An Efficient Procedure of Multi-Frequency Use for Image Reconstruction in Ultrasound Tomography

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Abstract. Distorted Born iterative method (DBIM) using multi-frequency information has been studied and applied in ultrasound tomography. However, the use of different frequencies in different iterations in the DBIM method is not used consistently. The value of the frequency hopping step is often chosen depending on the simulation builder or experimenter. Based on the multi-frequency combination technique, this paper suggests an effective multi-frequency combination procedure to improve the quality of reconstructing ultrasound images using fundamental tone and overtones (FTaOT). The numerical simulation results show that the normalized error of the suggested scheme is decrease by 45% in comparison with the dual-frequency method. Other multi-frequency combination scenarios are also simulated to demonstrate the feasibility of the proposed method.

Keywords: Ultrasound · DBIM · DF · Fundamental tone and overtones (FTaOT).

1 Introduction

In today's medicine, ultrasonic imaging has become an extensively employed tool in medicine thanks to its capability to diagnose and treat and many strong points like humble cost, non-invasive, pain-free, mobile and swift diagnosis. Ultrasonic imaging utilizes ultrasonic waves that is often used because of the evolution of sonar technique in 1910. Using the sonar theory, one of the widely used techniques is B-mode one. B-mode images represent changes in sound impedance function. Because of this change, it is possible to differentiate between different mediums in the area of inquisitiveness. Nevertheless, this scheme uses feedback of ultrasonic waves so it is impossible to identify very small objects. Another important phenomenon in ultrasound imaging is the scattering of ultrasound waves when encountering small structures compared to direction from the object. The scattered data will be used to reconstructed the object. Imaging technique based on scattering theory is called ultrasound tomography.

Most of the studies on tomography ultrasound have its origin in the Born approximation. The BIM and DBIM are commonly used in ultrasonic tomography [1]. In the BIM method, Green functions unalters during iterations, so the superiority of this approach is not being influenced by noise). But its disadvantage is that it has a large computational mass. In the DBIM method, Green functions are updated in each loop, so the speed of convergence in this method is faster than the BIM method. But its disadvantage is greatly influenced by noise. Most of researchers prefer to use DBIM. The major restriction of this method is the divergence of the DBIM method in strong scattering environments. In fact, the method of approximating Born supposes that the scattering signal is tiny and it can be neglected; this is only true in low scattering environments. With a strong scattering environment, this method is no longer true [2]. This issue can be defeated by using frequency combination techniques to create images of objects using sound contrast [3], [4]. In these works, the two frequencies are exploited to recover objects in N_{f1} and N_{f2} loops. The small value of f_1 makes sure the algorithm convergence to a level of contrast that is close to the actual value, but low quality of the spatial resolution. The high value of f_2 can enhance spatial resolution while maintaining convergence. In the works [5], [6], [9], [10], the authors propose solutions to fuse multiple frequencies to enhance the resolution of the ultrasonic image to the level that this technique can create images of biological tissues. However, the use of different frequencies in different loops in the DBIM method is not used consistently. The value of frequency hopping step is often chosen depending on the simulation builder or experimenter. Based on the multi-frequency combination technique, this paper suggests an effective multi-frequency combination procedure to improve the quality of restoring sliced ultrasound images using fundamental tone and overtones (FTaOT). The fundamental tone is used for the first loop in the DBIM method, and in turn, the next overtones are used for the next loops.

2 Distorted Born iterative method

Figure 1 exhibits the measurement system of the ultrasound tomography imaging model. The transmitters and receivers are arranged on a circle around the object (strange tumor) to collect scattering data from the object. At one time, only a pair of transmitter-receiver works and one corresponding measurement data value is obtained. DBIM method is applied to restore the sound contrast of the scattered object. Thanks to this, we can find out any target in the environment.

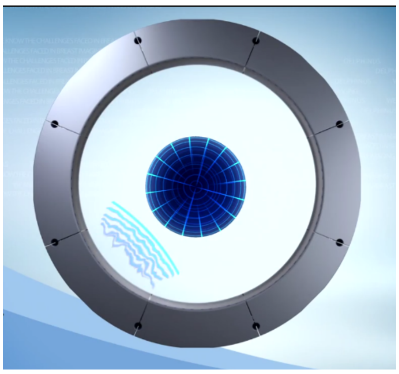

Fig. 1. Measurement configuration of ultrasound tomography imaging system [11]

Assuming that we consider an infinite space which includes a heterogeneous environment such as a water environment with a number of background waves k_0. Furthermore, there is an object $U(\vec{r})$ whose density is not changed and the number of

$$\bar{p} = \left(\bar{I} - \bar{C}.D(\bar{U})\right)p^{inc} \tag{1}$$

Points outside the object allow calculating scattering pressure with the size of $N_tN_r \times 1$:

$$\bar{p}^{sc} = \bar{B}.D(\bar{U}).\bar{p} \tag{2}$$

where $\bar{B}$ is the matrix with the coefficients being Green function $G_0(r,r')$ from each pixel to the receiver, $\bar{C}$ is the matrix with the coefficients being Green function $G_0(r,r')$ between pixels together, $\bar{I}$ is the unit matrix, and D(.) is the diagonal operator.

In equations (1) and (2), $\bar{p}$ and $\bar{U}$ are unknown. Therefore, we use the first-order Born approximation and equations (1) and (2) is re-expressed as follows [7].

$$\Delta p^{sc} = \bar{B}.D(\bar{p}).\Delta\bar{U} = \bar{M}.\Delta\bar{U} \tag{3}$$

in which $\bar{M} = \bar{B}.D(\bar{p})$. In fact, the unknown vector $\bar{U}$ has $N \times N$ varialbles, the number of this variables is identical to the number of pixels in the area of interest (ROI). The objective function can be calculated by the iterative method:

$$\bar{U}^{n} = \bar{U}^{(n-1)} + \Delta\bar{U}^{(n-1)} \tag{4}$$

where $\bar{U}^{n}$ and $\bar{U}^{(n-1)}$ is the objective function in the current iteration and the previous iteration; $\Delta\bar{U}$ can be estimated using Tikhonov method [8].

$$\Delta\bar{U} = \arg\min_{\Delta\bar{O}} \left\|\Delta\bar{p}^{sc}{}_t - \bar{M}_t\Delta\bar{U}\right\|_2^2 + \gamma\|\Delta\bar{U}\|_2^2 \tag{5}$$

where $\Delta\bar{p}^{sc}$ is a vector of size $(N_tN_r \times 1)$ which indicates the difference between a predictive and measurable ultrasonic signals; $\bar{M}_t$ is the system matrix of size $(N_tN_r \times N^2)$; and γ is the regularation parameter.

The implementation process of the DBIM method is shown in Algorithm 1.

Algorithm 1. Sound contrast reconstruction using DBIM
Select starting values: $\bar{U}_{(n)} = \bar{U}_{(0)}$ and $\bar{p}_0 = \bar{p}^{inc}$ using (7)
For $n = 1$ to N_{DBIM}, **do**
1. Compute $\bar{B}$ and $\bar{C}$
2. Compute p, $\bar{p}^{sc}$ corresponding to $\bar{U}_{(n)}$ using (1, 2)
3. Compute $\Delta\bar{p}^{sc}$ by (3)
4. Compute $\Delta\bar{U}_{(n)}$ by (5)
5. Compute $\bar{U}_{(n+1)} = \bar{U}_{(n)} + \Delta\bar{U}_{(n)}$
End For

3 The proposed method

The complexity of the ultrasonic imaging depends on the number of loops (N_{sum}), transmitter number (N_t) and receiver number (N_r). Assume that the total number of N_{sum} loops remains unchanged. Relative residual error (RRE) is used to evaluate the image recovery performance.

$$RRE = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{|C_{ij} - \hat{C}_{ij}|}{} \tag{6}$$

Physically, most oscillators when vibrating naturally produce a series of distinct frequencies, namely f_0, $2f_0$, $3f_0$, $4f_0$, $5f_0$, ... The lowest frequency (f_0) is called fundamental tone, higher frequencies ($2f_0$, $3f_0$, $4f_0$, $5f_0$, ...) is called overtones. It can be seen that, when the transmitter emits frequency f (the frequency used to create the image and we call it the fundamental tone), according to the physical mechanism, it also generates other frequencies (2f, 3f, 4f, ...) and we call them overtones. Therefore, in this paper, we use the frequency f emitted from the transmitter for the first loop and, naturally, we select the next overtones for the next loops. Thanks to that, the recovery image resolution will be improved gradually. From there, we have:

$$N_{sum} = N_{f1} + N_{f2} + N_{f3} + N_{f4} + N_{f5} + N_{f6} + N_{f7} + N_{f8}$$

The procedure of FTaOT method is presented in Algorithm 2, where $N_{f1} = N_{f2} = N_{f3} = N_{f4} = N_{f5} = N_{f6} = N_{f7} = N_{f8} = 1$.

Algorithm 2. The FTaOT-DBIM method
1. Choose initial values: $\overline{U}_{(n)} = \overline{U}_{(0)}$; $\overline{p}_0 = \overline{p}^{inc}$ sử dụng (7)
2. For n =1 to N_{f1}, do Algorithm 1, using f_1. End for.
3. For n =1 to N_{f2}, do Algorithm 1, using f_2. End for.
4. For n =1 to N_{f3}, do Algorithm 1, using f_3. End for.
5. For n =1 to N_{f4}, do Algorithm 1, using f_4. End for.
6. For n =1 to N_{f5}, do Algorithm 1, using f_5. End for.
7. For n =1 to N_{f6}, do Algorithm 1, using f_6. End for.
8. For n =1 to N_{f7}, do Algorithm 1, using f_7. End for.
9. For n =1 to N_{f8}, do Algorithm 1, using f_8. End for.
10. Calculate RRE using (6).

4 Numerical simulation and results

Simulation parameters: The frequencies f_1 = 1 MHz, f_2 = 2 MHz, f_3 = 3 MHz, f_4 = 4 MHz, f_5 = 5 MHz, f_6 = 6 MHz, f_7 = 7 MHz, f_8 = 8 MHz; Number of pixels N = 20; Number of transmitters N_t = 11; Number of receivers N_r = 22; Total number of loops N_{sum} = 8; Diameter of scattering area 7.3 mm; Sound contrast 30%; Gauss noise 10%; Distances from transmitter and receiver to center of object is 50 mm and 60 mm respectively.

The incident wave pressure is the zero-order Bessel beam in the two-dimensional space has the form:

$$\overline{p}^{inc} = J_0(k_0|r - r_k|) \quad (7)$$

where J_0 is zero-order Bessel function and $|r - r_k|$ is the distance between the transmitter and the k^{th} point in the ROI region.

Figure 2 compares the normalized error of the proposed method with traditional methods through loops. We can see that the normalized error of our proposed method is significantly reduced compared to traditional methods.

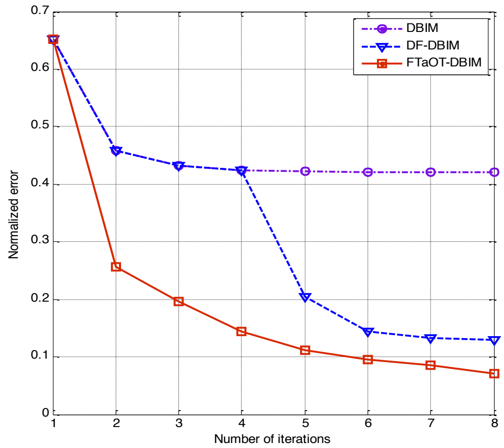

Fig. 2. Normalized errors of different methods through loops

After N_{sum} loops, normalized errors of the methods DBIM, DF-DBIM and FTaOT-DBIM respectively are 0.4205, 0.1293 and 0.0709. Therefore, the normalized error of our proposed method is reduced by 45% compared to the traditional DF-DBIM one.

Figure 3 shows the ideal object function that the ultrasound imaging system needs to recover. Figures 4, 5 and 6 show the recovered results of the DBIM, DF-DBIM and FTaOT-DBIM methods after N_{sum} loops. By visualization, we can see that background noise in our proposed method is lower than traditional methods and the recovered results by the proposed method are closer to the ideal object function than traditional methods.

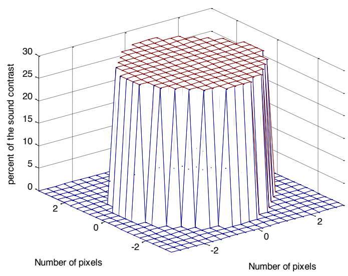

Fig. 3. Ideal object function (N=22)

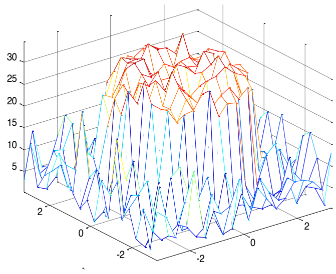

Fig. 4. Recovered result of the DBIM method after N_{sum} loops

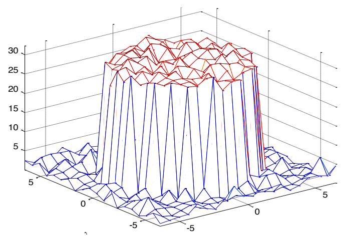

Fig. 5. Recovered result of the DF-DBIM method after N_{sum} loops

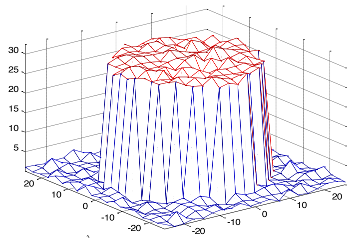

Fig. 6. Recovered result of the FTaOT-DBIM method after N_{sum} loops

The different scenarios of the DBIM method using multi-frequency information are shown in Table 1 and the normalized error of simulation scenarios after N_{sum} loops is shown in Table 2. We see that, the first scenario offers the smallest normalized error (0.0728) and this error value is still larger than the normalized error of the proposed images based on fundamental tone and overtones.

Table 1. Various scenarios of the DBIM method using multi-frequency information

Iterations	1	2	3	4	5	6	7	8
Scenario 1	f_1		f_2	f_3	f_4	f_5	f_6	f_7
Scenario 2	f_1			f_2	f_3	f_4	f_5	f_6
Scenario 3	f_1				f_2	f_3	f_4	f_5
Scenario 4	f_1					f_2	f_3	f_4
Scenario 5	f_1						f_2	f_3

Table 2. Normalized error of different scenarios of the DBIM method using multi-frequency information

Scenario	1	2	3	4	5
Error	**0.0728**	0.0781	0.0882	0.1157	0.1536

Conclusion

In this paper, we propose to apply the fundamental tone and overtones (the natural mechanism of oscillators) in the DBIM method based on multi-frequency information. The fundamental tone is used for the first loop in the DBIM method, and in turn, the next overtones are used for the next loops. The numerical simulation results prove that our proposed method is superior to the traditional methods of minimizing normalized errors and improving the resolution of recovered images.

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