(T)=\alpha_m(T)-\alpha_c(T),\\ &\rho_{mc}(T)=\rho_m(T)-\rho_c(T), &&\rho_{cm}(T)=\rho_c(T)-\rho_m(T),\\ &E_{cm}(T)=E_c(T)-E_m(T), &&\alpha_{cm}(T)=\alpha_c(T)-\alpha_m(T),\end{aligned} \tag{2}$$

Poisson's ratio ν is assumed to be constant.

The shell is reinforced by eccentrically oblique stiffeners with any inclination as Fig. 3. Stiffeners are arranged along two intersecting directions with the deviation angles between the direction of stiffeners and x axis are γ_x, γ_y, respectively. The width and thickness of stiffeners are denoted by e_x, h_x and e_y, h_y respectively; s_x, s_y are the spacing of the stiffeners. A_x, A_y are the cross-section areas of stiffeners and I_x, I_y are the second moments of cross-section areas of stiffeners.

The oblique stiffeners are also assumed to be functionally graded materials. The effective properties of oblique stiffeners are given by the power law distribution as

$$\begin{aligned}&[E_{sx},\alpha_{sx},\rho_{sx}]=[E_i,\alpha_i,\rho_i]+[E_{kl},\alpha_{kl},\rho_{kl}]\left(\frac{2z-h}{2h_x}\right)^{k_x},\\ &\quad h/2\le z\le h/2+h_x,\\ &[E_{sy},\alpha_{sy},\rho_{sy}]=[E_i,\alpha_i,\rho_i]+[E_{kl},\alpha_{kl},\rho_{kl}]\left(\frac{2z-h}{2h_y}\right)^{k_y},\\ &\quad h/2\le z\le h/2+h_y,\end{aligned} \tag{3}$$

with k_x, k_y are the volume fraction index of stiffeners ($0\le k_x, k_y<\infty$).

The sandwich double curved shallow shell is assumed to rest on elastic foundations of Pasternak model. The interaction between elastic foundations and the sandwich shell is described as

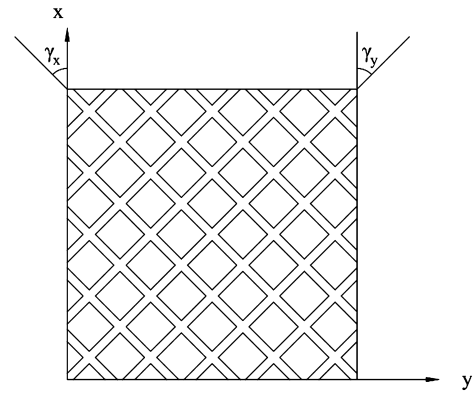

Fig. 3. Configuration of eccentrically oblique stiffeners of sandwich functionally graded double curved shallow shells.

$$q_e=k_1w-k_2\nabla^2w, \tag{4}$$

in which w is the deflection of the sandwich shell, k_1 (Pa/m) is Winkler foundation modulus and k_2 (Pa m) is the shear layer foundation stiffness of Pasternak model.

3. Theoretical formulation

According to the improved Donnell shell theory, the strains at the middle surface $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ and the curvatures $\chi_x, \chi_y, \chi_{xy}$ are expressed to the displacement components u, v, w in the x, y, z coordinate directions as [30]

$$\begin{aligned}&\varepsilon_x^0=\frac{\partial u}{\partial x}-\frac{w}{R_x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, &&\varepsilon_y^0=\frac{\partial v}{\partial y}-\frac{w}{R_y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\\ &\gamma_{xy}^0=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, &&\chi_x=\frac{1}{R_x}\frac{\partial u}{\partial x}+\frac{\partial^2 w}{\partial x^2},\\ &\chi_y=\frac{1}{R_y}\frac{\partial v}{\partial y}+\frac{\partial^2 w}{\partial y^2}, &&\chi_{xy}=\frac{1}{2R_x}\frac{\partial u}{\partial y}+\frac{1}{2R_y}\frac{\partial v}{\partial x}+\frac{\partial^2 w}{\partial x\partial y}.\end{aligned} \tag{5}$$

The strain components across the shell thickness at a distance z from the mid-plane are

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_x\\ \varepsilon_x\end{pmatrix}=\begin{pmatrix}\varepsilon_x^0-z\chi_x\\ \varepsilon_x^0-z\chi_y\\ \varepsilon_x^0-2z\chi_{xy}\end{pmatrix}. \tag{6}$$

Hooke law for a functionally graded sandwich double curved shallow shell, taking into account the thermal effect, is defined as

$$\begin{aligned}&(\sigma_x,\sigma_y)=\frac{E(z)}{1-\nu^2}\big[(\varepsilon_x,\varepsilon_y)+\nu(\varepsilon_x,\varepsilon_y)-(1+\nu)\alpha\Delta T(1,1)\big],\\ &\sigma_{xy}=\frac{E(z)}{2(1+\nu)}\gamma_{xy}.\end{aligned} \tag{7}$$

Because stiffeners are assumed to be thin and the distance between stiffeners is small, thermal stress in the stiffeners is ignored. Therefore, the stress-strain relations of the oblique stiffeners are given as follows

$$\sigma_\gamma^s=E_0(z)\varepsilon_\gamma, \tag{8}$$

where E_0 is Young's modulus of oblique stiffeners.

The force and moment resultants of the eccentrically oblique stiffened functionally graded sandwich double curved shallow shell are determined by Lekhnitskii's smeared technique [29] as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \chi_x \\ \chi_y \\ 2\chi_{xy} \end{Bmatrix} - \begin{bmatrix} \Phi_0/(1-\nu) \\ \Phi_0/(1-\nu) \\ 0 \\ \Phi_1/(1-\nu) \\ \Phi_1/(1-\nu) \\ 0 \end{bmatrix}, \tag{9}$$

in which the detail of coefficients A_{ij}, B_{ij}, D_{ij} $(ij = 11, 12, 16, 22, 26, 66)$, Φ_0, Φ_1 may be found in Appendix A.

The nonlinear equilibrium equations of eccentrically oblique stiffened functionally graded double curved shallow shells based on the improved Donnell shell theory are [30]

$$\begin{aligned} & N_{x,x} + N_{xy,y} - \frac{1}{R_x}(M_{xy,y} + M_{x,x}) = 0, \\ & N_{y,y} + N_{xy,x} - \frac{1}{R_y}(M_{xy,x} + M_{y,y}) = 0, \\ & M_{x,xx} + 2M_{,xyxy} + M_{y,yy} + \frac{N_x}{R_x} + \frac{N_y}{R_y} + \frac{\partial}{\partial x}(N_x w_{,x} + N_{xy} w_{,y}) \\ & \quad + \frac{\partial}{\partial y}(N_y w_{,y} + N_{xy} w_{,x}) - P_x h w_{,xx} + q - k_1 w + k_2 \nabla^2 w = 0, \end{aligned} \tag{10}$$

where P_x is axial compressive loads and q is an external pressure uniformly distributed on the surface of the shell.

Substitution of Eq. (5) into Eq. (9) yields the constitutive relations as

$$\begin{aligned} N_x &= F_{11}\frac{\partial u}{\partial x} + F_{12}\frac{\partial v}{\partial y} + F_{13}\frac{\partial u}{\partial y} + F_{14}\frac{\partial v}{\partial x} - F_{15} w \\ &\quad + \frac{1}{2}A_{11}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}A_{12}\left(\frac{\partial w}{\partial y}\right)^2 + A_{16}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + B_{11}\frac{\partial^2 w}{\partial x^2} + B_{12}\frac{\partial^2 w}{\partial y^2} + 2B_{16}\frac{\partial^2 w}{\partial x \partial y} - \frac{\Phi_o}{1-\nu}, \\ N_y &= F_{21}\frac{\partial u}{\partial x} + F_{22}\frac{\partial v}{\partial y} + F_{23}\frac{\partial u}{\partial y} + F_{24}\frac{\partial v}{\partial x} - F_{25} w \\ &\quad + \frac{1}{2}A_{21}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}A_{22}\left(\frac{\partial w}{\partial y}\right)^2 + A_{26}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + B_{21}\frac{\partial^2 w}{\partial x^2} + B_{22}\frac{\partial^2 w}{\partial y^2} + 2B_{26}\frac{\partial^2 w}{\partial x \partial y} - \frac{\Phi_o}{1-\nu}, \\ N_{xy} &= F_{31}\frac{\partial u}{\partial x} + F_{32}\frac{\partial v}{\partial y} + F_{33}\frac{\partial u}{\partial y} + F_{34}\frac{\partial v}{\partial x} - F_{35} w \\ &\quad + \frac{1}{2}A_{61}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}A_{62}\left(\frac{\partial w}{\partial y}\right)^2 + A_{66}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + B_{61}\frac{\partial^2 w}{\partial x^2} + B_{62}\frac{\partial^2 w}{\partial y^2} + 2B_{66}\frac{\partial^2 w}{\partial x \partial y}, \\ M_x &= F_{41}\frac{\partial u}{\partial x} + F_{42}\frac{\partial v}{\partial y} + F_{43}\frac{\partial u}{\partial y} + F_{44}\frac{\partial v}{\partial x} - F_{45} w \\ &\quad + \frac{1}{2}B_{11}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}B_{12}\left(\frac{\partial w}{\partial y}\right)^2 + B_{16}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2} + 2D_{16}\frac{\partial^2 w}{\partial x \partial y} - \frac{\Phi_1}{1-\nu}, \\ M_y &= F_{51}\frac{\partial u}{\partial x} + F_{52}\frac{\partial v}{\partial y} + F_{53}\frac{\partial u}{\partial y} + F_{54}\frac{\partial v}{\partial x} - F_{55} w \\ &\quad + \frac{1}{2}B_{21}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}B_{22}\left(\frac{\partial w}{\partial y}\right)^2 + B_{26}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + D_{21}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2} + 2D_{26}\frac{\partial^2 w}{\partial x \partial y} - \frac{\Phi_1}{1-\nu}, \\ M_{xy} &= F_{61}\frac{\partial u}{\partial x} + F_{62}\frac{\partial v}{\partial y} + F_{63}\frac{\partial u}{\partial y} + F_{64}\frac{\partial v}{\partial x} - F_{65} w \\ &\quad + \frac{1}{2}B_{61}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}B_{62}\left(\frac{\partial w}{\partial y}\right)^2 + B_{66}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &\quad + D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2} + 2D_{66}\frac{\partial^2 w}{\partial x \partial y}, \end{aligned} \tag{11}$$

with the detail of coefficients F_{ij} $(i = \overline{1 \div 6},\ j = \overline{1 \div 6})$ are given in Appendix B.

Introducing Eq. (11) into Eq. (10), the system of equilibrium equations of eccentrically oblique stiffened functionally graded sandwich double curved shallow shells are rewritten in terms of displacement components u, v, w as

$$\begin{aligned} & L_{11}(u) + L_{12}(v) + L_{13}(w) + P_1(w) = 0, \\ & L_{21}(u) + L_{22}(v) + L_{23}(w) + P_2(w) = 0, \\ & L_{31}(u) + L_{32}(u) + L_{33}(w) + P_3(w) + Q_3(u, v) + R_3(v, w) \\ & - P_x h w_{,xx} - k_1 w + k_2 \nabla^2 w = 0, \end{aligned} \tag{12}$$

with

$$\begin{aligned} L_{11}(u) &= I_{11} u_{,xx} + I_{12} u_{,yy} + I_{13} u_{,xy}, \\ L_{12}(v) &= I_{21} v_{,xx} + I_{22} v_{,yy} + I_{23} v_{,xy}, \\ L_{13}(w) &= -I_{31} w_{,x} - I_{32} w_{,y} + I_{33}(w_{,xx} w_{,y} + w_{,x} w_{,xy}) \\ &\quad + I_{34}(w_{,xy} w_{,y} + w_{,x} w_{,yy}) + I_{35} w_{,xxx} \\ &\quad + I_{36} w_{,yyy} + I_{37} w_{,xyy} + I_{38} w_{,xxy}, \\ P_1(w) &= I_{41}\left(w_{,x}^2\right)_{,y} + I_{42}\left(w_{,y}^2\right)_{,y} + \left(w_{,x}^2\right)_{,x} + I_{44}\left(w_y^2\right)_{,x}, \\ L_{21}(u) &= I_{51} u_{,xx} + I_{52} u_{,yy} + I_{53} u_{,xy}, \\ L_{22}(v) &= I_{61} v_{,xx} + I_{62} v_{,yy} + I_{63} v_{,xy}, \\ P_2(w) &= I_{71}\left(w_{,x}^2\right)_{,y} + I_{72}\left(w_y^2\right)_{,y} + I_{73}\left(w_{,x}^2\right)_{,x} + I_{74}\left(w_{,y}^2\right)_{,x}, \\ L_{23}(w) &= -I_{81} w_{,y} + I_{82} w_{,xxy} + I_{83} w_{,yyy} + I_{84} w_{,xyy} \\ &\quad - I_{85} w_{,x} + I_{86} w_{,xxx} + I_{87}(w_{,xy} w_{,y} + w_{,x} w_{,yy}) \\ &\quad + I_{88}(w_{,xx} w_{,y} + w_{,x} w_{,xy}), \\ L_{31}(u) &= I_{91} u_{,xxx} + I_{92} u_{,yyy} + I_{93} u_{,xxy} + I_{94} u_{,xyy} \\ &\quad + I_{95} u_{,x} + I_{96} u_{,y}, \\ L_{32}(v) &= I_{110} v_{,xxx} + I_{111} v_{,yyy} + I_{112} v_{,xyy} + I_{113} v_{,xxy} \\ &\quad + I_{114} v_{,y} + I_{115} v_{,x}, \\ L_{33}(w) &= -I_{120} w_{,xx} - 2I_{121} w_{,xy} - I_{122} w_{,yy} - I_{123} w \\ &\quad - w_{,xx}\frac{\Phi_0}{1-\nu} + D_{11} w_{,xxxx} + I_{124} w_{,xxyy} + I_{125} w_{,xxxy} \\ &\quad + I_{126} w_{,xyyy} + D_{22} w_{,yyyy} + I_{127} w_{,xx} + I_{128} w_{,yy} \\ &\quad + I_{129} w_{,xy} - w_{,yy}\frac{\Phi_0}{1-\nu}, \\ P_3(w) &= I_{130}(w_{,x})^2 + I_{131}(w_{,y})^2 + \frac{1}{2}B_{11}\left(w_{,x}^2\right)_{,xx} \\ &\quad + \frac{1}{2}B_{12}\left(w_{,y}^2\right)_{,xx} + \frac{1}{2}B_{21}\left(w_{,x}^2\right)_{,yy} + \frac{1}{2}B_{22}\left(w_{,y}^2\right)_{,yy} \\ &\quad + B_{61}\left(w_{,x}^2\right)_{,xy} + B_{62}\left(w_{,y}^2\right)_{,xy} + 2B_{66}(w_{,x} w_{,y})_{,xy} \end{aligned}$$

$$\begin{aligned}
&+B_{16}(w_{,x}w_{,y})_{,xx}+B_{26}(w_{,x}w_{,y})_{,yy}+I_{132}w_{,x}w_{,y}\\
&-w_{,x}^2I_{133}-ww_{,xx}I_{134}+\frac{1}{2}A_{11}\left(w_{,x}^3\right)_{,x}\\
&+\frac{1}{2}A_{12}\left(w_{,y}^2\right)_{,x}w_{,x}+\frac{1}{2}A_{12}w_{,y}^2w_{,xx}+A_{16}w_{,x}^2w_{,xy}\\
&+A_{16}\left(w_{,x}^2\right)_{,x}w_{,y}+B_{11}w_{,xxx}w_{,x}+B_{11}w_{,xx}^2\\
&+B_{12}w_{,xx}w_{,yy}+B_{12}w_{,x}w_{,xyy}+2B_{16}w_{,xx}w_{,xy}\\
&+2B_{16}w_{,x}w_{,xxy}+2B_{66}w_{,xyy}w_{,x}+\frac{1}{2}A_{61}\left(w_{,x}^2\right)_{,x}w_{,y}\\
&+B_{22}w_{,yyy}w_{,y}+B_{22}w_{,yy}^2+2B_{26}w_{,xyy}w_{,y}\\
&+2B_{26}w_{,xy}w_{,yy}-w_{,y}w_{,x}I_{139}-ww_{,xy}I_{139}^*\\
&+\frac{1}{2}A_{61}\left(w_{,x}^3\right)_{,y}+\frac{1}{2}A_{62}w_{,xy}w_{,y}^2\\
&+\frac{1}{2}A_{62}w_{,x}\left(w_{,y}^2\right)_{,y}+A_{66}\left(w_{,x}^2\right)_{,y}w_{,y}+A_{66}w_{,x}^2w_{,yy}\\
&+B_{61}w_{,xxy}w_{,x}+B_{61}w_{,xx}w_{,xy}+B_{62}w_{,yyy}w_{,x}\\
&+B_{62}w_{,yy}w_{,xy}+\frac{1}{2}A_{61}w_{,x}^2w_{,xy}+\frac{1}{2}A_{62}\left(w_{,y}^3\right)_{,x}\\
&+A_{66}w_{,xx}w_{,y}^2+A_{66}w_{,x}\left(w_{,y}^2\right)_{,x}+2B_{66}w_{,xy}w_{,xy}\\
&-w_{,x}w_{,y}I_{135}-ww_{,xy}I_{136}+B_{61}w_{,xxx}w_{,y}\\
&+B_{61}w_{,xx}w_{,xy}+B_{62}w_{,xyy}w_{,y}+B_{62}w_{,yy}w_{,xy}\\
&+2B_{66}w_{,xxy}w_{,y}+2B_{66}w_{,xy}w_{,xy}-w_{,y}^2I_{137}\\
&-ww_{,yy}I_{138}+\frac{1}{2}A_{21}\left(w_{,x}^2\right)_{,y}w_{,y}+\frac{1}{2}A_{21}w_{,x}^2w_{,yy}\\
&+\frac{1}{2}A_{22}\left(w_{,y}^3\right)_{,y}+A_{26}w_{,xy}w_{,y}^2+A_{26}w_{,x}\left(w_{,y}^2\right)_{,y}\\
&+B_{21}w_{,xxy}w_{,y}+B_{21}w_{,xx}w_{,yy}
\end{aligned}\tag{13}$$

in which I_{ij} $(i=\overline{1\div 9},\ j=\overline{1\div 6})$, I_{1kl}, $(k=\overline{1\div 3},\ l=\overline{0\div 9})$ and $Q_3(u,v)$, $R_3(u,v)$ are shown Appendix C.

Eq. (13) is the nonlinear equations in terms of variables u, v and w and they are used to investigate the nonlinear buckling and post-buckling of eccentrically oblique stiffened sandwich double curved shallow shell on elastic foundations in thermal environments.

4. Stability analysis

In the present study, four edges of the eccentrically oblique stiffened sandwich double curved shallow shell are assumed to be simply supported and immovable. The boundary conditions in this case are

$$\begin{aligned}
&w=w_{,xx}=v=u_{,x}=0 \quad \text{at } x=0,a,\\
&w=w_{,yy}=v=u_{,y}=0 \quad \text{at } y=0,b.
\end{aligned}\tag{14}$$

The approximate solutions of the system Eq. (12) satisfying the boundary conditions (14) may be assumed as [14,20,27]

$$\begin{aligned}
&u=U\cos\lambda_m x\sin\delta_n y,\\
&v=V\sin\lambda_m x\cos\delta_n y,\\
&w=W\sin\lambda_m x\sin\delta_n y,
\end{aligned}\tag{15}$$

where $\lambda_m=m\pi/a$, $\delta_n=n\pi/b$; m,n are odd natural numbers representing the number of half waves in the x and y directions and U,V,W are the amplitudes of displacements.

Substitution of Eq. (15) into Eq. (12) and then using Galerkin procedure for the resulting equations yields

$$\begin{aligned}
&l_{11}U+l_{12}V+l_{13}W+l_{14}W^2=0,\\
&l_{21}U+l_{22}V+l_{23}W+l_{24}W^2=0,\\
&l_{31}U+l_{32}V+l_{33}W+l_{34}W^2+l_{35}W^3+l_{36}UW+l_{37}VW\\
&\quad+q\frac{4}{\lambda_m\delta_n}-\frac{\Phi_0}{1-\nu}\frac{4}{\lambda_m\delta_n}\left(\frac{1}{R_x}+\frac{1}{R_y}\right)+\frac{ab}{4}P_xh\lambda_m^2W=0,
\end{aligned}\tag{16}$$

in which

$$\begin{aligned}
&l_{11}=\left(-\frac{ab}{4}\lambda_m^2I_{11}-\frac{ab}{4}\delta_n^2I_{12}\right),\\
&l_{12}=-\frac{ab}{4}\lambda_m\delta_nI_{23},\\
&l_{13}=-\frac{ab}{4}\lambda_m\left(I_{31}+\lambda_m^2I_{35}+\delta_n^2I_{37}\right),\\
&l_{14}=-\left(\frac{4}{9}\delta_nI_{34}+\frac{16}{9}\frac{\lambda_m^2}{\delta_n}I_{43}-\frac{8}{9}I_{44}\delta_n\right),\\
&l_{21}=-\frac{ab}{4}\lambda_m\delta_nI_{53},\\
&l_{22}=-\frac{ab}{4}\left(I_{61}\lambda_m^2+K_{62}\delta_n^2\right),\\
&l_{23}=-\frac{ab}{4}\delta_n\left(I_{81}+\lambda_m^2I_{82}+I_{83}\delta_n^2\right),\\
&l_{24}=\left(-\frac{4}{9}\lambda_mI_{88}+\frac{8}{9}\lambda_mI_{71}-\frac{16}{9}\frac{\delta_n^2}{\lambda_m}I_{72}\right),\\
&l_{31}=\frac{ab}{4}\lambda_m\left(\lambda_m^2I_{91}+\delta_n^2I_{94}-I_{95}\right),\\
&l_{32}=\frac{ab}{4}\delta_n\left(\delta_n^2I_{111}+\lambda_m^2I_{113}-I_{114}\right),\\
&l_{33}=\frac{ab}{4}\begin{pmatrix}\lambda_m^2I_{120}+\delta_n^2I_{122}-I_{123}+D_{11}\lambda_m^4\\ +I_{124}\lambda_m^2\delta_n^2+D_{22}\delta_n^4-I_{127}\lambda_m^2\\ -I_{128}\delta_n^2+(\lambda_m^2+\delta_n^2)\frac{\Phi_0}{1-\nu}\\ -k_1-k_2(\lambda_m^2+\delta_n^2)\end{pmatrix},\\
&l_{34}=\left(I_{51}^*\frac{\lambda_m}{\delta_n}+I_{52}^*\frac{\delta_n}{\lambda_m}+I_{53}^*\right),\\
&l_{35}=\begin{bmatrix}\frac{9ab}{128}\lambda_m^4A_{11}-\frac{ab}{128}A_{12}\delta_n^2\lambda_m^2\\ -\frac{ab}{64}A_{66}\delta_n^2\lambda_m^2+\frac{9ab}{128}\delta_n^4A_{22}\\ -\frac{ab}{64}(\frac{1}{2}A_{21}+A_{66})\lambda_m^2\delta_n^2\end{bmatrix},\\
&l_{36}=\frac{8}{9}\left[\left(A_{11}+B_{11}\frac{1}{R_x}\right)\frac{\lambda_m^2}{\delta_n}+\left(A_{21}+B_{21}\frac{1}{R_x}\right)\delta_n\right.\\
&\qquad\left.-\left(A_{66}+B_{66}\frac{1}{R_x}\right)\delta_n\right],\\
&l_{37}=\frac{8}{9}\left[\left(A_{22}+B_{22}\frac{1}{R_y}\right)\frac{\delta_n^2}{\lambda_m}+\left(A_{12}+B_{12}\frac{1}{R_y}\right)\lambda_m\right.\\
&\qquad\left.-\left(A_{66}+B_{66}\frac{1}{R_y}\right)\lambda_m\right].
\end{aligned}\tag{17}$$

Solving the first two equations of Eq. (16) for U and V yields

$$\begin{aligned}
&V=-\frac{(l_{13}l_{21}-l_{11}l_{23})}{(l_{12}l_{21}-l_{11}l_{22})}W-\frac{(l_{14}l_{21}-l_{11}l_{24})}{(l_{12}l_{21}-l_{11}l_{22})}W^2,\\
&U=-\frac{(l_{13}l_{22}-l_{12}l_{23})}{l_{11}l_{22}-l_{12}l_{21}}W-\frac{(l_{14}l_{22}-l_{12}l_{24})}{l_{11}l_{22}-l_{12}l_{21}}W^2.
\end{aligned}\tag{18}$$

Inserting Eq. (18) into the third equation of Eq. (16) we obtain the equation for determining nonlinear buckling and post-buckling of eccentrically oblique stiffened functionally graded sandwich double curved shallow shell on elastic foundations in thermal environments as

$$m_1 W + m_2 W^2 + m_3 W^3 + P_x h\lambda_m^2 \frac{ab}{4} W + q\frac{4}{\lambda_m \delta_n} - \frac{\Phi_0}{1-\nu}\frac{4}{\lambda_m \delta_n}\left(\frac{1}{R_x}+\frac{1}{R_y}\right) = 0, \tag{19}$$

with

$$\begin{aligned} m_1 &= \left(-l_{31}\frac{(l_{13}l_{22}-l_{12}l_{23})}{l_{11}l_{22}-l_{12}l_{21}} - l_{32}\frac{(l_{13}l_{21}-l_{11}l_{23})}{(l_{12}l_{21}-l_{11}l_{22})} + l_{33}\right),\\ m_2 &= \begin{pmatrix} -l_{31}\frac{(l_{14}l_{22}-l_{12}l_{24})}{l_{11}l_{22}-l_{12}l_{21}} - l_{32}\frac{(l_{14}l_{21}-l_{11}l_{24})}{(l_{12}l_{21}-l_{11}l_{22})} + l_{34} \\ -l_{36}\frac{(l_{13}l_{22}-l_{12}l_{23})}{l_{11}l_{22}-l_{12}l_{21}} - l_{37}\frac{(l_{13}l_{21}-l_{11}l_{23})}{(l_{12}l_{21}-l_{11}l_{22})} \end{pmatrix},\\ m_3 &= \left(l_{35} - l_{36}\frac{(l_{14}l_{22}-l_{12}l_{24})}{l_{11}l_{22}-l_{12}l_{21}} - l_{37}\frac{(l_{14}l_{21}-l_{11}l_{24})}{(l_{12}l_{21}-l_{11}l_{22})}\right). \end{aligned} \tag{20}$$

4.1. Eccentrically oblique stiffened functionally graded sandwich double curved shallow shell under uniform external pressure

Consider an eccentrically oblique stiffened functionally graded sandwich double curved shallow shell with immovable edges and subjected to uniform external pressure on the upper surface of the shell. In this case, Eq. (19) leads to

$$q = m_1\overline{W} + m_2\overline{W}^2 + m_3\overline{W}^3 + \frac{\Phi_0}{1-\nu}\left(\frac{1}{R_x}+\frac{1}{R_y}\right) - P_x h\lambda_m^3\delta_n\frac{ab}{16}\overline{W}, \tag{21}$$

with

$$\begin{aligned} m_1 &= \overline{a_1}\left(l_{31}\frac{(l_{13}l_{22}-l_{12}l_{23})}{l_{11}l_{22}-l_{12}l_{21}} + l_{32}\frac{(l_{13}l_{21}-l_{11}l_{23})}{(l_{12}l_{21}-l_{11}l_{22})} - l_{33}\right),\\ m_2 &= \overline{a_2}\begin{pmatrix} l_{31}\frac{(l_{14}l_{22}-l_{12}l_{24})}{l_{11}l_{22}-l_{12}l_{21}} + l_{32}\frac{(l_{14}l_{21}-l_{11}l_{24})}{(l_{12}l_{21}-l_{11}l_{22})} - l_{34} \\ +l_{36}\frac{(l_{13}l_{22}-l_{12}l_{23})}{l_{11}l_{22}-l_{12}l_{21}} + l_{37}\frac{(l_{13}l_{21}-l_{11}l_{23})}{(l_{12}l_{21}-l_{11}l_{22})} \end{pmatrix},\\ m_3 &= \overline{a_3}\left(-l_{35} + l_{36}\frac{(l_{14}l_{22}-l_{12}l_{24})}{l_{11}l_{22}-l_{12}l_{21}} + l_{37}\frac{(l_{14}l_{21}-l_{11}l_{24})}{(l_{12}l_{21}-l_{11}l_{22})}\right), \end{aligned} \tag{22}$$

and

$$\overline{a_1} = \frac{\lambda_m\delta_n}{4}h, \quad \overline{a_2} = \frac{\lambda_m\delta_n}{4}h^2, \quad \overline{a_3} = \frac{\lambda_m\delta_n}{4}h^3, \quad \overline{W} = W/h. \tag{23}$$

Eq. (21) is used to express post-buckling load – deflection curves of eccentrically oblique stiffened functionally graded double curved shallow shell resting on elastic foundations subjected to uniform external pressure.

4.2. Eccentrically oblique stiffened functionally graded sandwich double curved shallow shell under axial compressive loads

An immovable edges eccentrically oblique stiffened functionally graded double curved shallow shell subjected to axial compressive loads P_x uniformly distributed at two curved edges $x = 0, a$ in the absence of external pressure is considered. From Eq. (19) we have

$$P_x = m_1^* + m_2^*\overline{W} + m_3^*\overline{W}^2, \tag{24}$$

in which

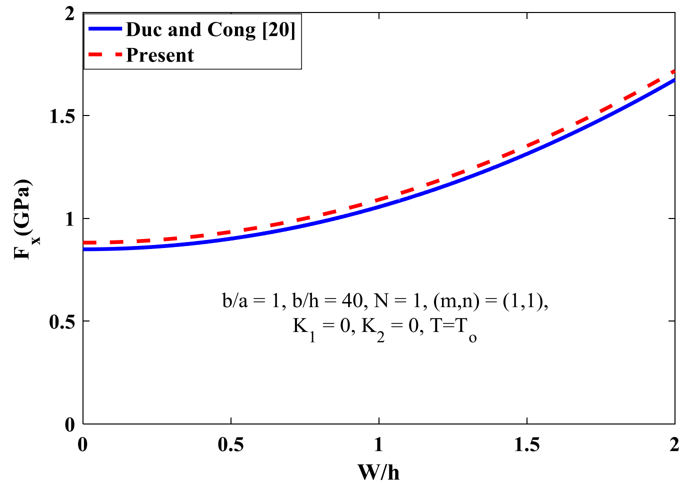

Fig. 4. Comparison on the post-buckling behaviors of eccentrically stiffened functionally graded plates subjected to axial compression.

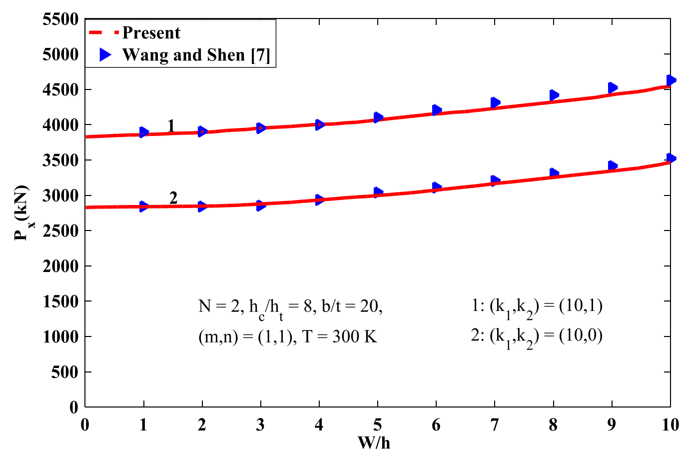

Fig. 5. Comparison of bending analysis of sandwich plates with functionally graded face sheets.

$$m_1^* = \frac{16m_1}{abh\lambda_m^3\delta_n}, \quad m_2^* = \frac{16m_2}{abh\lambda_m^3\delta_n}, \quad m_3^* = \frac{16m_3}{abh\lambda_m^3\delta_n}. \tag{25}$$

Eq. (24) is used to determine the post-buckling load – deflection curves of the eccentrically oblique stiffened functionally graded sandwich double curved shallow shell subjected to axial compressive loads.

The critical buckling compressive load of the sandwich shell may be obtained with $\overline{W} \to 0$ as

$$P_{xupper} = m_1^*. \tag{26}$$

5. Results and discussion

5.1. Validation

To validate the accuracy of the present results, comparisons are carried out for nonlinear post-buckling behaviors of eccentrically stiffened thin functionally graded plates subjected to axial compression in Fig. 4 and nonlinear bending analysis of thick sandwich plates with functionally graded face sheets resting on elastic foundations in thermal environments in Fig. 5 in this paper with results of Duc and Cong [20] based on the classical plate theory and Wang and Shen [7] using the higher order shear deformation plate theory, respectively. Obviously, good agreements are obtained between the present and existing predictions. The small differences could be explained by the differences in theories.

Table 1
Effects of deviation angle of oblique stiffeners (γ_x, γ_y) on the critical buckling compressive loads P_{xcr} (GPa) of functionally graded sandwich double curved shallow shells.

(γ_x, γ_y)	$(\frac{\pi}{4}, \frac{\pi}{4})$	$(\frac{\pi}{6}, \frac{\pi}{6})$	$(\frac{\pi}{2}, 0)$
1A	11.53	6.302	4.66
1B	15.67	10.07	7.646

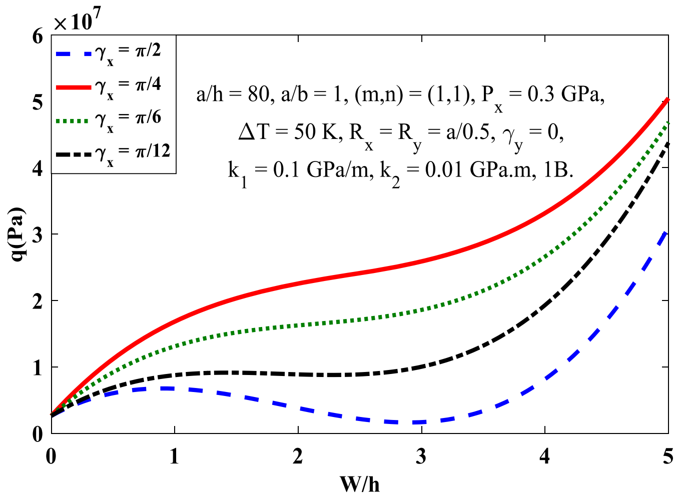

Fig. 6. Effects of deviation angle of oblique stiffeners γ_x on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure.

5.2. Critical buckling loads and nonlinear post-buckling curves

In this section, we consider functionally graded sandwich double curved shallow shell that consists of aluminum (metal) and alumina (ceramic) to investigate the influences of deviation angle of stiffeners, types of sandwich shells, elastic foundations, temperature increment and geometrical parameters on the critical buckling load and nonlinear post-buckling curves of the sandwich shells. The material properties of the constituents are $E_m = 70$ GPa, $\alpha_m = 23 \times 10^{-6}$ K^{-1}, $E_c = 380$ GPa, $\alpha_c = 7.4 \times 10^{-6}$ K^{-1} whereas Poisson's ratio is chosen to be 0.3. The geometrical parameters of the oblique stiffeners are $h_x = h_y = 0.003$ m, $s_x = s_y = 0.1$ m, $e_x = e_y = 0.02$ m.

Problem 1: Influences of deviation angle of oblique stiffeners

Table 1 considers the effects of deviation angle of oblique stiffeners (γ_x, γ_y) on the critical buckling loads P_{xcr} of functionally graded sandwich double curved shallow shells subjected to axial compression. Three cases of deviation angle are considered. As expected, the critical buckling compressive load of the sandwich shell which is reinforced by oblique stiffeners with deviation angle $(\gamma_x, \gamma_y) = (\frac{\pi}{4}, \frac{\pi}{4})$ is highest and the critical buckling compressive load of the sandwich shell which is reinforced by oblique stiffeners with deviation angle $(\gamma_x, \gamma_y) = (\frac{\pi}{2}, 0)$ is lowest of all. The comparison of the critical buckling compressive load between two types of sandwich shells 1A and 1B is also shown in Table 1. It is easy to see that the critical buckling load of the sandwich shell with type 1B is higher than one of the sandwich shell with type 1A.

Figs. 6 and 7 show the effect of deviation angle γ_x on the nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells in case of type 1B subjected to external pressure and axial compression, respectively. As can be observed, the load carrying capacity of the sandwich shell with the deviation angle $\gamma_x = \pi/2$ is the lowest and the load carrying capacity of the sandwich shell with the deviation angle $\gamma_x = \pi/4$ is the highest of all. In other words, the oblique stiffeners enhance the mechanical properties of the sandwich shells more than the orthogonal stiffeners.

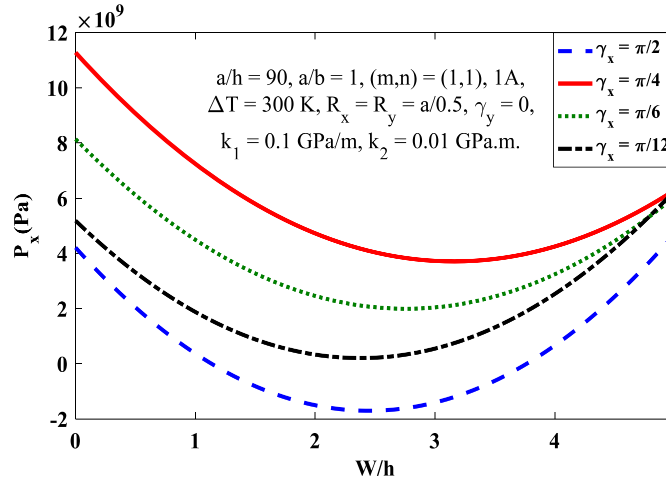

Fig. 7. Effects of deviation angle of oblique stiffeners γ_x on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to axial compression.

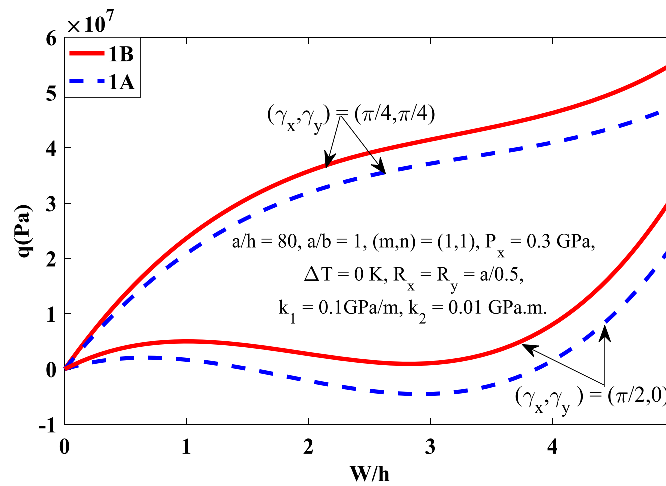

Fig. 8. Nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure in two cases of types 1A and 1B.

Problem 2: Influences of types of functionally graded sandwich double curved shallow shells

Figs. 8 and 9 compare the nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure and axial compression between two types 1A and 1B, respectively. The results show that the load carrying capacity of the sandwich shell with type 1B is higher than one of the sandwich shell with type 1A.

Problem 3: Influences of elastic foundations

Table 2 indicates the effects of elastic foundations on the critical buckling compressive loads of the functionally graded sandwich double curved shallow shells in case of type 1B. The deviation angles of oblique stiffeners are chosen as $(\gamma_x, \gamma_y) = (\frac{\pi}{6}; \frac{\pi}{6})$, the geometrical parameters are $a/R_x = a/R_y = 1/5$, $h_t = h/4$ and the temperature increment is $\Delta T = 300$ K. The results from this table show that the elastic foundations have positive influences on the critical buckling load of the sandwich shells. The critical buckling compressive load of the functionally graded sandwich shell

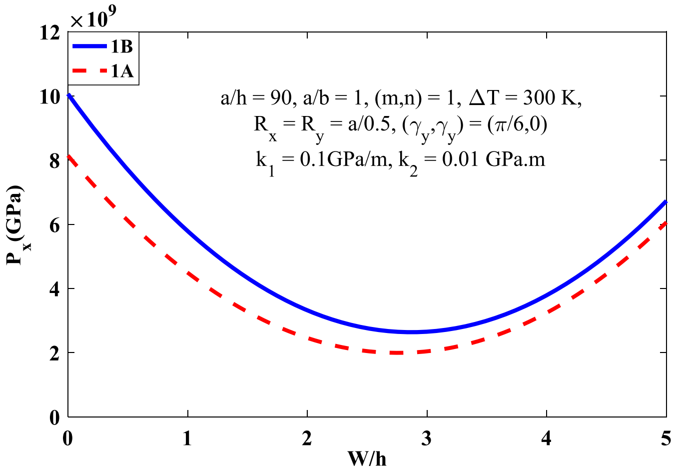

Fig. 9. Nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to axial compression in two cases of types 1A and 1B.

Table 2
Effects of elastic foundations on the critical buckling compressive loads P_{xcr} (GPa) of functionally graded sandwich double curved shallow shells.

(m, n) \ k_1 (GPa/m), k_2 (GPa m)	0, 0	0.1, 0	0.1, 0.01
(1, 1)	7.424	9.066	10.07
(3, 3)	5.415	8.512	9.787
(1, 5)	4.121	7.059	9.102
(3, 7)	3.954	6.623	8.832

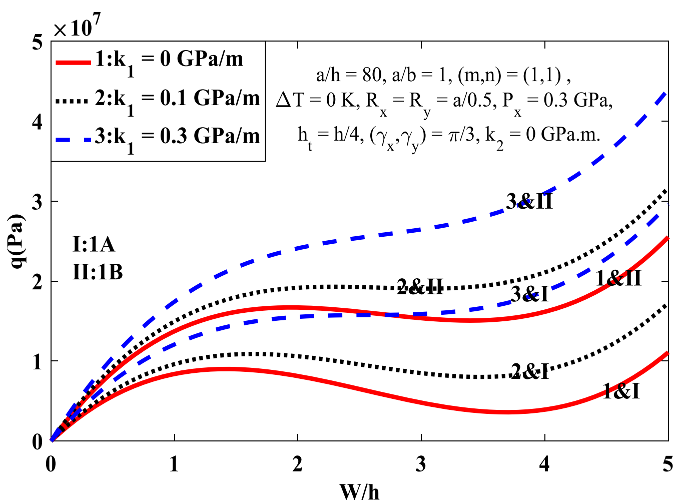

Fig. 10. Effect of the Winkler foundation on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure.

rises considerably when increasing the coefficients k_1 and k_2 of elastic foundations. The effect of modes (m, n) on the critical buckling load of the sandwich shells is also given in Table 2. It is seen that the sandwich shell with modes $(m, n) = (1, 1)$ has the highest value of the critical buckling load.

Figs. 10 and 11 illustrate the effects of the Winkler foundation modulus k_1 and the shear layer foundation stiffness of Pasternak model k_2 on the nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure, respectively. Clearly, post-buckling load carrying capacity of the sandwich shallow shells increases significantly due to the support of elastic foundations. Furthermore, the effect of Pasternak foundation with coefficient k_2

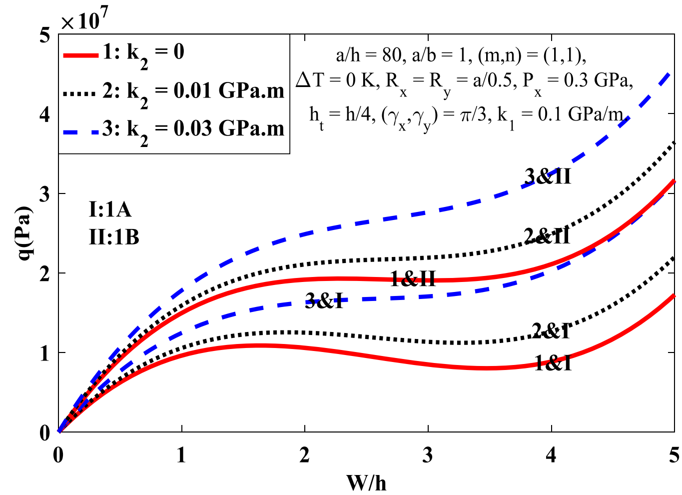

Fig. 11. Effect of the Pasternak foundation on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to external pressure.

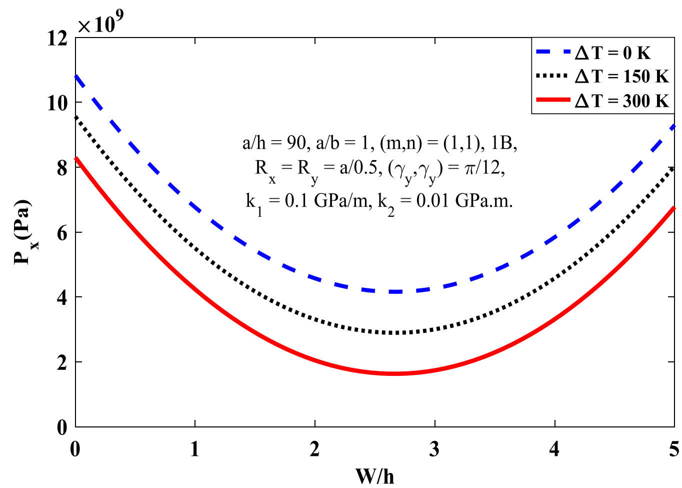

Fig. 12. Effects of temperature increment on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells.

on the load carrying capacity of the sandwich shell is stronger than one of Winkler foundation with coefficient k_1.

Problem 4: Influences of temperature increment and pre-loaded axial compression

Fig. 12 illustrates the effects of temperature increment ΔT on the nonlinear post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells under axial compression with $a/h = 90$, $a/b = 1$, $m = n = 1$, $a/R_x = a/R_y = 0.5$. As can be observed, the temperature increment has a negative influence on the load carrying capacity of the sandwich shell in post-buckling state. Specifically, the load carrying capacity of the sandwich shell decreases along with the increase of temperature increment.

Fig. 13 describes the effects of pre-loaded axial compression P_x on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to uniformly external pressure in two cases of distribution of functionally graded face sheets and homogeneous core. Although the effect of P_x is quite weak, it can be seen that the load carrying capacity of the sandwich shell reduces when P_x increases. Again, the results from Fig. 13 also show that the load

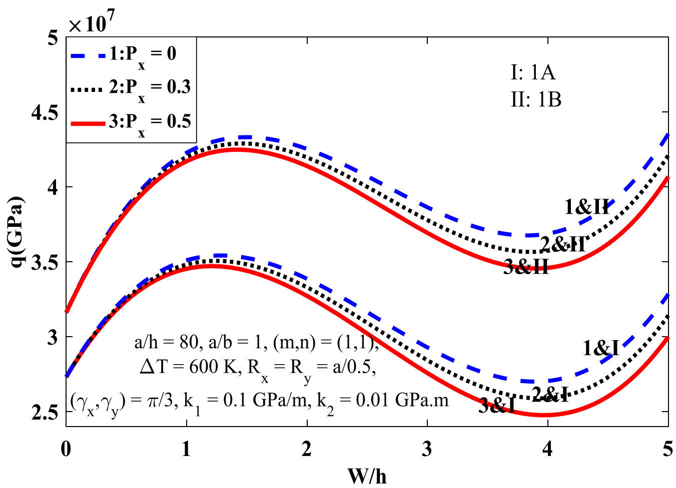

Fig. 13. Effects of pre-loaded axial compression on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells.

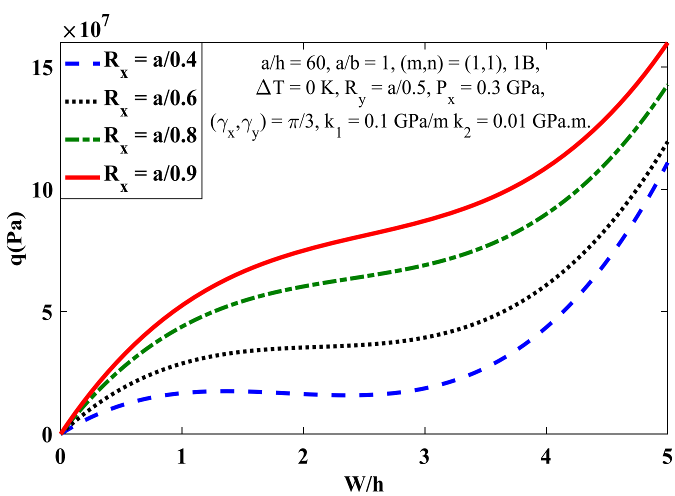

Fig. 14. Effects of ratio a/R_x on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells.

carrying capacity of the sandwich shells with type 1B is higher than one of the sandwich shells with type 1A.

Problem 5: Influences of geometrical parameters

The effect of a/R_x ratio on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to uniformly external pressure is shown in Fig. 14. Clearly, the higher the ratio a/R_x is, the higher the loading capacity of the sandwich shells is. This conclusion is easy to explain because an increase of a/R_x ratio makes the sandwich shells become shallower. Consequently, the load carrying capacity of the shells rises significantly.

Fig. 15 presents the effect of a/b ratio on the post-buckling curves of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells subjected to uniformly external pressure. There cases of a/b ratio: 1, 1.5, 2 are considered. As can be seen, the load carrying capacity of the sandwich shells increase when reducing a/b ratio.

6. Conclusion remarks

This paper presents an analytical approach to study the nonlinear buckling and post-buckling of eccentrically oblique stiffened

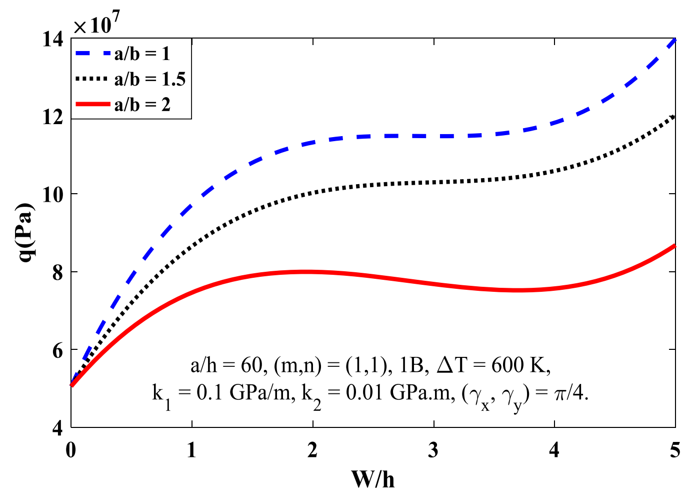

Fig. 15. Effects of a/b ratio on the post-buckling behaviors of eccentrically oblique stiffened sandwich functionally graded double curved shallow shells.

sandwich functionally graded double curved shallow shells subjected to axial compressive load, uniform external pressure and thermal loads. The shell is reinforced by different oblique stiffeners with any deviation angle to the coordinate axis. The basic equations are established based on improved Donnell shell theory and Lekhnitskii's smeared stiffener technique then solved by Galerkin method. The effects of geometrical parameters, oblique stiffeners, temperature increment, elastic foundations and types of distribution on the critical loads and loads – deflection curves of the shells are examined. The following conclusions are obtained from numerical results:

- The effect of oblique stiffeners on the load carrying capacity and the critical buckling load of the eccentrically oblique stiffened sandwich functionally graded double curved shallow shells is stronger than orthogonal stiffeners. Specifically, the shell with the deviation angle $(\gamma_x, \gamma_y) = (\pi/4, \pi/4)$ has the best load carrying capacity and critical buckling load.
- The sandwich functionally graded shell with type of distribution 1A (functionally graded face sheets and metal core) is better than the sandwich functionally graded shell with type of distribution 1B (functionally graded face sheets and ceramic core). The critical buckling load and the load carrying capacity of the shell with type 1A are higher than ones of the shell with type 1B.
- The elastic foundations have positive effect on the buckling and post-buckling behaviors of the eccentrically oblique stiffened sandwich functionally graded double curved shallow shells. Furthermore, the effect of Pasternak foundation is stronger than Winkler one.
- The critical buckling load and the load carrying capacity of the sandwich shell decrease significantly by the presence of temperature.
- The geometrical parameters have considerable influences on the nonlinear buckling and post-buckling behaviors of functionally graded sandwich shell.

Conflict of interest statement

The authors declare no conflict of interest.

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Appendix A

$$(A_{11}, A_{12}) = (\overline{A_{11}}, \overline{A_{12}}) + \frac{E_{1x}A_x}{s_x}\left(\cos^4_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{1y}A_y}{s_y}\left(\cos^4_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

$$(A_{22}, A_{16}) = (\overline{A_{22}}, 1) + \frac{E_{1x}A_x}{s_x}\left(\sin^4_{\gamma_x}, \sin_{\gamma_x}\cos^3_{\gamma_x}\right) + \frac{E_{1y}A_y}{s_y}\left(\sin^4_{\gamma_y}, \sin_{\gamma_y}\cos^3_{\gamma_y}\right),$$

$$(A_{26}, A_{66}) = (1, \overline{A_{66}}) + \frac{E_{1x}A_x}{s_x}\left(\sin^3_{\gamma_x}\cos_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{1y}A_y}{s_y}\left(\sin^3_{\gamma_y}\cos_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

$$(B_{11}, B_{12}) = (\overline{B_{11}}, \overline{B_{12}}) + \frac{E_{2x}A_x}{s_x}\mathrm{e}_x\left(\cos^4_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{2y}A_y}{s_y}\mathrm{e}_y\left(\cos^4_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

$$(B_{22}, B_{16}) = (\overline{B_{22}}, 1) + \frac{E_{2x}A_x}{s_x}\mathrm{e}_x\left(\sin^4_{\gamma_x}, \sin_{\gamma_x}\cos^3_{\gamma_x}\right) + \frac{E_{2y}A_y}{s_y}e_y\left(\sin^4_{\gamma_y}, \sin_{\gamma_y}\cos^3_{\gamma_y}\right),$$

$$(B_{26}, B_{66}) = (1, \overline{B_{66}}) + \frac{E_{2x}A_x}{s_x}\mathrm{e}_x\left(\sin^3_{\gamma_x}\cos_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{2y}A_y}{s_y}\mathrm{e}_y\left(\sin^3_{\gamma_y}\cos_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

$$(D_{11}, D_{12}) = (\overline{D_{11}}, \overline{D_{12}}) + \frac{E_{3x}I_x}{s_x}\left(\cos^4_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{3y}I_y}{s_y}\left(\cos^4_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

$$(D_{22}, D_{16}) = (\overline{D_{22}}, 1) + \frac{E_{3x}I_x}{s_x}\left(\sin^4_{\gamma_x}, \sin_{\gamma_x}\cos^3_{\gamma_x}\right) + \frac{E_{3y}I_y}{s_y}\left(\sin^4_{\gamma_y}, \sin_{\gamma_y}\cos^3_{\gamma_y}\right),$$

$$(D_{26}D_{66}) = (1, \overline{D_{66}}) + \frac{E_{3x}I_x}{s_x}\left(\sin^3_{\gamma_x}\cos_{\gamma_x}, \sin^2_{\gamma_x}\cos^2_{\gamma_x}\right) + \frac{E_{3y}I_y}{s_y}\left(\sin^3_{\gamma_y}\cos_{\gamma_y}, \sin^2_{\gamma_y}\cos^2_{\gamma_y}\right),$$

and

$$\overline{A_{11}} = \overline{A_{22}} = \frac{E_1}{1-\nu^2}, \qquad \overline{A_{12}} = \nu\overline{A_{11}}, \qquad \overline{A_{66}} = \frac{E_1}{2(1+\nu)},$$

$$\overline{B_{11}} = \overline{B_{11}} = \frac{E_2}{1-\nu^2}, \qquad \overline{B_{12}} = \nu\overline{B_{11}}, \qquad \overline{B_{66}} = \frac{E_2}{2(1+\nu)},$$

$$\overline{D_{11}} = \overline{D_{11}} = \frac{E_3}{1-\nu^2}, \qquad \overline{D_{12}} = \nu\overline{D_{11}}, \qquad \overline{D_{66}} = \frac{E_3}{2(1+\nu)},$$

$$(E_1, E_2, E_3) = \int_{-h/2}^{h/2} \left(1, z, z^2\right) E_{sh}(z)dz,$$

$$(E_{1i}, E_{2i}, E_{3i}) = \int_{h/2}^{h/2+h_i} \left(1, z, z^2\right) E_{si}(z)dz,$$

$$(\Phi_0, \Phi_1) = \int_{-h/2}^{h/2} E_{sh}(z)\alpha_{sh}(z)\Delta T(1, z)dz, \quad i = x, y.$$

Appendix B

$$F_{11} = A_{11} + B_{11}\frac{1}{R_x}, \qquad F_{12} = A_{12} + B_{12}\frac{1}{R_y},$$

$$F_{13} = A_{16} + B_{16}\frac{1}{R_x}, \qquad F_{14} = A_{16} + B_{16}\frac{1}{R_y},$$

$$F_{15} = A_{11}\frac{1}{R_x} + A_{12}\frac{1}{R_y}, \qquad F_{21} = A_{21} + B_{21}\frac{1}{R_x},$$

$$F_{22} = A_{22} + B_{22}\frac{1}{R_y}, \qquad F_{23} = A_{26} + B_{26}\frac{1}{R_x},$$

$$F_{24} = A_{26} + B_{26}\frac{1}{R_y}, \qquad F_{25} = A_{21}\frac{1}{R_x} + A_{22}\frac{1}{R_y},$$

$$F_{31} = A_{61} + B_{61}\frac{1}{R_x}, \qquad F_{32} = A_{62} + B_{62}\frac{1}{R_y},$$

$$F_{33} = A_{66} + B_{66}\frac{1}{R_x}, \qquad F_{34} = A_{66} + B_{66}\frac{1}{R_y},$$

$$F_{35} = A_{61}\frac{1}{R_x} + A_{62}\frac{1}{R_y}, \qquad F_{41} = B_{11} + D_{11}\frac{1}{R_x},$$

$$F_{42} = B_{12} + D_{12}\frac{1}{R_y}, \qquad F_{43} = B_{16} + D_{16}\frac{1}{R_x},$$

$$F_{44} = B_{16} + D_{16}\frac{1}{R_y}, \qquad F_{45} = B_{11}\frac{1}{R_x} + B_{12}\frac{1}{R_y},$$

$$F_{51} = B_{21} + D_{21}\frac{1}{R_x}, \qquad F_{52} = B_{22} + D_{22}\frac{1}{R_y},$$

$$F_{53} = B_{26} + D_{26}\frac{1}{R_x}, \qquad F_{54} = B_{26} + D_{26}\frac{1}{R_y},$$

$$F_{55} = B_{21}\frac{1}{R_x} + B_{22}\frac{1}{R_y}, \qquad F_{61} = B_{61} + D_{61}\frac{1}{R_x},$$

$$F_{62} = B_{62} + D_{62}\frac{1}{R_y}, \qquad F_{63} = B_{66} + D_{66}\frac{1}{R_x},$$

$$F_{64} = B_{66} + D_{66}\frac{1}{R_y}, \qquad F_{65} = B_{61}\frac{1}{R_x} + D_{62}\frac{1}{R_y}.$$

Appendix C

$$I_{11} = A_{11} - \frac{D_{11}}{R_x^2}, \qquad I_{12} = A_{66} - \frac{D_{66}}{R_x^2},$$

$$I_{13} = A_{16} + A_{61} - \frac{1}{R_x^2}(D_{61} + D_{16}),$$

$$I_{21} = A_{16} + B_{16}\left(\frac{1}{R_y} - \frac{1}{R_x}\right) - \frac{D_{16}}{R_xR_y},$$

$$I_{22} = A_{62} + B_{62}\left(\frac{1}{R_y} - \frac{1}{R_x}\right) - \frac{D_{62}}{R_xR_y},$$

$$I_{23} = A_{12} + A_{66} + (B_{12} + B_{66})\left(\frac{1}{R_y} - \frac{1}{R_x}\right) - \frac{1}{R_xR_y}(D_{12} + D_{66}),$$

$$I_{31}=\left(A_{11}-B_{11}\frac{1}{R_x}\right)\frac{1}{R_x}+\left(A_{12}-\frac{1}{R_x}B_{12}\right)\frac{1}{R_y},$$
$$I_{32}=\left(A_{61}-\frac{1}{R_x}B_{61}\right)\frac{1}{R_x}+\left(A_{62}-\frac{D_{62}}{R_x}\right)\frac{1}{R_y},$$
$$I_{33}=A_{16}-\frac{1}{R_x}B_{16},\qquad I_{34}=A_{66}-\frac{1}{R_x}B_{66},$$
$$I_{35}=\left(B_{11}-\frac{1}{R_x}D_{11}\right),\qquad I_{130}=\frac{1}{2R_x}A_{11}+\frac{1}{2R_y}A_{21},$$
$$I_{131}=\frac{1}{2R_x}A_{12}+\frac{1}{2R_y}A_{22},\qquad I_{132}=\frac{A_{16}}{R_{,x}}+\frac{A_{26}}{R_y},$$
$$I_{133}=A_{11}\frac{1}{R_x}+A_{12}\frac{1}{R_y},\qquad I_{134}=A_{11}\frac{1}{R_x}+A_{12}\frac{1}{R_y},$$
$$I_{135}=A_{61}\frac{1}{R_x}+A_{62}\frac{1}{R_y},\qquad I_{136}=A_{61}\frac{1}{R_x}+A_{62}\frac{1}{R_y},$$
$$I_{137}=A_{21}\frac{1}{R_x}+A_{22}\frac{1}{R_y},\qquad I_{138}=A_{21}\frac{1}{R_x}+A_{22}\frac{1}{R_y},$$
$$I_{139}=A_{61}\frac{1}{R_x}+A_{62}\frac{1}{R_y},\qquad I_{140}\left(A_{61}\frac{1}{R_x}+A_{62}\frac{1}{R_y}\right),$$
$$I^*_{51}=\frac{8}{9}\left(\frac{3}{2R_x}A_{11}+\frac{1}{2R_y}A_{21}+A_{12}\frac{1}{R_y}\right),$$
$$I^*_{52}=\frac{8}{9}\left(\frac{1}{2R_x}A_{12}+\frac{3}{2R_y}A_{22}+A_{21}\frac{1}{R_x}\right),$$
$$I^*_{53}=\frac{16}{9}\frac{\lambda_m^3}{\delta_n}B_{11}+\frac{4}{9}B_{12}\lambda_m\delta_n+\frac{4}{9}B_{21}\lambda_m\delta_n+\frac{16}{9}\frac{\delta_n^3}{\lambda_m}B_{22}-\frac{8}{9}B_{66}\lambda_m\delta_n$$
$$I_{36}=B_{62}-\frac{1}{R_x}D_{62},\qquad I_{37}=B_{12}+2B_{66}-\frac{1}{R_x}(2D_{66}+D_{12}),$$
$$I_{38}=2B_{16}+B_{61}-\frac{1}{R_x}(D_{61}+2D_{16}),$$
$$I_{41}=\frac{1}{2}A_{61}-\frac{1}{2R_x}B_{61},\qquad I_{42}=\frac{1}{2}A_{62}-\frac{1}{2R_x}B_{62},$$
$$I_{43}=\frac{1}{2}A_{11}-\frac{1}{2R_x}B_{11},\qquad I_{44}=\frac{1}{2}A_{12}-\frac{1}{2R_x}B_{12},$$
$$I_{51}=A_{61}+B_{61}\left(\frac{1}{R_x}-\frac{1}{R_y}\right)-\frac{D_{61}}{R_yR_x},$$
$$I_{52}=A_{26}+B_{26}\left(\frac{1}{R_x}-\frac{1}{R_y}\right)-\frac{D_{26}}{R_xR_y},$$
$$I_{53}=A_{21}+A_{66}+(B_{21}+B_{66})\left(\frac{1}{R_x}-\frac{1}{R_y}\right)-\frac{(D_{66}+D_{21})}{R_xR_y},$$
$$I_{61}=\left(A_{66}-\frac{D_{66}}{R_y^2}\right),\qquad I_{62}\left(A_{22}-\frac{D_{22}}{R_y^2}\right),$$
$$I_{63}\left[A_{26}+A_{62}-\frac{(D_{62}+D_{26})}{R_y^2}\right],$$
$$I_{71}=\frac{1}{2}A_{21}-\frac{1}{2R_y},\qquad I_{72}=\frac{1}{2}A_{22}-\frac{1}{2R_y}B_{22},$$
$$I_{73}=\frac{1}{2}A_{61}-\frac{1}{2R_y}B_{61},\qquad I_{74}=\frac{1}{2}A_{62}-\frac{1}{2R_y}B_{62},$$
$$I_{81}=A_{21}\frac{1}{R_x}-\frac{1}{R_y}\left(B_{21}\frac{1}{R_x}+B_{22}\frac{1}{R_y}-A_{22}\right),$$
$$I_{82}=B_{21}+2B_{66}-\frac{1}{R_y}(D_{21}+2D_{66}),$$
$$I_{83}=B_{22}-\frac{1}{R_y}D_{22},\qquad I_{84}=2B_{26}+B_{62}-\frac{1}{R_y}(D_{62}+2D_{26}),$$
$$I_{85}=A_{61}\frac{1}{R_x}+A_{62}\frac{1}{R_y}-\frac{1}{R_y}\left(B_{61}\frac{1}{R_x}+D_{62}\frac{1}{R_y}\right),$$
$$I_{86}=B_{61}-\frac{1}{R_y}D_{61},$$
$$I_{87}=A_{26}-\frac{1}{R_y}B_{26},\qquad I_{88}=A_{66}-\frac{1}{R_y}B_{66},$$
$$I_{91}=B_{11}+D_{11}\frac{1}{R_x},\qquad I_{92}=B_{26}+D_{26}\frac{1}{R_x},$$
$$I_{93}=B_{16}+D_{16}\frac{1}{R_x}+2\left(B_{61}+D_{61}\frac{1}{R_x}\right),$$
$$I_{94}=B_{21}+D_{21}\frac{1}{R_x}+2\left(B_{66}+D_{66}\frac{1}{R_x}\right),$$
$$I_{95}=\frac{1}{R_x}\left(A_{11}+B_{11}\frac{1}{R_x}\right)+\frac{1}{R_y}\left(A_{21}+B_{21}\frac{1}{R_x}\right),$$
$$I_{96}=\frac{1}{R_x}\left(A_{16}+B_{16}\frac{1}{R_x}\right)+\frac{1}{R_y}\left(A_{26}+B_{26}\frac{1}{R_x}\right),$$
$$I_{110}=B_{16}+D_{16}\frac{1}{R_y},\qquad I_{111}=B_{22}+D_{22}\frac{1}{R_y},$$
$$I_{112}=B_{26}+D_{26}\frac{1}{R_y}+2\left(B_{62}+D_{62}\frac{1}{R_y}\right),$$
$$I_{113}=B_{12}+D_{12}\frac{1}{R_y}+2\left(B_{66}+D_{66}\frac{1}{R_y}\right),$$
$$I_{114}=\frac{1}{R_x}\left(A_{12}+B_{12}\frac{1}{R_y}\right)+\frac{1}{R_y}\left(A_{22}+B_{22}\frac{1}{R_y}\right),$$
$$I_{115}=\frac{1}{R_x}\left(A_{16}+B_{16}\frac{1}{R_y}\right)+\frac{1}{R_y}\left(A_{26}+B_{26}\frac{1}{R_y}\right),$$
$$\begin{aligned}Q_3(u,w)&=u_{,xx}w_{,x}\left(A_{11}+B_{11}\frac{1}{R_x}\right)+u_{,x}w_{,xx}\left(A_{11}+B_{11}\frac{1}{R_x}\right)\\&\quad+u_{,y}w_{,xx}\left(A_{16}+B_{16}\frac{1}{R_x}\right)u_{,yx}w_{,y}\left(A_{21}+B_{21}\frac{1}{R_x}\right)\\&\quad+u_{,x}w_{,yy}\left(A_{21}+B_{21}\frac{1}{R_x}\right)+u_{,yy}w_{,y}\left(A_{26}+B_{26}\frac{1}{R_x}\right)\\&\quad+u_{,y}w_{,yy}\left(A_{26}+B_{26}\frac{1}{R_x}\right)+u_{,xx}w_{,y}\left(A_{61}+B_{61}\frac{1}{R_x}\right)\\&\quad+u_{,x}w_{,xy}\left(A_{61}+B_{61}\frac{1}{R_x}\right)+u_{,xy}w_{,y}\left(A_{66}+B_{66}\frac{1}{R_x}\right)\\&\quad+u_{,y}w_{,xy}\left(A_{66}+B_{66}\frac{1}{R_x}\right)+u_{,xy}w_{,x}\left(A_{61}+B_{61}\frac{1}{R_x}\right)\\&\quad+u_{,x}w_{,xy}\left(A_{61}+B_{61}\frac{1}{R_x}\right)+u_{,yy}w_{,x}\left(A_{66}+B_{66}\frac{1}{R_x}\right)\\&\quad+u_{,y}w_{,xy}\left(A_{66}+B_{66}\frac{1}{R_x}\right)+u_{,xy}w_{,x}\left(A_{16}+B_{16}\frac{1}{R_x}\right),\end{aligned}$$
$$\begin{aligned}R_3(v,w)&=v_{,y}w_{,xx}\left(A_{12}+B_{12}\frac{1}{R_y}\right)+v_{,xy}w_{,x}\left(A_{12}+B_{12}\frac{1}{R_y}\right)\\&\quad+v_{,xx}w_{,x}\left(A_{16}+B_{16}\frac{1}{R_y}\right)+v_{,x}w_{,xx}\left(A_{16}+B_{16}\frac{1}{R_y}\right)\\&\quad+v_{,yy}w_{,y}\left(A_{22}+B_{22}\frac{1}{R_y}\right)+v_{,y}w_{,yy}\left(A_{22}+B_{22}\frac{1}{R_y}\right)\end{aligned}$$

$$+ v_{,xy}w_{,y}\left(A_{26} + B_{26}\frac{1}{R_y}\right) + v_{,x}w_{,yy}\left(A_{26} + B_{26}\frac{1}{R_y}\right)$$
$$+ v_{,xy}w_{,y}\left(A_{62} + B_{62}\frac{1}{R_y}\right) + v_{,y}w_{,xy}\left(A_{62} + B_{62}\frac{1}{R_y}\right)$$
$$+ v_{,xx}w_{,y}\left(A_{66} + B_{66}\frac{1}{R_y}\right) + v_{,x}w_{,xy}\left(A_{66} + B_{66}\frac{1}{R_y}\right)$$
$$+ v_{,yy}w_{,x}\left(A_{62} + B_{62}\frac{1}{R_y}\right) + v_{,y}w_{,yx}\left(A_{62} + B_{62}\frac{1}{R_y}\right)$$
$$+ v_{,xy}w_{,x}\left(A_{66} + B_{66}\frac{1}{R_y}\right) + v_{,x}w_{,xy}\left(A_{66} + B_{66}\frac{1}{R_y}\right).$$

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