EEG Source Localization: A New Multiway Temporal-Spatial-Spectral Analysis

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***Abstract*—Accurate localization of epileptogenic zone is highly meaningful for epilepsy diagnosis and treatment in general and removal of the epileptogenic region in epilepsy surgery in particular. In this paper, we present a robust method for electroencephalography (EEG) source localization based on a new multiway temporal–spatial–spectral (TSS) analysis for epileptic spikes via graph signal processing and multiway blind source separation. Instead of using the temporal behavior of the EEG distributed sources, we first apply the graph wavelet transform to the spatial variable for epilepsy tensor construction in order to exploit latent information of the spatial domain. We then apply the tensorial multiway blind source separation method for estimating the sources and hence localizing them. Numerical experiments on both synthetic and real data are carried out to evaluate the effectiveness of the TSS analysis and to compare it with two state-of-the-art types of analysis: space–time–frequency (STF) and space–time–wave–vector (STWV). Experimental results show that the proposed method is promising for epileptic source estimation and localization.**

***Index Terms*—Electroencephalography (EEG), source localization, epileptic spikes, multiway blind source separation, tensor decomposition, graph signal processing.**

I. Introduction

Epilepsy is a chronic disorder of the nervous system in the brain due to abnormal, excessive discharges of nerve cells. There are approximately 50 million people diagnosed of epilepsy and 2.4 million people detected signs of the disorder each year in the world. This makes epilepsy is one of the most common neurological disorders [1].

Electroencephalography (EEG) is one of the most accessible tools for epilepsy diagnosis and treatment. It records electrical activities in the brain by measuring voltage fluctuations of neurons. From EEG recordings of patients with epilepsy, neurologists can detect specific epileptic biomarkers (such as seizures, spikes and sharp waves) and the epileptogenic region deep in the brain that initiates these epileptic EEG sources. Accurate localization of epileptogenic zone is highly meaningful for epilepsy diagnosis and treatment in general and removal of the epileptogenic region in epilepsy surgery in particular. An automatic system for EEG source localization is thus desirable.

Given EEG signals recorded at the scalp, the localization of EEG sources of interest deep in the brain is known as an underdetermined ill-posed inverse problem where we may want to estimate and localize the electrical activities of interest from only the temporal–spatial measurements. In the last decades, there have been a number of studies on EEG source localization. The reader is invited to view good surveys in [2]–[4]. Well-known methods for EEG source localization include: minimum norm, variations of low resolution electromagnetic tomography (LORETA), recursive multiple signal classification (MUSIC) and independent component analysis (ICA), to name a few. These conventional methods, however, require strong assumptions on the source distribution, and yield either blur or too sparse estimates. To overcome these drawbacks, multiway (i.e. tensor) based analysis provide good alternatives.

Tensor representation and decomposition have become a useful approach for multiway data analysis [5] and EEG signals in particular [6]. In the context of (focal) epilepsy, there are two main types of multiway representation for EEG signals, including space–time–frequency (STF) and space–time–wave–vector (STWV) [3], [7]. The STF representation is to transform EEG signals recorded at each electrode into the frequency domain using time–frequency tools such as windowed Fourier transform and continuous wavelet transform. The STF-based analysis has been applied for seizure oneset localization [8], epileptic seizure modeling [9] and epileptic spike detection [10]. The STWV representation is to transform EEG signals at each time instance into the spatial–frequency domain, i.e., applying a 3D windowed Fourier transform to the spatial variable instead of the temporal variable as in the STF-based analysis [11]. The STWV-based analysis helps localize extended EEG sources in the brain. However, these two types of analysis do have some drawbacks. The former would not permit the separation of multiple sources simultaneously when correlated signals are within more than one component. The latter requires strong assumptions on the distributed sources such as the smoothness and sparsity in the spatial distribution, which may not be met in practice [7]. These drawbacks inspire us to look for a more robust type of multiway analysis.

Recently, graph signal processing (GSP), seen as intersection of graph theory and computational harmonic analysis, has emerged as a new tool for efficiently analyzing structural data in general [12] and brain signals in particular [13], [14]. Given the ambiently measured temporal–spatial EEG data, we can construct graph signals on the brain network that naturally enables correlation analysis among different brain regions over the time. Therefore, the use of GSP can reveal latent information of the brain signals and hence aid to detect activities of interest. This motivates us, in this paper, to look for a GSP-based model for multiway analysis of EEG signals and thus facilitate epileptogenic zone localization. This model for EEG data to be proposed in Section III-A was preliminarily reported in [15] and is here given with more details and further applied to the problem of EEG source localization.

II. EEG Data Model

Assuming a source space with K dipole signals in the brain is measured by N nodes of an EEG electrode array during T time samples. According to [3], a generative model for EEG data $\mathbf{X} \in \mathbb{R}^{N \times T}$ can be expressed as follows:

$$\mathbf{X} = \mathbf{G}\mathbf{S} + \mathbf{N}, \tag{1}$$

where $\mathbf{S} \in \mathbb{R}^{K \times T}$ is the source matrix, $\mathbf{G} \in \mathbb{R}^{N \times K}$ is the lead field matrix that models the propagation of the signals, and $\mathbf{N} \in \mathbb{R}^{N \times T}$ presents artifacts and noise.

For source estimation, we may want to separate extended sources from background activities, thus the data model of (1) can be rewritten as

$$\mathbf{X} = \underbrace{\sum_{k=1}^{K} \sum_{i_k \in \Omega_k} \mathbf{g}_{i_k} \mathbf{s}_{i_k}^T}_{\mathbf{X}_s} + \underbrace{\sum_{j \notin \{\Omega_k\}_{k=1}^{K}} \mathbf{g}_j \mathbf{s}_j^T}_{\mathbf{X}_b} + \mathbf{N}, \tag{2}$$

where Ω_k denotes the set of dipoles of the k-th extended source, $\mathbf{g}_k$ is the lead field vector at the k-th dipole, and $\mathbf{s}_k$ is the k-th row signal vector of $\mathbf{S}$. It is noted that signals from dipoles belonging to the same source are supposed to be "equal", since the activities of interest from an EEG source are highly synchronized in general [3]. Therefore, the EEG data model $\mathbf{X}$ in (2) can be approximated as

$$\mathbf{X} \approx \sum_{k=1}^{K} \mathbf{h}_k \mathbf{s}_k^T + \mathbf{X}_b + \mathbf{N} = \mathbf{H}\mathbf{S} + \mathbf{X}_b + \mathbf{N}, \tag{3}$$

where the spatial mixing vector $\mathbf{h}_k$ is defined by the sum of the lead field vectors $\mathbf{g}_{i_k}$ in Ω_k, that is,

$$\mathbf{h}_k = \sum_{i_k \in \Omega_k} \mathbf{g}_{i_k} = \mathbf{G}\mathbf{c}_k, \tag{4}$$

with $\mathbf{c}_k$ is the indicator vector,

$$\mathbf{c}_k[r] = \begin{cases} 1 & \text{if } \; r \in \Omega_k, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The main objective of EEG source localization is to estimate the unknown source matrix, $\mathbf{S}$, and the source position matrix, $\mathbf{C} = [\mathbf{c}_1 \; \mathbf{c}_2 \ldots \mathbf{c}_K]$, from the EEG data, $\mathbf{X}$.

III. Proposed Method of EEG Source Localization

Generally, a scheme for source localization consists of three main stages: (i) data representation, (ii) blind source separation and (iii) source localization. In this work, we adapt the scheme for EEG source localization in the context of epilepsy, as described next.

A. EEG Tensor Representation via Graph Signal Processing

Given the ambiently measured temporal–spatial EEG data, $\mathbf{X} \in \mathbb{R}^{N \times T}$, we now construct graph signals to represent the time-evolving EEG brain graph/network.

At each time sample t, considering an EEG graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of N vertices presenting N nodes of the EEG electrode array, and $\mathcal{E}$ is the set of edges presenting the connection among the vertices. In graph theory, associated with $\mathcal{G}$ are two special matrices: the adjacency matrix, $\mathbf{A}(t) \in \mathbb{R}^{N \times N}$, and the normalized Laplacian matrix, $\mathbf{L}(t) \in \mathbb{R}^{N \times N}$.

An element $\mathbf{A}(t)[i,j]$ of $\mathbf{A}(t)$ is nonnegative and represents the edge weight of vertices i and j. In this work, for estimating the edge weights, we apply a synchronization measure based on correlation coefficient [16], which is defined as follows:

$$\mathbf{A}(t)[i,j] = \frac{1}{T} \frac{(\mathbf{x}_i(t) - \bar{\mathbf{x}}_i(t))(\mathbf{x}_j(t) - \bar{\mathbf{x}}_j(t))}{\sigma_{\mathbf{x}_i(t)} \sigma_{\mathbf{x}_j(t)}}, \tag{6}$$

where $\mathbf{x}_i(t)$ is the row vector of $\mathbf{X}$ that represents the signal recorded at the i-th vertex of the EEG graph, $\bar{\mathbf{x}}_i(t)$ and $\sigma_{\mathbf{x}_i}(t)$ are the mean and variance of $\mathbf{x}_i(t)$ respectively. The Laplacian matrix is defined as

$$\mathbf{L}(t) = \mathbf{I} - \mathbf{D}(t)^{-1/2} \mathbf{A}(t) \mathbf{D}(t)^{-1/2}, \tag{7}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{D}(t)$ is the degree matrix of $\mathbf{A}(t)$.

In GSP, a graph signal $\mathbf{f}(t) \in \mathbb{R}^{N \times 1}$ is constructed from $\mathbf{X}$ as the t-th column vector of $\mathbf{X}$. Taking eigenvalue decomposition (EVD) of the Laplacian matrix $\mathbf{L}(t)$, we obtain

$$\mathbf{L}(t) \overset{\text{EVD}}{=} \mathbf{F}(t) \boldsymbol{\Sigma}(t) \mathbf{F}^*(t). \tag{8}$$

The eigenvalues of $\mathbf{L}(t)$ carry the notion of "graph frequency" and the eigenvector matrix $\mathbf{F}(t)$ is responsible for the graph Fourier transform (GFT) [12].

Therefore, the wavelet coefficients of $\mathbf{f}(t)$ in spectral graph domain is given by

$$\mathbf{W}_{\mathbf{f}(t)}[a,s] = \langle \mathbf{f}(t), \boldsymbol{\psi}_{a,s}(t) \rangle, \tag{9}$$

where the wavelet $\boldsymbol{\psi}_{a,s}(t)$ is defined as [17]:

$$\boldsymbol{\psi}_{a,s}(t)[n] = \sum_{i=1}^{N} g(s\lambda_i) \mathbf{F}^*(t)[a,i] \mathbf{F}(t)[n,i], \tag{10}$$

with $g(\cdot)$ being the wavelet generating kernel.

Finally, we propose a temporal-spatial-spectral representation of EEG data by mapping the wavelet coefficients to a three-way tensor, as follows:

$$\boldsymbol{\mathcal{X}}(t,a,s) = \sum_{i=1}^{N} g(s\lambda_i) \mathbf{F}^*(t)[a,i] \mathbf{F}(t)[n,i] \mathbf{f}(t)[i]. \tag{11}$$

For a closer look at graph wavelet transform (GWT) and a fast implementation, we refer the readers to [17] for further information.

B. Source Estimation by Multiway Blind Source Separation

Let us consider the unconstrained Tucker model for decomposing the tensor $\boldsymbol{\mathcal{X}}$, as given by

$$\boldsymbol{\mathcal{X}} \overset{\text{Tucker}}{=} \boldsymbol{\mathcal{F}} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \times_3 \mathbf{U}_3, \tag{12}$$

where $\boldsymbol{\mathcal{F}} \in \mathbb{R}^{K\times K\times K}$ is the core tensor, $\mathbf{U}_1, \mathbf{U}_2$ and $\mathbf{U}_3$ are the orthogonal loading factors.

For an EEG tensor, Tucker decomposition may not be of interest because the loading factors $\mathbf{U}_i$ do not carry important information about the EEG sources. We may want to obtain meaningful factors $\widehat{\mathbf{U}}_i$, instead. Since $\mathbf{U}_i$ and $\widehat{\mathbf{U}}_i$ share the same subspace, the relationship between $\mathbf{U}_i$ and $\widehat{\mathbf{U}}_i$ can be expressed in the following way:

$$\widehat{\mathbf{U}}_i = \mathbf{U}_i \mathbf{P}_i \mathbf{Q}_i, \tag{13}$$

where $\mathbf{P}_i$ the permutation matrix and $\mathbf{Q}_i$ is the scaling matrix.

We now propose to adapt multiway blind source separation (MBSS) [18] in order to extract the loading factors of the tensor $\boldsymbol{\mathcal{X}}$ and hence obtain the EEG sources.

A simple and flexible approach is to exploit separately each mode-i unfolding matrix of $\boldsymbol{\mathcal{X}}$. From (12), the mode-i unfolding matrix of $\boldsymbol{\mathcal{X}}$, denoted by $\mathbf{X}^{(i)}$, can be given by

$$\mathbf{X}^{(i)} = \mathbf{U}_i {\mathbf{B}_i}^T, \tag{14}$$

where $\mathbf{B}_i = \mathbf{F}^{(i)}(\otimes_{j\neq i}\mathbf{U}_j)^T$ and provides a specific Kronecker structure, $\mathbf{F}^i$ is the mode-i unfolding matrix of the core tensor $\boldsymbol{\mathcal{F}}$.

Thanks to the uniqueness of BSS models, it is easy to obtain a more meaningful factor $\widehat{\mathbf{U}}_i$ by taking

$$\widehat{\mathbf{U}}_i = \Psi_i(\mathbf{X}^{(i)}) = \Psi_i(\mathbf{U}_i\mathbf{B}_i^T) = \mathbf{U}_i\mathbf{P}_i\mathbf{D}_i, \tag{15}$$

where $\Psi_i(\cdot)$ denotes a specific BSS algorithm.

Then, different sources with specific physical meaning can be extracted from different modes of the EEG tensors whose decomposed factors respectively characterize the temporal, spatial and spectral domains of the EEG signals. Particularly, the MBSS method gives us the following:

$$\begin{aligned} &\boldsymbol{\mathcal{X}} \overset{\text{MBSS}}{=} \widehat{\boldsymbol{\mathcal{F}}} \times_1 \mathbf{U}_{\text{temporal}} \times_2 \mathbf{U}_{\text{spatial}} \times_3 \mathbf{U}_{\text{spectral}}, \\ &\mathbf{X}^{(1)} = \mathbf{U}_{\text{temporal}} \widehat{\mathbf{F}}^{(1)} (\mathbf{U}_3 \otimes \mathbf{U}_2)^T, \end{aligned} \tag{16}$$

where $\widehat{\boldsymbol{\mathcal{F}}}$ is a new core tensor and $\widehat{\mathbf{F}}^{(1)}$ is its mode-i unfolding matrix. The new temporal characteristics $\mathbf{U}_{\text{temporal}}$ of $\boldsymbol{\mathcal{X}}$ already provides a good approximate $\hat{\mathbf{S}}$ of the source matrix $\mathbf{S}$.

C. EEG Source Localization using LORETA

Once the three-way tensor $\boldsymbol{\mathcal{X}}$ has been decomposed into multiple components with different EEG sources using the MBSS, localization of the sources can then be implemented, by first estimating the mixing matrix $\mathbf{H}$ and then computing the source position matrix $\mathbf{C}$. We propose to do so in our method of TSS-based analysis.

It is noted that the spatial and spectral variables (in terms of channels and graph wavelet scales) are interdependent, e.g. both are characterized for the spatial domain. Thus, performing MBSS on an EEG tensor does not result in a bilinear model in the graph spectrum and space. Consequently, the spatial factor obtained by MBSS may result in incorrect EEG source localization.

A simple approach to approximate $\mathbf{H}$ from the data, $\mathbf{X}$, and the estimated sources, $\hat{\mathbf{S}}$, is such that

$$\widehat{\mathbf{H}} = \mathbf{X}\hat{\mathbf{S}}^{\dagger}, \tag{17}$$

where $(\cdot)^{\dagger}$ denotes the Moore–Penrose pseudo-inverse operator [3]. Since the number of sources is generally smaller than the number of time samples, i.e. $K \ll T$, the pseudo-inverse matrix of $\hat{\mathbf{S}}$ can be computed efficiently.

In order to estimate the positions of the sources, we can solve the following optimization [7]:

$$\arg\min_{\mathbf{c}_i} \|\hat{\mathbf{h}}_i - \widehat{\mathbf{G}}\mathbf{c}_i\|_2^2 + \lambda\|\mathbf{L}\mathbf{Z}\mathbf{c}_i\|_2^2, \ i \in \{1,\ldots,K\}, \tag{18}$$

where $\hat{\mathbf{h}}_i$ is the i-th column of $\widehat{\mathbf{H}}$, $\widehat{\mathbf{G}} = [\hat{\mathbf{g}}_1, \ldots, \hat{\mathbf{g}}_K]$ is the numerical lead field matrix which can be calculated by using the FieldTrip toolbox[1], $\mathbf{L}$ is the Laplacian matrix defined above in (7), and $\mathbf{Z}$ is a diagonal matrix with $\mathbf{Z}[i,i] = \|\hat{\mathbf{g}}_i\|_2^{-1}$. In particular, the first term of (18) is referred to as the fit between the surface vector recovered from the estimated source and the measurement, while the second term is an ℓ_2-norm regularization about smooth source distributions.

Thanks to the cortical LORETA algorithm [3], the close-form solution of (18) is given by

$$\mathbf{c}_i = (\mathbf{Z}\mathbf{L}^T\mathbf{L}\mathbf{Z})^{-1}\widehat{\mathbf{G}}^T\big(\widehat{\mathbf{G}}(\mathbf{Z}\mathbf{L}^T\mathbf{L}\mathbf{Z})^{-1}\widehat{\mathbf{G}} + \lambda\mathbf{I}\big)^{-1}\hat{\mathbf{h}}_i, \tag{19}$$

for $i = 1, 2\ldots, K$. Finally, a threshold value can be set for the dipole amplitude to obtain the source location where a node belongs to the distributed source if its strength exceeds the value.

IV. Experiments

In order to evaluate the effectiveness of the proposed TSS-based analysis for EEG source localization, both synthetic and real EEG datasets are used in the study. The TSS-based analysis is compared with the state-of-the-art STF-based analysis and STWV-based analysis.

A. EEG Datasets

1) Synthetic Data: We used the Brainstorm software[2] to generate the synthetic EEG data. For consistency with the real EEG data, to be described later, the synthetic data were generated for 19 electrodes, a sampling frequency $f_s = 256$ Hz, epochs of the same length of 100 time samples (or 400 ms. The EEG source space was referred to as the inner cortical surface. The lead field matrix $\mathbf{G} \in \mathbb{R}^{19\times 19626}$ was automatically calculated by Brainstorm, where the grid contains 19626 triangles.

In order to generate the distributed sources, the neuronal population-based model was used to generate epileptic spike-like signals $\mathbf{X}_e$ as well as background activities $\mathbf{X}_b$ in the

[1] http://www.fieldtriptoolbox.org

[2] https://neuroimage.usc.edu/brainstorm

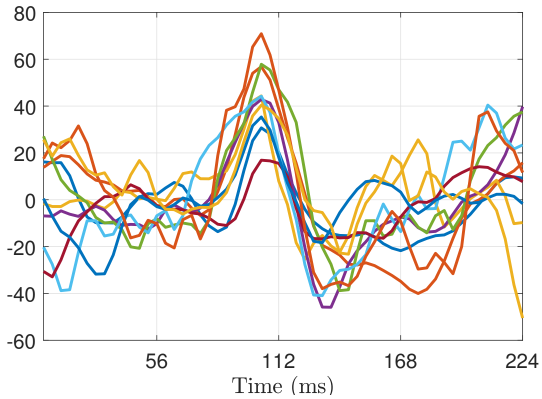


Fig. 1: Simulated epileptic spikes

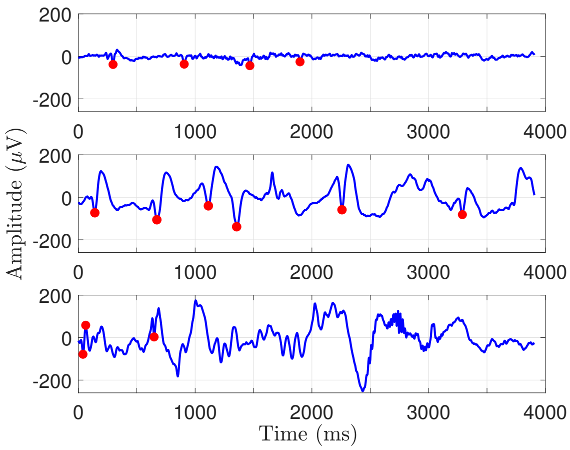


Fig. 2: Real epileptic spikes from three patients with epilepsy in our EEG dataset.

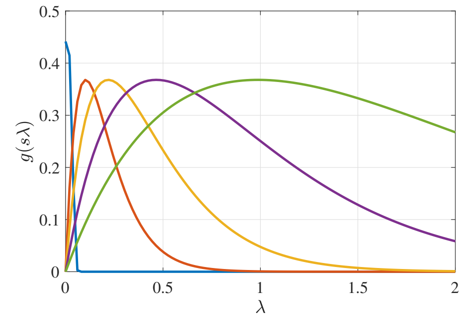
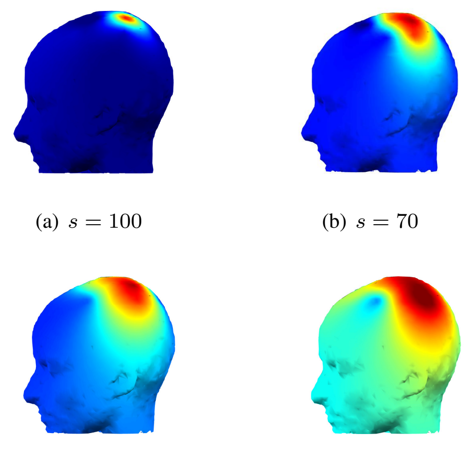


Fig. 3: Wavelet kernel $g(s\lambda)$ for different values of the wavelet scale s.



(a) $s = 100$

(b) $s = 70$

(c) $s = 30$

(d) $s = 10$

Fig. 4: Graph wavelets residing in the 7-th (P3) vertex of the EEG graph.

brain [19], see Figure 1. The white noise matrix $\mathbf{N}$ was generated from the Gaussian distribution $\mathcal{N}(0, \sigma)$.

2) Real Data: The EEG data from patients already diagnosed of epilepsy were recorded by using the international standard 10-20 system with 19 electrodes and a sampling frequency of 256 Hz. Epileptic spikes were manually identified by a neurologist from Vietnam National Children's Hospital. Standard filters for pre-processing EEG signals were used: a lowpass filter with the cutoff frequency of 70 Hz, a highpass filter with the cutoff frequency of 0.5 Hz, and a notched filter to notch the frequency of 50 Hz for removing the electricity grid frequency. Figure 2 illustrates some real epileptic spikes in our EEG dataset.

B. EEG Tensor Representation

We constructed the temporal–spectral–spatial epileptic tensors as follows. First, for each epileptic spike, a data sample is presented by an EEG segment of 100 points around the location of the spike. As such, we have 100 graph signals $\{\mathbf{f}\}_{i=1}^{100}, \mathbf{f}_i \in \mathbb{R}^{19\times 1}$ representing the time-evolving EEG graph in the epileptic epoch. Then, the GWT was applied to derive the vertex-frequency representation of each graph signal. Here, we obtained 100 graph wavelet coefficient matrices of size $19 \times N_{\text{scale}}$ presenting EEG graph spectral features and the number of wavelet scales was selected at $N_{\text{scale}} = 100$. Finally, we concatenated the 100 coefficient matrices into a three-way tensor $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{100\times 19\times 100}$.

In order to generate spectral graph wavelets, we used the Mexican hat kernel, i.e. $g(s\lambda) = s\lambda e^{-s\lambda}$, where λ denotes the eigenvalue of the Laplacian matrix $\mathbf{L}$. Figure 3 illustrates the wavelet kernel with different values of the wavelet scale s and Figure 4 shows the resulting spectral graph wavelets centered at the P3 vertex on the EEG graph.

C. EEG Source Estimation using MBSS

Two epileptic EEG segments from the same patient were used to provide the evidence of applying MBSS for EEG source estimation. We can observe from the first EEG segment in Figure 5(a) that the first component S1 in the spatial mode of the EEG tensor was centered at occipital lobe in the brain. The component S1 suggested that a brain activity can be generated deep in the brain and near electrodes O_1 and O_2. Let us take a closer look at its signature, F1, in the spectral graph domain. We can detect that the activity took place in low spectral wavelet scales indicating that it has a low frequency content. In contrast, the second component S2 exhibited a very high frequency activity. It was also centered at the frontal lobe in the brain. Therefore, it may be the signature of another activity. Similarly, for the second EEG segment, MBSS helped us separate the two different components, see Figure 5(b). Specifically, we can see that the two first components S1 that were obtained from the two EEG segments have similar signatures in all the three domains. It would therefore recommend that this activity is due to epilepsy and the spatial factor can help localize its epileptogenic zone. Besides, the second component S2 from segment 2 can be referred to as a background activity.

D. EEG Source Localization

Figure 6 shows the source localization results using three different methods of multiway analysis: our proposed method (TSS), and the state-of-the-art ones (STF, STWV) on the synthetic EEG data with two distributed sources, 19 electrodes, 100 time samples and the signal-to-noise ratio of $\mathrm{SNR} = 5$ dB. In particular, the two sources were centered at the P3 and F4 vertex respectively, see Figure 6a for the ground truth. For a fair comparison, MBSS was used for EEG source separation and LORETA for EEG source localization across the three different methods of analysis.

We can see from Figure 6 that our TSS-based analysis (Figure 6b) yielded the best localization result in terms of the number of correctly detected dipoles and the sparsity of estimated sources. The STF-based analysis failed to localize the two sources simultaneously. The STWV-based analysis did accurately localize the two sources, but with a detected surface area larger than that by the TSS-based analysis.

V. Conclusions

In this work, we have introduced a new multiway analysis for EEG data which can enhance the ability to separate and localize extended sources in the EEG data. By exploiting the brain structure, we first generated graph signals from the temporal–spatial EEG measurement and then converted the signals into the spectral graph domain using the GWT, which is a GSP tool. We then constructed a new temporal–spatial–spectral tensor representation for the measurement. From that, we applied MBSS to extract meaningful loading factors of the three-way EEG tensors and hence separate the EEG sources. In order to locate the source positions, the cortical LORETA algorithm was used. Experimental results indicated that the proposed multiway TSS analysis using GSP and tensorial MBSS allowed us to not only extract features from multiple domains of the EEG data but also to be able to localize the epileptic spikes. The proposed TSS-based analysis also yielded a more robust result of EEG source localization than the results obtained by the state-of-the-art types of analysis, STF and STWV.

Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.04-2019.14.

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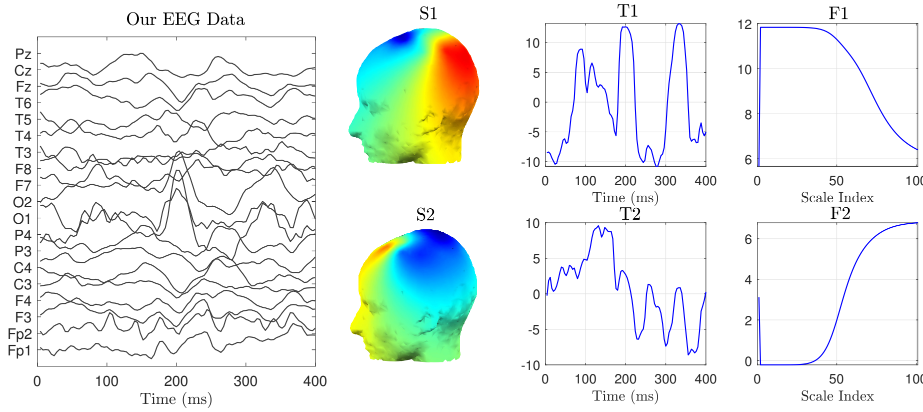
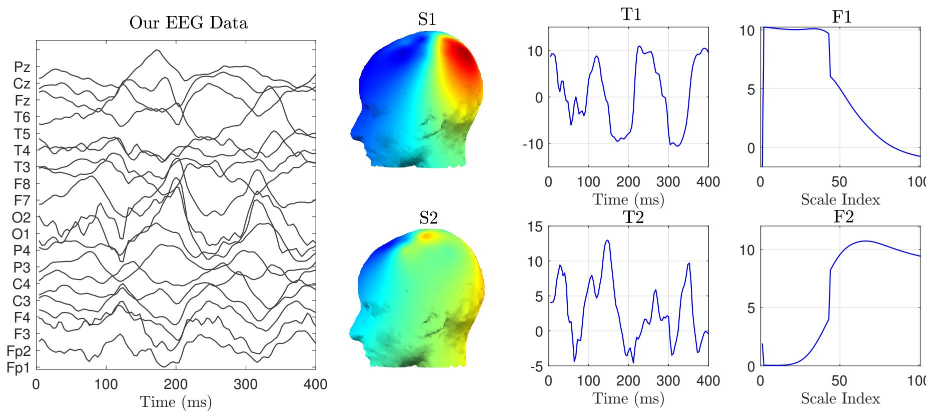
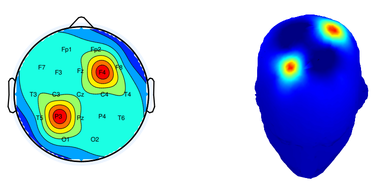


(a) Segment 1

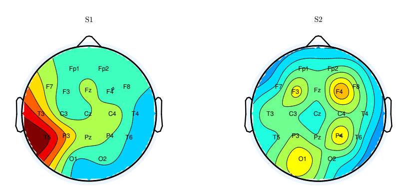


(b) Segment 2

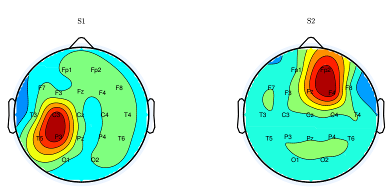
Fig. 5: Tensorial multiway blind source separation (MBSS) on two EEG segments of the same patient.



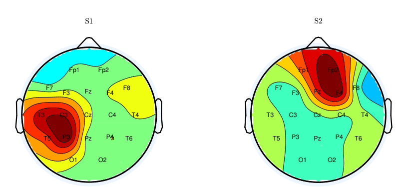
(a) Grounth truth



(b) STF



(c) TSS



(d) STWV

Fig. 6: A performance comparison of EEG source localization methods: TSS vs state-of-the-arts (STF, STWV).