Strict Frequency Reuse Algorithm in Downlink 3GPP Random Cellular Networks

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Abstract

In this work, a mathematical network model which follows on the recommendations of 3GPP to evaluate the downlink Long Term Evolution (LTE) network utilizing Strict Frequency Reuse (FR) scheme is introduced. The network modelling bases on the establishment phase and communications of the FR scheme. The user average coverage probability is derived and analysed under Rayleigh fading environment and furthermore the closed-form formulations of the performance are found using Gauss Quadratures. Through the Monte Carlo simulation, it is proved that the proposed analytical approach is more accurate than other approaches in the literature. Furthermore, this paper stated that the overall system can achieve the better performance with a higher number of Cell-Edge Users (CEUs), which contrasts with other works in the literature.
Keywords: Poisson Cellular Network, coverage probability, throughput, strict frequency reuse, Rayleigh fading.

1. Introduction

In Long Term Evolution Network (LTE), Strict Frequency Reuse (Strict FR) is a basic interference mitigation technique that was introduced to reduce InterCell Interference (ICI). Under Strict FR, the allocated frequency resources are partitioned into $f_c + f_e$ Resource Blocks (RBs). While f_c RBs are called as Cell-Center (CC) RBs and shared between all cells, f_e RBs are partitioned equally into Δ groups of Cell-Edge (CE) RBs in which each group is used privately within Δ cells as shown in Figure 1. Thus, Δ is called a FR factor. Hence, each cell has permission to use $f_c + \frac{f_e}{\Delta}$ RBs [1].

Throughout the recent releases in [2, 3], 3rd Generation Partnership Project (3GPP) recommended that the operation of the FR scheme should contain two separate phases. During the establishment phase, by comparing the pre-defined SINR threshold with the measured SINR on the downlink control channel, the Base Stations (BSs) classify the associated users into Cell-Center Users (CCUs) and Cell-Edge Users (CEUs). CCUs with the high measured SINRs will be served on the CC RBs at a low power level while CEUs with the low reported SINRs will be served on the CE RBs at a high power level. During the communication phase, the data is exchanged between the user and its associated BS. In other documents, 3GPP also stated that the time period of the

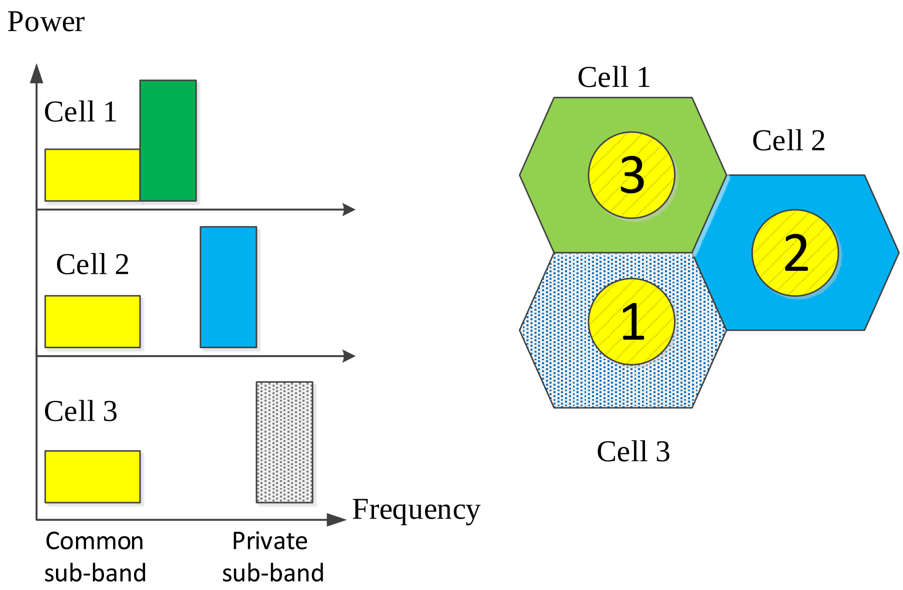

Figure 1: An example of Strict FR

establishment phase is adjustable and depends on the network operations while the communication phase takes place every time slot [4, 5].

Recently, the random cellular network model in which the BSs are distributed according to a spatial Poisson Point Process (PPP) has been utilized to evaluate the network performance [6, 7, 8, 9]. The most well-known work on performance analysis of Strict FR in random cellular networks was presented in [10]. In [11], the closed-form of the user performance was derived under Rayleigh-Lognormal fading channels. However, in these works the recommendations of 3GPP were not strictly followed such as the two-phase model was not defined for the CCU. Although the deployment of FR schemes has not been standardised, the standards of LTE have been finalised by 3GPP [3, 5, 12]. Hence, following 3GPP recommendations, such as downlink and uplink signal measurement for user classification purpose during the establishment phase, to model FFR is an essential step to provide a better performance examination of the FR scheme in the random cellular networks.

Following the results in [10, 13], the authors in [14] stated that the optimal value of SINR threshold can be selected at the coverage threshold. This conclusion is not able to convince the readers since Reference [14] based on the analytical approach in [13] which assumed that the received SINR of a given user between two phases, denoted by $SINR^{(o)}$ and $SINR$, are independent Random Variables (RVs). Thus, the joint probability $\mathbb{P}(SINR > \hat{T}, SINR^{(o)} < T)$ was simply obtained by $\mathbb{P}(SINR > \hat{T})\mathbb{P}(SINR^{(o)} < T)$ in which T and $\hat{T}$ are SINR threshold and coverage threshold respectively. However, since both $SINR$ and $SINR^{(o)}$ are functions of the distances from the user to the interfering BSs which are RVs and do not change between two phases, $SINR^{(o)}$ and $SINR$ are correlated RVs. Therefore, this assumption is not reasonable for the downlink PPP network model[15]. In work [15], the random cellular networks were employed to model the two-phase operation of Strict FR in uplink LTE networks.

This work follows on the 3GPP recommendations for LTE networks to model the downlink cellular network using Strict FR. This work can be distinguished with other works in the literature by following aspects: *First*, compared to the study in [10, 13], this work investigates the 3GPP recommendations for cellular network operations. The analytical model that recently developed for uplink in [15] to examine the network performance is deployed. The analytical and simulation results presented in Section 4.1 indicates that our approach is more accurate than others in the literature. *Second*, compared to our work in [15], in this paper, the model for downlink is developed. It is noted that the establishment phase as well as communication phase base on the SINR on the control channels and data channels respectively. The main difference between the downlink and uplink can be described as follows: *(i)* In the case of uplink, the user can share the control channel with the particular users and data channels with other users. Thus, during two phases the user in uplink experiences interferences from different sources. *(ii)* In the case of downlink, both these channels are suffered interference from the same neighbouring BSs since each BS can transmit on the whole RBs to serve its connected users. Thus, the modelling and mathematical analysis of this work and the work in [15] have significant differences.

This work makes contributions in terms of the network modelling, mathematical expressions of performance metrics and performance analysis. The main contributions are highlighted as bellows:

- The cellular network using Strict FR is modelled. The performance metrics in terms of the number of CCU and CEU, average coverage probability are analysed.
- Throughout the analysis and Monte Carlo simulation, it is indicated that the proposed approach is more accurate than others in the literature.
- The analytical results in this paper indicate that more users are served as CEUs, higher network performance is achieved. Our conclusion is more reasonable than that in [14] because while the ICI is unlikely to change when the transmit power increases, the user achieves higher performance if it is served by a higher transmit power.

2. Network model

A downlink model of a single-tier PPP network where the locations of BSs follow a spatial Point Poisson Process (PPP) distribution with a density of λ is considered. It is supposed that the typical user is located at the origin and has a connection with the nearest BS at distance r.

Thus, R is a RV whose Probability Density Function (PDF) was derived in [6]:

$$f_R(r) = 2\pi\lambda r e^{-\lambda\pi r^2} \tag{1}$$

It is assumed that the number of users in each cell is greater than the number of RBs. Thus, all RBs are employed to serve the active users. Consequently, each user is affected by ICI originating from all adjacent BSs.

2.1. Frequency Reuse Algorithm

Define $P^{(z)}$ as the transmit power on RB z which also presents the serving power of user z, in which $z = (c, e)$ correspond to the Cell-Center and Cell-Edge. Denote ϕ as the transmit power ratio between the transmit power on the CE and CC RBs ($\phi > 1$), then $P^{(e)} = \phi P^{(c)}$. The set of BSs that produce interference to the control channel in a typical cell is denoted by $\theta^{(c)}$. Thus, the density of BSs in $\theta^{(c)}$ is approximated by λ. Since the CC RBs are shared between cells, $\theta^{(c)}$ consists of interfering BSs of a CCU. Meanwhile, CE RBs are private resources within a group of Δ cell, each CEU experiences interference from the BSs whose set is denoted by $\theta^{(e)}$ and density is $\frac{\lambda}{\Delta}$.

Establishment phase. The users measure and report the received SINRs on the downlink control channels [2, 3] for user classification purpose. Every BS is continuously transmitting downlink control information, and subsequently each control channel experiences the ICI from all adjacent BSs. Furthermore, since all BSs are assumed to transmit on the control channels at the CC power, the interference of the measured SINR during this phase is given by

$$I = \sum_{j\in\theta^{(c)}} P g_{jz}^{(o)} r_{jz}^{-\alpha} \tag{2}$$

where $g_{jz}^{(o)}$ and r_{jz} are the channel power gain and distance between BS j and user z, respectively.

Communication phase. The power of interference $I^{(z)}$ of user z is

$$I^{(z)} = \sum_{j\in\theta^{(z)}} P_j^{(z)} g_{jz} r_{jz}^{-\alpha} \tag{3}$$

The received SINR of user z from the serving BS at distance r is given by

$$SINR(\phi^{(z)}, r) = \frac{\phi^{(z)} P g r^{-\alpha}}{\sigma^2 + I_{FR}^{(z)}} \tag{4}$$

in which σ^2 is noise power, g and r are the channel power gain and the distance between user z and its connected BS, respectively.

3. Average Coverage Probability

3.1. Definition

The average coverage probability expressions of the CCU as well as CEU are defined and derived in this section. Define $SINR^{(o)}(1,r)$ as the measured SINR on the downlink control channel during the establishment phase. The user classification follows the following rules

$$\begin{cases} SINR^{(o)}(1,r) > T, \text{then user is CCU} \\ SINR^{(o)}(1,r) < T, \text{then user is CEU} \end{cases} \tag{5}$$

in which T is the pre-defined SINR threshold.

During the communication phase, the CCU is under the network coverage when its downlink SINRs during this phase, denoted by $SINR(1,r)$, is greater than the coverage threshold $\hat{T}$. Hence, the average coverage probability is defined as a conditional probability:

$$\mathcal{P}^{(c)}(T,\lambda) = \mathbb{P}\left(SINR(1,r) > \hat{T} | SINR^{(o)}(1,r) > T\right) \tag{6}$$

The definition of the CCU average coverage probability in Equation 6 differs from the previous works because the operation of Strict FR in this work follows the recommendations of 3GPP.

Second, the CEU average coverage probability can be obtained from the following definition:

$$\mathcal{P}^{(e)}(T,\lambda) = \mathbb{P}\left(SINR(\phi,r) > \hat{T} | SINR^{(o)}(1,r) < T\right) \tag{7}$$

3.2. CCU and CEU Average Coverage Probability

In this section, the average coverage probability of the CCU and CEU are derived by solving the Equations 6 and 7.

Theorem 3.1. *The CCU average coverage probability is obtained by*

$$\mathcal{P}^{(c)}(T,\lambda) = \frac{\int_0^\infty r e^{-\pi\lambda r^2 - \frac{T+\hat{T}}{SNR} r^\alpha} \mathcal{L}(T,\hat{T},\lambda) dr}{\int_0^\infty r e^{-\pi\lambda r^2 - \frac{T}{SNR} r^\alpha} \mathcal{L}(T,0,\lambda) dr} \tag{8}$$

where $\mathcal{L}(T,\hat{T},\lambda) = e^{-2\pi\lambda r^2 \int_1^\infty \left[1 - \frac{1}{(1+Tx^{-\alpha})(1+\hat{T}x^{-\alpha})}\right] x dx}$.

The average coverage probability in Equation (8) is approximated by

$$\mathcal{P}^{(c)}(T,\lambda) \approx \frac{\sum_{j=1}^{N_{GL}} w_j e^{-\frac{(T+\hat{T})}{SNR}\zeta_j^\alpha} \mathcal{L}^{(j)}\left(T,\hat{T},\lambda\right)}{\sum_{j=1}^{N_{GL}} w_j e^{-\frac{T}{SNR}\zeta_j^\alpha} \mathcal{L}^{(j)}(T,0,\lambda)} \tag{9}$$

in which $\mathcal{L}^{(j)}(T,\hat{T},\lambda) \approx e^{-\pi\lambda\zeta_j^2 \left(\frac{2}{\alpha} \frac{T^{1+\frac{2}{\alpha}} - \hat{T}^{1+\frac{2}{\alpha}}}{T-\hat{T}} \frac{\pi}{\sin\left(\frac{2\pi}{\alpha}\right)}\right)} e^{-\pi\lambda\zeta_j^2 \left(\sum_{i=1}^{N_G} \frac{ci}{2} \frac{(T+\hat{T})\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + T\hat{T}}{\left(\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + T\right)\left(\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + \hat{T}t^\alpha\right)}\right)}$

Proof 3.2. *See Appendix Appendix A.*

Theorem 3.3. *The CEU average coverage probability is obtained by*

$$\mathcal{P}^{(e)}(T,\lambda) = \frac{2\pi\lambda \int_0^\infty re^{-\pi\lambda r^2} \begin{bmatrix} e^{-\frac{Tr^\alpha}{\phi SNR}} \mathcal{L}(\hat{T}, \frac{\lambda}{\Delta}) \\ -e^{-\left(\frac{\hat{T}}{\phi}+T\right)\frac{r^\alpha}{SNR}} \mathcal{L}(T,\hat{T},\lambda) \\ \times \mathcal{L}\left(T, \frac{(\Delta-1)\lambda}{\Delta}\right) \end{bmatrix} dr}{1 - 2\pi\lambda \int_0^\infty re^{-\pi\lambda r^2 - \frac{Tr^\alpha}{SNR}} \mathcal{L}(T,0,\lambda)dr} \tag{10}$$

In the case of $\hat{T} \neq T$, the approximation expression can be obtained by

$$\mathcal{P}^{(e)}(T,\lambda) \approx \frac{\sum_{j=1}^{N_{GL}} w_j \begin{bmatrix} e^{-\frac{T}{\phi SNR}\zeta_j^\alpha} \mathcal{L}^{(j)}(\hat{T}, \frac{\lambda}{\Delta}) \\ -e^{-\left(\frac{\hat{T}}{\phi}+T\right)\frac{\zeta_j^\alpha}{SNR}} \mathcal{L}^{(j)}\left(\hat{T}, \frac{\hat{T}}{\Delta(\hat{T}-T)}\right) \\ \times \mathcal{L}^{(j)}\left(T, \lambda - \frac{\hat{T}\lambda}{\Delta(\hat{T}-T)}\right) \end{bmatrix}}{1 - \sum_{j=1}^{N_{GL}} w_j e^{-\frac{T}{SNR}\zeta_j^\alpha} \mathcal{L}^{(j)}(T,0,\lambda)} \tag{11}$$

Proof 3.4. *See Appendix Appendix B*

3.3. Average Coverage Probability of the typical user

A user at a distance of r from its associated BS can be served as a CCU with a probability of $\mathbb{P}(SINR^{(o)}(1,r) > T|r)$ or a CEU with a probability of $\mathbb{P}(SINR^{(o)}(1,r) < T|r)$. Thus, the coverage probability is given by:

$$\begin{aligned} \mathcal{P}(T,\lambda|r) = \; & \mathbb{P}(SINR^{(o)}(1,r) > T|r)\mathbb{P}(SINR(1,r) > T|r) \\ & + \mathbb{P}(SINR^{(o)}(1,r) < T|r)\mathbb{P}(SINR(\phi,r) < T|r) \end{aligned} \tag{12}$$

in which $\mathbb{P}(SINR(1,r) > T|r)$ and $\mathbb{P}(SINR(\phi,r) < T|r)$ are the coverage probabilities of the CCU and CEU at a distance of r from their serving BS.

The average coverage probability of the typical user can be obtained by taking the conditional coverage probability $\mathcal{P}(T,\lambda|r)$ over the network

$$\mathcal{P}(T,\lambda) = 2\pi\lambda \int_0^\infty r^2 e^{-\pi\lambda r^2} \mathcal{P}(T,\lambda|r)dr \tag{13}$$

In interference-limited network ($\sigma_G \approx 0$ or $SNR \to \infty$) . By using the results in Section 3.2, the average coverage probability of the typical user is given by Equation 14 on the top of next page.

Since the coverage threshold $\hat{T}$, frequency reuse factor and the path loss exponent α in a real network are usually determined, the first fraction in Equation 14 is a constant number.

Since $\tau(T) = \frac{1}{1+Tt^{-\alpha/2}}$ is a monotonically decreasing function with respect T and $0 < \tau(T), \tau(\hat{T}) < 1$, $\forall \phi > 1, T > 0, \hat{T} > 0$. Moreover, $\int_1^\infty \left[1 - \tau(T)\tau(\hat{T})\right] dt$ and $\int_1^\infty \left[1 - \tau(T)\right] dt$ are monotonically increasing functions. Therefore, the second fraction in Equation 14 reduce with increments of T. Consequently, the probability $\mathcal{P}(T,\epsilon)$ increases with SINR threshold T.

$$
\begin{aligned}
\mathcal{P}(T,\lambda) &= \int_0^\infty re^{-\pi\lambda r^2} e^{-2\pi\lambda r^2 \int_1^\infty \left[1-\tau(T)\tau(\hat{T})\right]dt} dr \\
&\quad + 2\pi\lambda \int_0^\infty re^{-\pi\lambda r^2} \left[\begin{array}{l} e^{-\frac{\pi\lambda r^2}{\Delta}\int_1^\infty \left[1-\tau(\hat{T})\right]dt} \\ \quad -e^{-\frac{2\pi\lambda r^2}{\Delta}\int_1^\infty \left[1-\tau(T)\tau(\hat{T})\right]dt} \\ \qquad \times e^{-\frac{2\pi\lambda(\Delta-1)r^2}{\Delta}\int_1^\infty [1-\tau(T)]dt} \end{array} \right] dr \\
&= \frac{1}{1+\frac{1}{\Delta}\int_1^\infty \left[1-\tau(\hat{T})\right]dt} - \frac{\frac{\Delta-1}{\Delta}\int_1^\infty \tau(T)\left[1-\tau(\hat{T})\right]dt}{\left[\begin{array}{r} \left(1+\frac{1}{\Delta}\int_1^\infty \left[1-\tau(T)\tau(\hat{T})\right]dt + \frac{\Delta-1}{\Delta}\int_1^\infty [1-\tau(T)]\right) \\ \times\left(1+\int_1^\infty \left[1-\tau(T)\tau(\hat{T})\right]dt\right) \end{array} \right]}
\end{aligned} \tag{14}
$$

in which $\tau(T) = \frac{1}{1+Tt^{-\alpha/2}}$.

When $T \to 0$ which is equivalent that all users are classified as CEUs , $\tau(T)$ and consequently the second fractions in Equations 14 reaches 0. In this case, the average coverage probability of the typical user $\mathcal{P}(T,\lambda)$ reaches the maximum values

$$
\mathcal{P}(T,\lambda) = \frac{1}{1+\frac{1}{\Delta}\int_1^\infty \left[1-\tau(\hat{T})\right]dt} \tag{15}
$$

Under Strict FR, the interfering BSs and the serving BS of the user are always the same for both users served on CC and CE RBs. However, the user on a CC RB is affected by interference originating from BSs with a density of λ, while the density of interfering BSs is only $\frac{\lambda}{\Delta}$ in the case of the user on a CE RB.

From the discussion above, it can be concluded that the optimal values of SINR threshold are selected so all users are classified as CEUs. However, the selection of SINR threshold in practical networks should be depended on the required user performance as well as overall power consumption of the BSs.

4. Simulation and Discussion

The numerical and simulation results are used in this section to validate the accuracy of our analysis approach, in which the author first compare our analytical results with the well-known results in [10, 13] in terms of average coverage probability, then effects of SINR threshold, SNR on the network performance are analysed.

4.1. Validation of the proposed model

The analytical results, which are conducted for $SNR = 10$ dB, $\alpha = 3.5$, are compared to the Monte Carlo simulation and the corresponding results in [10] and [13].

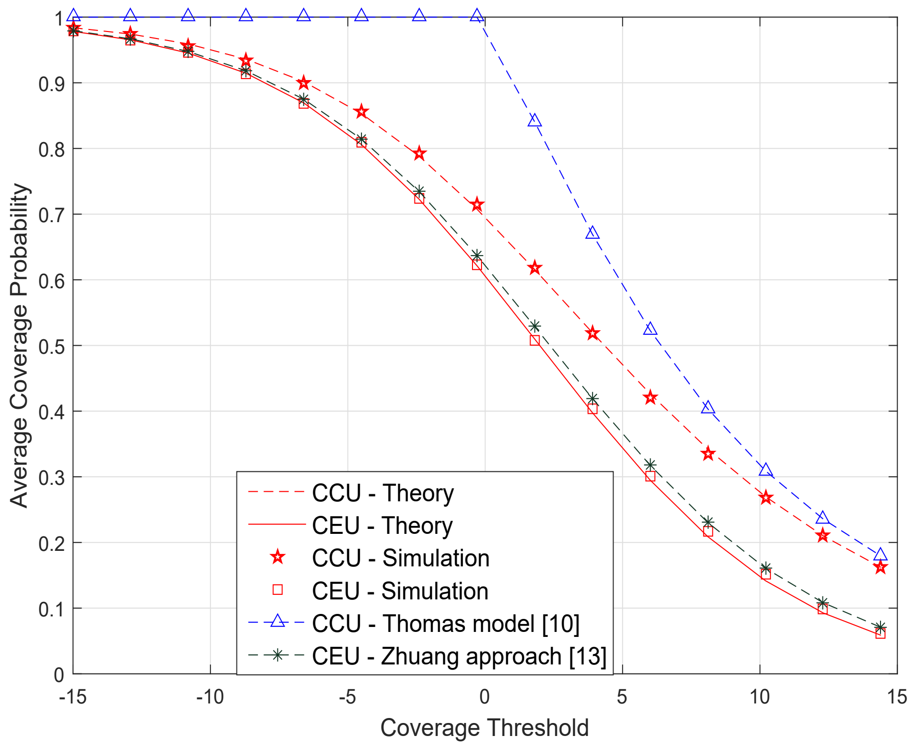

Figure 2: Comparison between Theoretical Analysis and Simulation Results

Discussion on the Results in [10]

Since [10] assumed that the user generates the signal for user classification and data exchange at the same time, the user was under the network coverage if the received SINR is greater than both coverage threshold $\hat{T}$ and SINR threshold T. Thus, the CCU average coverage probability, $\mathcal{P}_c$, was defined as $\mathbb{P}(SINR > T|SINR > \hat{T})$. Therefore, $\mathcal{P}_c = 1$ if $\hat{T} > T$.

Discussion on the Results in [13]

In [13], it was assumed that the interference between the establishment phase and communication phase are independent RVs, thus the joint probability in the numerator of Equation B.1 was evaluated as $\mathbb{P}(SINR^{(o)} < T, SINR^{(e)} > \hat{T}|r) = \mathbb{P}(SINR^{(o)} < T|r)\mathbb{P}(SINR^{(e)} > \hat{T}|r)$. However, in downlink cellular networks, the user during both establishment phase and communication phase is experienced interference from the same BSs, thus the interference during both phases are functions of the distance from the user to adjacent BSs. Consequently, $SINR^{(o)}$ and $SINR^{(e)}$ are correlated random variables. As a results, there are also gaps between the result in [13] and simulation results.

4.2. Effects of SNR on the network performance

Now, the performance of users in the cases of $T = -5$ dB and $T = 5$ dB are considered. In the case of Strict FR where the user only affected by interference originating from the BSs transmitting

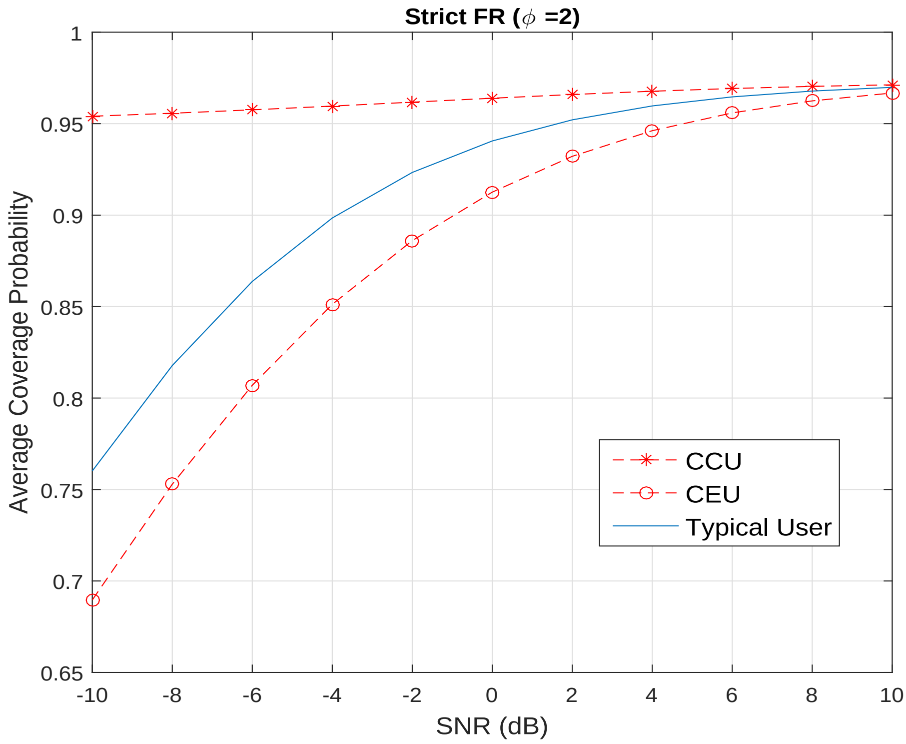

Figure 3: User Performance with $T = -5$ dB and $\hat{T} = -15$ dB, $\phi = 2$

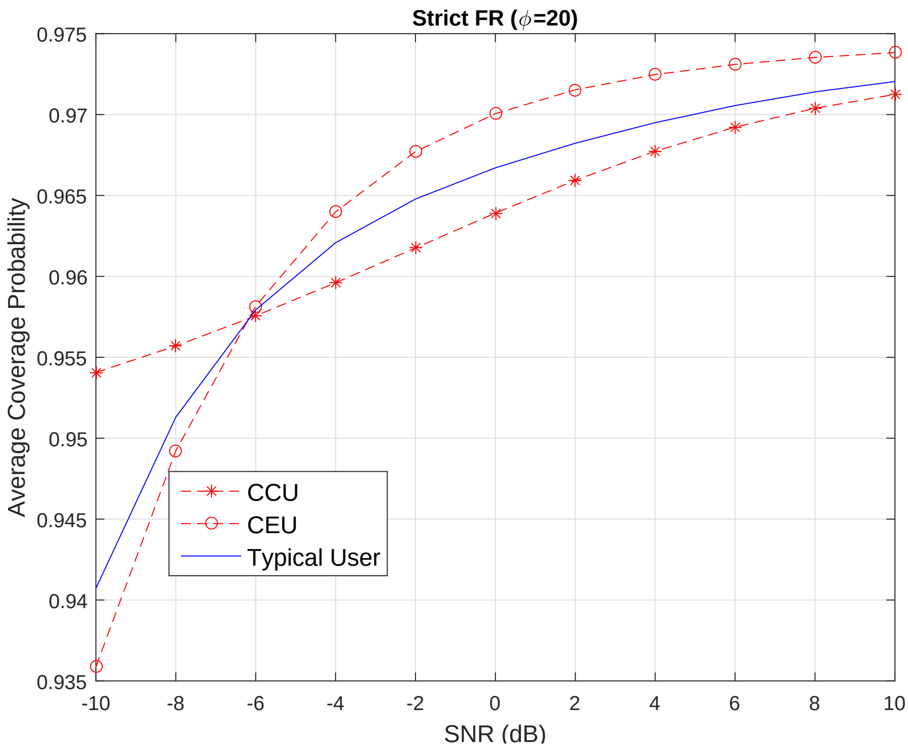

Figure 4: User Performance with $T = 5$ dB and $\hat{T} = -15$ dB, $\phi = 20$

at the same power with the serving power, an increment of SNR leads to an increase in average coverage probability as shown Figures 3 and 4.

An increase in the transmit power ratio is equivalent to an increment in the serving signal of the CEU. In addition, when SNR creases by δ, the serving transmit power of the CCU increases by the same amount while that of the CEU is $\phi\delta$. Hence, it is obvious from the figure that higher transmit power ratio and SNR provides more benefits for the CEU than for the CCU.

It is reminded that the transmit power of the serving BS and interfering BSs of the CCU are the same, i.e. P, and do not depend on the transmit power ratio. Hence, the average coverage probabilities of the CCU are the same in both cases of $\phi = 2$ and $\phi = 20$ as shown in Figures 3 and 4.

5. Conclusion

In this work, the model of the downlink PPP network was modelled according to 3GPP documents. The two-phase operation was proposed for both CCU and CEU in which the user classification procedure takes place during the establishment phase and the data transfer is taken play during the communication phase. The performance metrics in terms of CCU and CEU classification probability, average coverage probability are derived in Rayleigh fading channels. The Gauss - Laguerre and Gauss - Legendre are utilised to derive the closed-form expressions of the performance matrices. In contrast to the previous work which stated that the system achieves the highest performance when SINR Threshold T equals the coverage threshold $\hat{T}$, this paper stated that more users are served as CEUs, higher performance is achieved.

Appendix A. Theorem 1 - CCU under Strict FR

The CCU average coverage probability can be derived by following these steps

$$\begin{aligned}\mathcal{P}^{(c)}(T,\epsilon) &= \frac{\mathbb{P}\left(\frac{P^{(c)}gr^{-\alpha}}{\sigma^2+I^{(c)}} > \hat{T}, \frac{P^{(c)}g^{(o)}r^{-\alpha}}{\sigma^2+I} > T\right)}{\mathbb{P}\left(\frac{Pgr^{-\alpha}}{\sigma^2+I} > T\right)} \\ &= \frac{\int_0^\infty re^{-\pi\lambda r^2} e^{-\frac{(T+\hat{T})\sigma^2}{P^{(c)}r^{-\alpha}}} \mathbb{E}\left[e^{-\frac{\hat{T}I^{(c)}}{P^{(c)}r^{-\alpha}} - \frac{TI}{P^{(c)}r^{-\alpha}}}\right] dr}{\int_0^\infty re^{-\pi\lambda r^2} \left(e^{-\frac{T\sigma^2}{P^{(c)}r^{-\alpha}}} \mathbb{E}\left[-\frac{TI}{P^{(c)}r^{-\alpha}} dr\right]\right) dr}\end{aligned} \tag{A.1}$$

Considering the numerator of Equation (A.1), the expectation is the joint Laplace transform of I and $I^{(c)}$, denoted by $\mathcal{L}(T,\hat{T},\lambda)$ and joint evaluated at T and $\hat{T}$.

$$\begin{aligned}\mathcal{L}(T,\hat{T},\lambda) &= \mathbb{E}\left[e^{-T\sum_{j\in\theta} r^\alpha g_{jz} r_{jz}^{-\alpha} - \hat{T}\sum_{j\in\theta^{(c)}} r^\alpha g_{jz}^{(o)} r_{jz}^{-\alpha}}\right] \\ &= \prod_{j\in\theta} \mathbb{E}\left[\frac{1}{1+Tr^\alpha r_{jz}^{-\alpha}} \frac{1}{1+\hat{T}r^\alpha r_{jz}^{-\alpha}}\right]\end{aligned} \tag{A.2}$$

where Equation (A.2) since all channel gains are independent Rayleigh RVs.

Employing the properties of PGF and the change of variable $x = (r_{jz}/r)^2$, Equation (A.2) becomes

$$\mathcal{L}(T, \hat{T}, \lambda) = e^{-\pi\lambda r^2 \int_1^\infty \left[1 - \frac{1}{\left(1+Tx^{-\alpha/2}\right)\left(1+\hat{T}x^{-\alpha/2}\right)} dt\right] dx} \tag{A.3}$$

Approximate $\mathcal{L}(T, \hat{T}, \lambda)$. Denote $\upsilon(T, \hat{T}, \lambda)$ as the integral of $\mathcal{L}(T, \hat{T}, \lambda)$, hence

$$\begin{aligned}
\upsilon(T, \hat{T}, \lambda) &= \int_1^\infty \frac{\left(T + \hat{T} + T\hat{T}x^{-\frac{\alpha}{2}}\right) x^{-\frac{\alpha}{2}}}{\left(1 + Tx^{-\frac{\alpha}{2}}\right)\left(1 + \hat{T}x^{-\frac{\alpha}{2}}\right)} dx \\
&= \int_0^\infty \frac{\left(T + \hat{T} + T\hat{T}x^{-\frac{\alpha}{2}}\right) x^{-\frac{\alpha}{2}}}{\left(1 + Tx^{-\frac{\alpha}{2}}\right)\left(1 + \hat{T}x^{-\frac{\alpha}{2}}\right)} dx - \int_0^1 \frac{\left(T + \hat{T} + T\hat{T}x^{-\frac{\alpha}{2}}\right) x^{-\frac{\alpha}{2}}}{\left(1 + Tx^{-\frac{\alpha}{2}}\right)\left(1 + \hat{T}x^{-\frac{\alpha}{2}}\right)} dx \\
&= I_0 - I_1
\end{aligned} \tag{A.4}$$

Let $\gamma = x^{-\frac{\alpha}{2}}$, and if $\hat{T} \neq T$, I_0 equals

$$I_0(t) = \frac{2}{\alpha} \frac{1}{T - \hat{T}} \int_0^\infty \left[\frac{T^2\gamma^{-\frac{2}{\gamma}}}{1 + T\gamma} - \frac{\hat{T}^2\gamma^{-\frac{2}{\gamma}}}{1 + \hat{T}\gamma}\right] d\gamma$$

The integral is partitioned into two integrals which can be computed by letting $\gamma_1 = T\gamma$ and $\gamma_2 = \hat{T}\gamma$, and utilising the Gamma function properties. Hence, $I_0(t)$ is obtained by

$$I_0(t) = \frac{2}{\alpha} \frac{T^{1+\frac{2}{\alpha}} - \hat{T}^{1+\frac{2}{\alpha}}}{T - \hat{T}} \frac{\pi}{\sin\left(\frac{2\pi}{\alpha}\right)} \tag{A.5}$$

The integral $I_1(t)$ can be approximated by employing Gauss - Legendre Quadrature

$$I_1(t) = \sum_{i=1}^{N_G} \frac{ci}{2} \frac{\left(T + \hat{T}\right)\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + T\hat{T}t^\alpha}{\left(\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + T\right)\left(\left(\frac{x_n+1}{2}\right)^{\frac{\alpha}{2}} + \hat{T}\right)} \tag{A.6}$$

Finally, employing the properties of Gauss - Laguerre Quadrature, $\upsilon(T, \hat{T}, \lambda)$ is approximated by

$$\upsilon(T, T, \lambda) = \sum_{j=1}^{N_{GL}} \frac{w_j}{2} \left(I_0(\zeta_j) - I_1(\zeta_j)\right) \tag{A.7}$$

Approximate CCU Average Coverage Probability. It is observed from Equation (8) that the average coverage probability formulation has a suitable form of Gauss - Laguerre Quadrature. Thus, it approximately equals by

$$\mathcal{P}^{(c)}(T,\epsilon) \approx \frac{\sum_{j=1}^{N_{GL}} w_j e^{-\frac{(T+\hat{T})}{SNR}\zeta_j^{\alpha}} \mathcal{L}^{(i)}\left(T,\hat{T},\lambda\right)}{\sum_{j=1}^{N_{GL}} w_j e^{-\frac{T}{SNR}\zeta_j^{\alpha/2}} \mathcal{L}_I^{(i)}(T,0,\lambda)} \tag{A.8}$$

The Theorem 3.1 is proved.

Appendix B. Theorem 2 - CEU under Strict FR

The CCU average coverage probability expression is derived based on approach in [16] with notice that the density of interfering BSs is λ during establishment phase and λ/Δ during communication phase. Hence,

$$\begin{aligned}\mathcal{P}^{(e)}(T,\epsilon) =& \frac{\mathbb{P}\left(\frac{P^{(e)}gr^{-\alpha}}{\sigma^2+I^{(e)}} > \hat{T}, \frac{P^{(c)}g^{(o)}r^{-\alpha}}{\sigma^2+I} < T\right)}{\mathbb{P}\left(\frac{Pgr^{-\alpha}}{\sigma^2+I} < T\right)} \\ =& 2\pi\lambda \frac{\int_0^{\infty} re^{-\pi\lambda r^2}\mathbb{E}\left[e^{-\frac{\hat{T}\left(\sigma^2+I^{(e)}\right)}{P^{(e)}r^{-\alpha}}}\left(1-e^{-\frac{T\left(\sigma^2+I\right)}{P^{(c)}r^{-\alpha}}}\right)\right]dr}{1-\int_0^{\infty} 2\pi\lambda re^{-\pi\lambda r^2}e^{-\frac{T\sigma^2}{P^{(c)}r^{-\alpha}}}\mathbb{E}\left[-\frac{TI}{P^{(c)}r^{-\alpha}}dr\right]dr}\end{aligned} \tag{B.1}$$

The expected value of the numerator can be separated into two expectations in which the first one is evaluated using the same approach as the second one,
i.e. $E_2 = \mathbb{E}\left[e^{-\frac{\hat{T}I^{(e)}}{P^{(e)}r^{-\alpha}}}e^{-\frac{TI}{P^{(c)}r^{-\alpha}}}\right]$ can be computed based on the following steps

$$\begin{aligned}E_2 &= \mathbb{E}\left[e^{-\hat{T}\sum_{j\in\theta^{(e)}} r_{je}^{-\alpha}r^{\alpha}g_{je}}e^{-T\sum_{j\in\theta} r_{jc}^{-\alpha}r^{\alpha}g_{jc}^{(o)}}\right] \\ &= \mathbb{E}\left[\prod_{j\in\theta^{(e)}} e^{-\hat{T}r_{je}^{-\alpha}r^{\alpha}g_{je}}\prod_{j\in\theta} e^{-Tr_{jc}^{-\alpha}r^{\alpha}g_{jc}^{(o)}}\right] \\ &= \mathbb{E}\left[\prod_{j\in\theta^{(e)}} \frac{1}{1+Tr_{je}^{-\alpha}r^{\alpha}}\frac{1}{1+\hat{T}r_{je}^{-\alpha}r^{\alpha}}\prod_{j\in\theta\backslash\theta^{(e)}} \frac{1}{1+Tr_{jc}^{-\alpha}r^{\alpha}}\right]\end{aligned} \tag{B.2}$$

Since $\theta^{(e)}$ and $\theta^{(c)}$ are independent Poisson Process, and employing the properties of Probability Generating Function (PGF) with notes that the densities of BSs in $\theta^{(e)}$ and $\theta\backslash\theta^{(c)}$ are $\frac{\lambda}{\Delta}$ and $\frac{\Delta-1}{\Delta}\lambda$ respectively, the expectation equals

$$\begin{aligned}&= e^{-\frac{2\pi\lambda}{\Delta}\int_r^{\infty}\left[1-\frac{1}{1+Tr_{je}^{-\alpha}r^{\alpha}}\frac{1}{1+\hat{T}r_{je}^{-\alpha}r^{\alpha}}\right]r_{je}dr_{je}}e^{-\frac{2\pi\lambda(\Delta-1)}{\Delta}\int_r^{\infty}\left[1-\frac{1}{1+Tr_{jc}^{-\alpha}r^{\alpha}}\right]r_{jc}dr_{jc}} \\ &= e^{-\frac{2\pi\lambda r^2}{\Delta}\int_1^{\infty}\left[1-\frac{1}{1+Tt^{-\alpha/2}}\frac{1}{1+\hat{T}t^{-\alpha/2}}\right]dt}e^{-\frac{2\pi\lambda(\Delta-1)r^2}{\Delta}\int_1^{\infty}\left[1-\frac{1}{1+Tt^{-\alpha/2}}\right]dt}\end{aligned} \tag{B.3}$$

in which Equation B.3 follows changes of variable $t = (r_{je}/r)^2$ for the first integral and $t = (r_{jc}/r)^2$ for the second integral.

Substituting Equation A.3 into Equation B.3, Equation 10 is proved.

In the case of $\hat{T} \neq T$, it is obtained

$$\begin{aligned}
&= e^{-\frac{\pi\lambda}{\Delta} r^2 \frac{\hat{T}}{\hat{T}-T} \int_1^\infty \left[\frac{\hat{T}t^{-\alpha/2}}{1+\hat{T}t^{-\alpha/2}} - \frac{Tt^{-\alpha/2}}{1+Tt^{-\alpha/2}}\right] dt} e^{-\pi\lambda r^2 \int_1^\infty \frac{Tt^{-\alpha/2}}{1+Tt^{-\alpha/2}} dt} \\
&= e^{-\left(\lambda - \frac{\hat{T}\lambda}{\Delta(\hat{T}-T)}\right)\pi r^2 \int_1^\infty \frac{Tt^{-\alpha/2}}{1+Tt^{-\alpha/2}} dt} e^{-\frac{\hat{T}\lambda}{\Delta(\hat{T}-T)}\pi r^2 \int_1^\infty \frac{t^{-\alpha/2}}{1+\hat{T}t^{-\alpha/2}} dt} \\
&= \mathcal{L}\left(T, \lambda - \frac{\hat{T}\lambda}{\Delta(\hat{T}-T)}\right) \mathcal{L}\left(\hat{T}, \frac{\hat{T}\lambda}{\Delta(\hat{T}-T)}\right) \qquad \text{(B.4)}
\end{aligned}$$

in which $\mathcal{L}(T, 0, \lambda) = e^{-\pi\lambda r^2 \int_1^\infty \left[1 - \frac{1}{1+Tt^{-\alpha/2}}\right] dt}$

By substituting (B.4) into (B.3), the Theorem 3.3 is proved.

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