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MATCHED FIELD PROCESSING FOR SOURCE LOCALIZATION BASED ON AN APPROACH OF RIEMANNIAN GEOMETRY

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Abstract: *The matched field processing (MFP) for source localization has long history, and remains a viable area of research as well as application of SONAR. Some methods such as empirical mode decomposition, adaptive MFP, compressive MFP and MFP using Riemannian geometry have been introduced recently in order to increase the performance of conventional MFP. In case of ocean environment variability, there are many modeled field replicas thus the number of degree of freedom is increased, consequently the true source selection becomes more complexity. In this paper, we presents a MFP using an approach of Riemannian geometry in which Riemannian distance is obtained in close-form from a new isometric mapping and 20 modeled field replicas that are received in simulation from variable sound speeds. On the basis of the proposed MFP and simulation results, the source localization could be found in a more realistic manner.*

Keywords: *SONAR, Matched Field Processing, Riemannian Distance, Sine-Isometric Mapping*

1. INTRODUCTION

The Matched Field Processing for source localization has long history, and remains a viable area of research as well as application of SONAR. The development of MFP is from the conventional to other MFPs [1-5] in order to increase its reliability and resolution and to avoid mismatch conditions. Some methods such as empirical mode decomposition, adaptive MFP, compressive MFP and MFP using Riemannian geometry have been introduced recently [6-9] but they are not covered the ocean variability scenario completely. When ocean environment variability or lack of ocean information or mismatch of ocean environment leading to many modeled field replicas (the number of degree of freedom is increased), as a result the true source selection becomes more complexity.

The fact that Cross Spectral Density Matrices (CSDMs) which are not randomly but Hermite and positive definite, form a manifold that each CSDM is a point on it. One always uses Euclidean distance (ED) to compare the similarity between two CSDMs or two points in a manifold. However, Riemannian distance (RD) has been suggested recently due to the principle of Riemannian geometry appropriate to the curvature of acoustic ray [10]. In this case, RD is a comparison of geodesic distance between two points in a manifold. To reduce the complexity of RD calculation, we use an isometric mapping i.e., the mapping from the tangent space of a manifold to the tangent space of an Euclidean subspace. Firstly, this idea is the methodology of pattern classification [11], then it is applied to the problem of acoustic source inversion in ocean waveguide [9].

In this paper, we use a new isometric mapping which is called Sine Isometric Mapping (SIM) $\widetilde{m} = \sin m$,
where
m is a point in the tangent space of a manifold and $\widetilde{m}$ is the corresponding point in the tangent space of an Euclidean space. Thanks to the isometric mapping, the minimum distance of all the parameterization paths $l(\vartheta)$ connecting two fix points (a,b) leading to the corresponding RD in a manifold. Then the Riemannian matched field processor is defined using the minimum of the RD in order to locate the true source position in a more realistic manner.

The paper is organized as follows. Part 2 introduces Riemannian distance and SIM. The matched field processors based on Riemannian geometry are described in Part 3. Some simulations are given in Part 4. Finally, we conclude the paper in Part 5.

2. SINE-ISOMETRIC MAPPING AND RIEMANNIAN DISTANCE

2.1 CSDM MATRIX manifold

An CSDM manifold (M, g_m) is a manifold M which consists of CSDM matrices and is equipped with inner product (Riemannian metric) g_m on the tangent space $T_M(m)$ .Given the inner g_m product $T_M(m)$ on, each point m that varies smoothly from point to point in the sense that if X and Y are differentiable vector fields on *M*, then $m \mapsto g_m(X\big|_m, Y\big|_m)$ is a smooth function.

2.2 SINE-ISOMETRIC mapping (SIM)

Theorem 1:

Let the mapping $f: E \mapsto M$ be such that $m = \sin(m)$ where

$\tilde{m} \in \tilde{E}, m \in M$ M is CSDM atrix manifold and

$\tilde{E}$ is subspace of Euclidean space.

If the Riemannian metric on M is given by

$$g_m(a,b) = \langle a,c \rangle \quad (1)$$

where

$a,b \in T_M(m)$ and c is a matrix such $\cos^2 m.c = b$ that.Then $T_M(m)$ and $T_{\tilde{E}}(\tilde{m})$ are isometric.

Proof of theorem 1:

Consider $l(r)$ and $\tilde{l}(r)$ as respectively curves in M and $\tilde{E}$ such that $l(0) = m \in M$ and $\tilde{l}(0) = \tilde{m} \in \tilde{E}$ with $\tilde{l}(r) = sinl(r)$

Differentiating both sides with respect to *r*, we obtain

$$\widetilde{l(r)}' = [sinl(r)]' = cosl(r).l(r)' \quad (2)$$

At r =0, thus $\tilde{m}' = cosm.m'$. In addition, $m' \in T_M(m)$ and $\tilde{m}' \in T_{\tilde{E}}(\tilde{m})$ we have

$$\tilde{a} = cosm.a \text{ and } \tilde{b} = cosm.b \quad (3)$$

Define an operator $L_{\tilde{m}}$ on $T_{\tilde{E}}(\tilde{m})$ such that $L_{\tilde{m}}(\tilde{a}) = cosm.a$.

Then, (3) becomes $\tilde{a} = L_{\tilde{m}}(a), \tilde{b} = L_{\tilde{m}}(b)$.

Since $\tilde{m}$ is Hermitian, $L_{\tilde{m}}$ and $L_{\tilde{m}}^{-1}$ is also Hermitian. Thus

$$\langle \tilde{a}, \tilde{b} \rangle = \langle L_{\tilde{m}}^{-1}(a), L_{\tilde{m}}^{-1}(b) \rangle = \langle a, L_{\tilde{m}}^{-2}(b) \rangle = \langle a,c \rangle \quad (4)$$

where

$c = L_{\tilde{m}}^{-2}(b)$

In another way, $b = L_{\tilde{m}}^{2}(c) = L_{\tilde{m}}(L_{\tilde{m}}(c)) = L_{\tilde{m}}(cosm.c) = cos^2 m.c$ (5)

Therefore, $\langle \tilde{a}, \tilde{b} \rangle = g_m(a,b)$ or $T_M(m)$ and $T_{\tilde{E}}(\tilde{m})$ are isometric.

2.3 Riemannian distance fom SIM

Suppose that we have two points m_a and m_b on M. Parameterization of smooth curved path on M connecting m_a and m_b is a smooth function $\gamma : \theta \mapsto M$ in which $\theta \in R$ and θ is an open interval with their limits $\theta_a \le \theta \le \theta_b$

In general, the length of the path between the two points is calculated as

$$l(m) = \int_{\theta_a}^{\theta_b} \sqrt{g_m(m', m')}\, d\theta \quad (6)$$

where

$m' = \dfrac{dm}{d\theta}$ and $g_m(m', m')$ is an inner product in the tangent space, $T_M(m)$ at m on M.

The Riemannian distance between the two points is defined as the length of the geodesic,i.e.,

$$d_R(m_a, m_b) = \min_{m(\theta):[\theta_a,\theta_b] \mapsto M} \{ l(m(\theta)) \} \quad (7)$$

The length of a geodesic connecting m_a and m_b ı M has the same length of geodesic connecting $\tilde{m}_a$ and $\tilde{m}_b$ in $\tilde{E}$ as a result of SIM.

Therefore we derived the Riemannian distance as follows

$$d_{SIM}^2(m_a, m_b) = \min_{\substack{\tilde{m}_a = sinm_a \\ \tilde{m}_b = sinm_b}} \langle \widetilde{m_a}, \widetilde{m_b} \rangle = \min_{\substack{\tilde{m}_a = sinm_a \\ \tilde{m}_b = sinm_b}} \| \widetilde{m_a} - \widetilde{m_b} \|^2 \quad (8)$$

$$d_{SIM}^2(m_a, m_b) = \min_{\substack{\tilde{m}_a = sinm_a \\ \tilde{m}_b = sinm_b}} tr\left[(\widetilde{m_a} - \widetilde{m_b})(\widetilde{m_a} - \widetilde{m_b})^H \right]$$

$$= tr(sin(m_a)^2) + tr(sin(m_b)^2) - 2tr(sin(m_a)sin(m_b)) \quad (9)$$

where

tr is trace operator for a matrix as in [12].

3. MATCHED FIELD PROCESSORS BASED ON RIEMANNIAN GEOMETRY

An acoustic pressure field on a vertical array of N sensors with locations $p_a = (r_a, z_a), a = 1, N$ and from the true source coordinate $p_s = (r_s, z_s)$ is given by

$$F_{p_s}(p_s, p_a) = S.G(p_s, p_a) + W(p_a) \quad (10)$$

where

S is a spectral component of the source,

G is Green function which is calculated by Normal mode model and

W represents uncorrelated additive ambient noise.

The cross-spectral density matrix is written as

$$\bar{R}_{p_s} = \sum_{m=1}^{M} \left[F_{p_s} \right]_m \left[F_{p_s} \right]_m^H \quad (11)$$

Normalization of CSDM using Frobenius norm, we have

$$R_{p_s} = \frac{\bar{R}_{p_s}}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{M} \left| (\bar{R}_{p_s})_{mn} \right|^2}} \quad (12)$$

The Frobenius norm define that $\|A\|_F^2 = \sum_{ij} a_{ij} = tr(AA^H)$ whe-re a_{ij} is element of matrix **A** and H is the transpose conjugate [12]. The corresponding normalization of CSDM of modeled $\mathbf{R}_{\hat{p}}$ field replica from estimated source $\hat{p} = (\hat{r}, \hat{z})$ coordinate

denoted by

The matched field processor based on Riemannian Geometry is received by obtaining the space coordinates of modeled field replicas which are scanning over all modeled field replicas position $\hat{p} = (\hat{r}, \hat{z})$ with a subject constraint of minimization of specific Riemannian distance.

According to [9] we have three matched field processors which are based on Riemannian Geometry as follows

$$(\hat{r}_s, \hat{z}_s)_{d_1} = \underset{\hat{p}}{argmin} \sqrt{tr(R_{p_s}) + tr(R_{\hat{p}}) - 2tr(R_{p_s} R_{\hat{p}})} \quad (13)$$

$$(\hat{r}_s, \hat{z}_s)_{d_2} = \underset{\hat{p}}{argmin} \sqrt{tr(R_{p_s}) + tr(R_{\hat{p}}) - 2tr(R_{p_s}^{1/2} R_{\hat{p}}^{1/2})} \quad (14)$$

$$(\hat{r}_s, \hat{z}_s)_{d_3} = \underset{\hat{p}}{argmin} \sqrt{\sum_k ln^2 \lambda_k} \quad (15)$$

where

λ_k is eigenvalues of $R_{p_s}^{-1} R_{\hat{p}}$

On the basis the Riemannian distance of SIM (Part 2.3), the proposed Riemanian matched field processor is written as

$$(\hat{r}_s,\hat{z}_s)_{d_{SIM}} = \underset{\hat{\mathbf{p}}}{\mathbf{argmin}}\sqrt{\mathbf{tr}[(\sin\mathbf{R}_{\mathbf{p}_s})^2]+\mathbf{tr}[(\sin\mathbf{R}_{\hat{\mathbf{p}}})^2]-2\mathbf{tr}[\sin(\mathbf{R}_{\mathbf{p}_s})\sin(\mathbf{R}_{\hat{\mathbf{p}}})]} \tag{16}$$

In another way, the diagram of matched field processors based on Riemannian geometry is described in Fig. 1 as follows

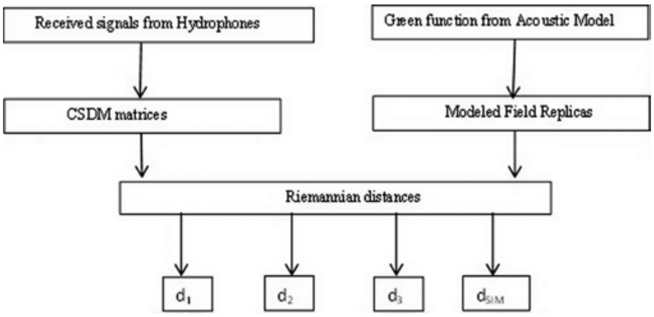

Fig. 1: Classification of Matched Field Processors based on Riemannian geometry

4. SIMULATIONS

4.1 Acoustic model

The acoustic model in this paper using Normal mode model, in this case the acoustic pressure from [13] is given by

$$\boldsymbol{F}(\boldsymbol{r},\boldsymbol{z}) = \frac{i}{\boldsymbol{\rho}(\boldsymbol{z}_s)\sqrt{8\pi \boldsymbol{r}}} e^{-i\frac{\pi}{4}} \sum_{m=1}^{M} \boldsymbol{\psi}_m(\boldsymbol{z}_s)\boldsymbol{\psi}_m(\boldsymbol{z}) \frac{e^{ik_m r}}{\sqrt{\boldsymbol{k}_m}} \tag{17}$$

where
r is range,
z is depth,
z_s is the depth of the source,
ρ is sea water density,
ψ_m is amplitude of mth mode, and
k_m is mth eigenvalue (wavenumber).

4.2 Input acoustic data

Passive array data SONAR from SACLANTC1993 North Elba experiment available in Internet was used for processing [14]. The vertical underwater acoustic array data was collected in shallow-water off the Italia west coast by the NATO SACLANT Center in La Spezia, Italy. The original SACLANT time series has been converted to a series of MATLAB .mat files each of which contains a matrix "dat" that is 48 sensors by 64K data points long. Each file represents about 1 minute of data. The vertical array consists of 48 hydrophones with spacing 2 m between elements at total aperture length 94 m (18.7 m to 112.7 m in depth). The source emitted PRN signal with center frequency of 170 Hz.

The Sound Speed Profile (SSP) from [14] is described in Fig. 2.

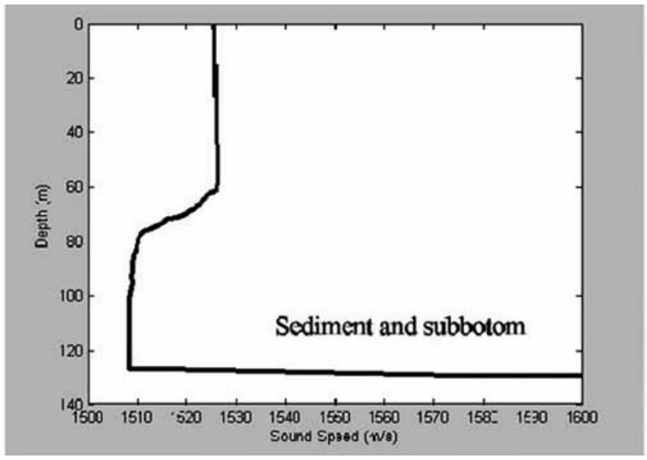

Fig. 2: SSP of SACLANTC 1993 North Elba

4.3 Simulation results

-Each simulation uses 10 replicas of SONAR array data which provided by SACLANC and SNR level of 10 dB and the number of snapshot is greater than 30 samples. Twenty modeled field replicas are obtained from variable sound speeds that changed to depth according to SSP as depicted in Fig. 2 (In reality modeled field replicas could be caused from other factor such as internal-wave, bottom parameter mismatch and others). From Fig. 3 it can be seen that the true source can be detected at depth of 60 m and range of 6000 m if 20 modeled field replicas and 10 data replicas were used for the proposed Riemannian matched field processor. In Fig. 4 the performance of the proposed Riemannian matched field processor used only 6 modeled field replicas shows that beside the true source location there are a number of spurious peak locations which are corresponding to ocean variability or mismatch conditions. Since the true source location is higher than other spurious peak locations, one can find it. However, we could not detect the source in the case in Fig. 5 when only 3 modeled field replicas were used and all peak location are almost equally. It is always the case of MFP when the number of modeled field replicas could not provide enough fluctuation of ocean environment.

The comparison of Riemannian matched field processors is not easy since the complexity of physical interpretation of Riemannian distance. However, the performance of conventional MFP is equivalent to the low level of Riemannian matched field processor (is not shown here).

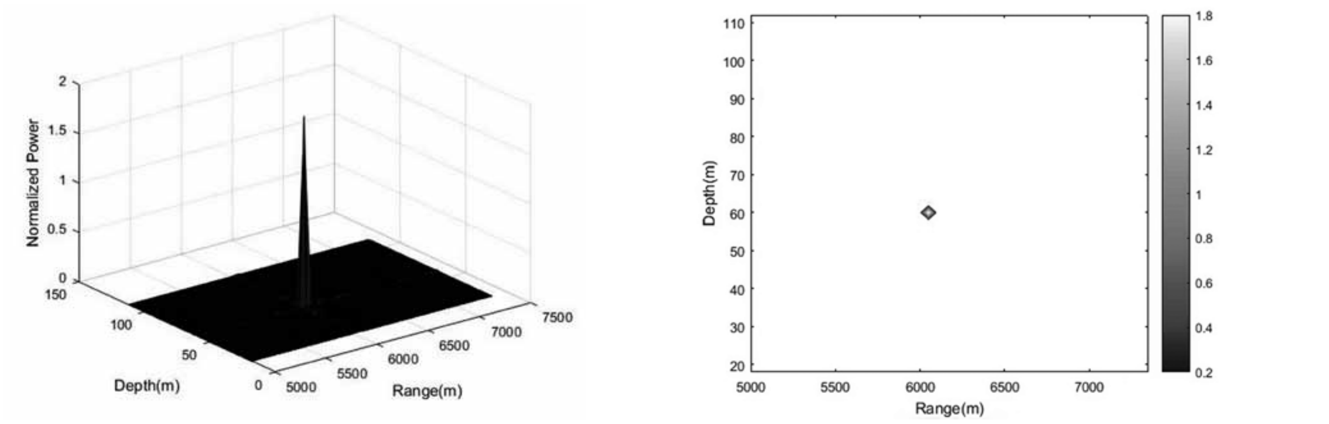

Fig. 3: Riemannian ambiguity surface for 20 modeled field replicas and 10 data replicas, SNR=10dB, No of snapshot> 30 samples

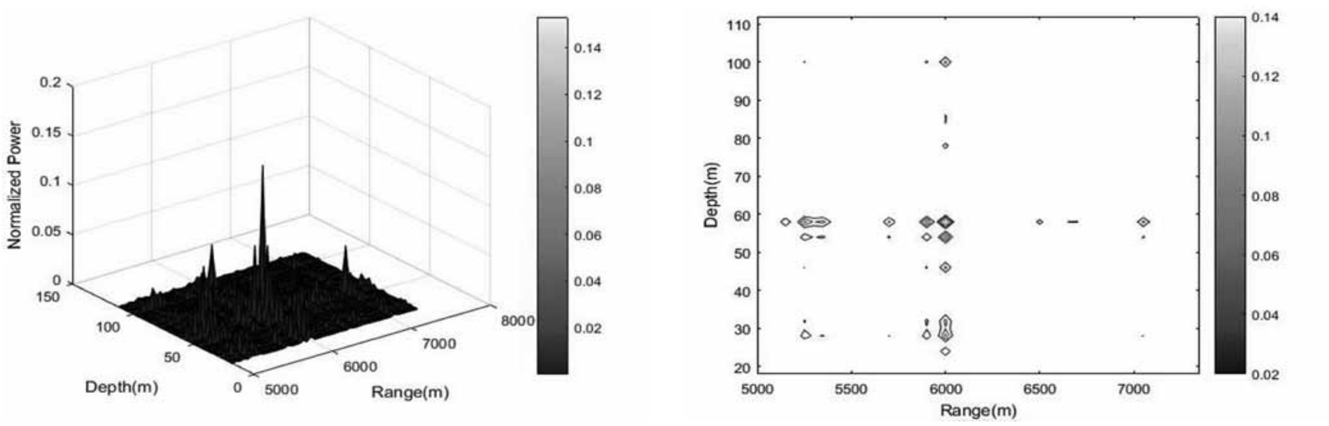

Fig. 4: Riemannian ambiguity surface for 6 modeled replicas and 10 data replicas, SNR=10 dB, No of snapshot>30 samples

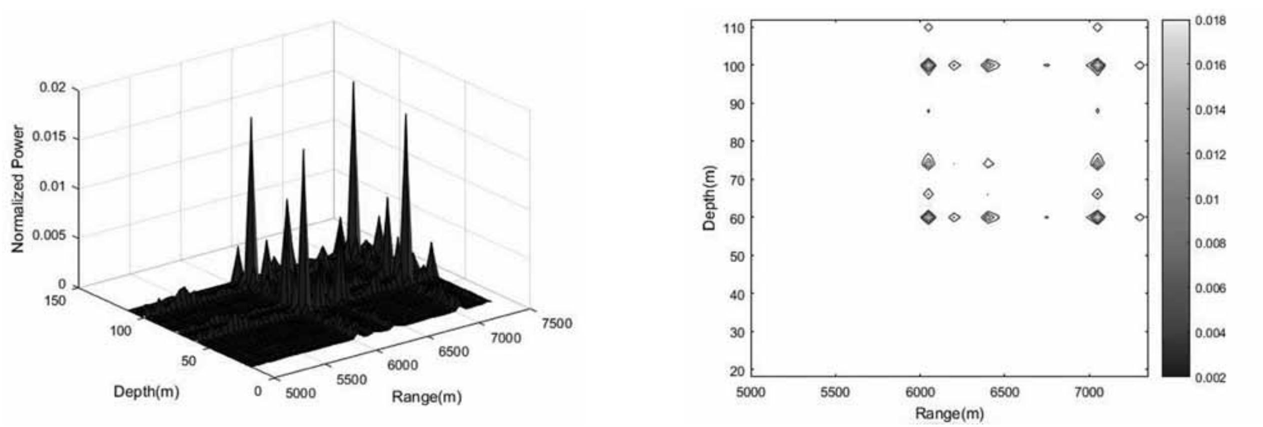

Fig. 5: Riemannian ambiguity surface for 3 modeled replicas and 10 data replicas, SNR=10dB, No of snapshot>30 samples

5. CONCLUSION

In view of Riemannian Geometry, Sine-Isometric Mapping (SIM) is introduced as well as the RD from SIM is derived. Then the Riemannian MFP based on the RD is analyzed and validated by simulation in the case of variable modeled field replicas. The simulation results show that the true source could be found in a more realistic manner and it can be detected more precisely if the more modeled field replicas are used.

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