An Efficient Column Generation Approach for Solving the Routing and Spectrum Assignment Problem in Elastic Optical Networks

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***Abstract*—Routing and spectrum assignment (RSA) is an essential problem in designing, operating and managing elastic optical networks to achieve spectrum efficiency and thus, efficient algorithms for solving the RSA has been of crucial importance. The conventional Mixed Integer Linear Programming (MILP) formulation has a critical drawback of scalability and hence has been applicable to only small data instances while heuristic-based approach is prone to locally optimal solutions without guarantees for global optimality. In order to mitigate the scalability issue of the traditional MILP models and possibly low-quality solutions from heuristic, we investigate an approach based on the column generation (CG) method for solving the RSA problem by presenting an efficient CG-based formulation and numerically evaluate it on various realistic network topologies with full mesh traffic. The performance of our CG-based approach is benchmarked with the typical heuristic, First-Fit algorithm, and it has been revealed that our CG proposal can provide better solutions in most cases and the solution gap could be up to more than 20%.**

***Index Terms*—elastic optical networks, integer linear programming, routing and spectrum allocation, column generation, heuristic algorithms**

I. INTRODUCTION

The coming into popularities of increasing data-intensive services such as cloud computing, big data applications and the advent of Augmented Reality/Virtual Reality gives rise to the unprecedented traffic growth in optical core networks. Due to the limited spectrum bandwidth, various technological and algorithmic solutions have been developed to achieve greater spectrum efficiency [1]–[8]. Nevertheless the traditional technologies for core networks based on the fixed transmission scheme (i.e., fixed grid wavelength division multiplexing) has been shown to be spectrally ineffective and hence, may cause the so-called capacity crunch [9]–[12]. In this context, the arrival of elastic optical networks (EONs) enabled by the use of advanced transmission and modulation formats, spectrum-selective switching technologies and flexible frequency spacing paves the new way for provisioning traffic requests in a cost and energy-efficient manner, marking a major departure from the conventional approach based on fixed-grid WDM technologies [13]–[19].

A fundamental problem in designing, operating and managing EONs is the solving of routing and spectrum assignment (RSA) for traffic demands. Specifically, for each demand, it involves the finding of suitable physical path between the source and destination, and provide the adequate spectrum allocation subjected to contiguity, continuity and non-overlapping constraints. RSA problem has been proved to be NP-hard and thus, seeking the globally optimal solution for large-scale scenarios in terms of network size, traffic sets and frequency width is indeed computationally challenging due to the proliferation of variables and constraints. In the literature, there are two approaches for solving RSA problem where the first one is exact method, often based on mixed-integer programming and the second one is approximation algorithm based on heuristic/meta-heuristics. The former approach has the capability of providing optimal solutions or solutions with known quality while the solution delivered by heuristic-based approach could be rapidly obtained and yet with unknown quality [10], [20], [21].

To cope with a huge number of generated variables and constraints for large instances, decomposition techniques have been widely used and among such techniques, column generation (CG) is an efficient method allowing significant reduction of number of variables in the formulation. Specifically, the problem formulated with CG is initiated with a small set of admissible columns and then it can be dynamically added new columns and/or constraints according to the solving of the so-called pricing problem so that leading to the improvement of objective function [22], [23]. Nevertheless, the use of CG for modeling and solving the RSA problem has remained inadequately been investigated [24], [25] and this paper is a contribution to fill this gap. In Section II, we therefore present an efficient CG-based formulation for the RSA problem aiming at finding good sets of light-paths, avoiding the pre-computing and managing a large set of variables while maintaining the high quality of solutions. Our proposal is then benchmarked extensively with the most popular heuristic, First-Fit algorithm, on various realistic networks and full mesh traffic in Section III. Finally, Section IV is dedicated to conclusion and future works.

II. Problem Formulation

We consider the network which is represented by graph $G = (V, E)$: V is the set of optical nodes and E is the set of fiber links. In each link $e \in E$, the same band-width (i.e., optical frequency spectrum) is available and it is divided into the set $S = \{s_1, s_2, ..., s_{|S|}\}$ of fixed frequency width. D denotes the set of node-to-node (traffic) demands which must be realized in the network. Each demand $d \in D$ is represented by its source node $s(d)$ and destination node $t(d)$ and is characterized by a demand bit-rate $k(d)$ in Gbps.

We will use the following notations to formulate the problem:

- V is the nodes set.
- E is the links set
- D is the traffic demands set
- S is the set of all frequency slices, $S = \{s_1, s_2, ..., s_{|S|}\}$.
- $L(d)$ is the set of feasible light-paths for demand d.
- L is the set of all feasible light-paths for all demand (i.e., $L = \cup_{d \in D} L(d)$).
- $L(e, s)$ represents the set of light-paths passing through link e and using slice s.
- $E(l)$ is the set of links of light-path l.
- $S(l)$ is the set of slices of light-path l.
- $d(l)$ represents demand satisfied by light-path l.

Now, we define two family of binary variables:

$$x_{dl} = \begin{cases} 1 & \text{if demand } d \text{ uses light-path } l, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_s = \begin{cases} 1 & \text{if slice } s \text{ is used in any link of the network} \\ 0 & \text{otherwise.} \end{cases}$$

The formulation is based on the notation of link light-path in which a light-path (also called *optical path*) is represented by a pair (p, c), where p is a routing path and c is a frequency channel. The routing consists of links connecting the source node to the destination node while the frequency channel is a set of contiguous spectrum slices assigned to the light-path—the spectrum contiguity constraint. For instance, a frequency channel c of capacity n must be in the form $c = \{s_i, s_{i+1}, ..., s_{i+n-1}\}$ for some i between 1 and $|S| - (n-1)$. Note that, the frequency channel c must be the same on links belonging to the routing path and such property is called the spectrum continuity constraint. We assume that for each demand $d \in D$, the set of feasible light-paths $L(d)$ is given. Finally, we denote by L the set of all feasible light-paths, say $L = \cup_{d \in D} L(d)$.

In this work, the objective of solving the RSA problem is to optimally identify one light-path for each demand subject to constraints including spectrum continuity, spectrum contiguity and the uniqueness of spectrum slice usage—no two demands use the same slice on the same link—so that the number of used spectrum slices is minimized. For each light-path $d \in D$, we consider decision variable $x_{dl}, l \in L(d)$, which equals to 1 if the light-path l is chosen and carries the traffic of demand d, and equals to 0 otherwise. The utilization of slice s in the network is characterized by a binary variable $y_s, s \in S$. The formulation of RSA can be expressed as an integer linear programming problem

$$\min \sum_{s \in S} y_s$$

subject to

$$\sum_{l \in L(d)} x_{dl} = 1, \forall d \in D, \quad (1)$$

$$\sum_{l \in L(e,s)} x_{d(l),l} \leq y_s, \forall e \in E, \forall s \in S, \quad (2)$$

$$x_{dl} \in \{0, 1\}, \forall d \in D, \forall l \in L(d), \quad (3)$$

$$y_s \in \{0, 1\}, \forall s \in S. \quad (4)$$

In this formulation, $L(e, s)$ is the set of light-paths that pass through the link e and use the slice s, $L(d)$ is the set of feasible light-paths for demand d, and $d(l)$ is the demand satisfied by the feasible light-path l. The goal is to minimize the number of actually used slices (say, the sum of variables y_s in the objective function). Constraint (1) requires that each demand will use precisely one feasible light-path. Constraint (2) enforces that there are no collisions of the assigned resources, i.e., no two light-paths use the same slice on the same link if a slice s is used.

Because of the constraints (1), we can replace the conditions $x_{dl} \in \{0, 1\}, d \in D, l \in L(d)$, by the following ones $x_{dl} \in \mathbb{N}, d \in D, l \in L(d)$. The linear programming relaxation of this problem (called the Master Problem (MP)) which removes the integrality constraint of each variable, can be written as

$$\min \sum_{s \in S} y_s$$

subject to

$$\sum_{l \in L(d)} x_{dl} = 1, \forall d \in D, \quad (5)$$

$$y_s - \sum_{l \in L(e,s)} x_{d(l),l} \geq 0, \forall e \in E, \forall s \in S. \quad (6)$$

$$y_s \leq 1, \forall s \in S, \quad (7)$$

$$x_{dl} \geq 0, \forall d \in D, \forall l \in L(d), \quad (8)$$

$$y_s \geq 0, \forall s \in S. \quad (9)$$

In what follows, we will describe the methodology of column generation approach for solving this problem. By taking $L^1(d) \subset L(d), d \in D$, and $L^1 = \bigcup_{d \in D} L^1(d)$, we firstly consider the Restricted Master Problem corresponding to this subset of light-paths, denoted by MP(L^1)

$$\min \sum_{s\in S} y_s$$

subject to

$$\sum_{l\in L^1(d)} x_{dl} = 1, \forall d \in D, \quad (10)$$

$$y_s - \sum_{l\in L^1(e,s)} x_{d(l),l} \geq 0, \forall e \in E, \forall s \in S, \quad (11)$$

$$y_s \leq 1, \forall s \in S, \quad (12)$$

$$x_{dl} \geq 0, \forall d \in D, l \in L^1(d), \quad (13)$$

$$y_s \geq 0, \forall s \in S. \quad (14)$$

The dual program of MP(L^1), denoted by D(L^1), is

$$\max \sum_{d\in D} \lambda_d + \sum_{s\in S} \mu_s$$

subject to

$$\sum_{e\in E} \pi_{es} \leq 1 + \mu_s, \forall s \in S, \quad (15)$$

$$\lambda_d - \sum_{e\in E(l)} \sum_{s\in S(l)} \pi_{es} \leq 0, \forall d \in D, \forall l \in L^1(d), \quad (16)$$

$$\lambda_d \in \mathbb{R}, \forall d \in D, \quad (17)$$

$$\pi_{es} \geq 0, \forall e \in E, \forall s \in S, \quad (18)$$

$$\mu_s \leq 0, \forall s \in S. \quad (19)$$

In this program, λ_d is the dual variable related to the satisfying constraints (10) of demand d, μ_s is the dual variable related to the utilization constraints (12) of slice s, and π_{es} is the dual variable corresponding to the constraint (11) about using slice s on edge e.

Suppose that

$$(\bar{\lambda}, \bar{\mu}, \bar{\pi}) = (\bar{\lambda}_1, \bar{\lambda}_2, ..., \bar{\lambda}_D, \bar{\mu}_1, \bar{\mu}_2, ..., \bar{\mu}_S, \bar{\pi}_1, \bar{\pi}_2, ..., \bar{\pi}_{|E|*|S|})$$

is an optimal solution of the dual problem D(L^1). Then, the following condition is satisfied

$$\bar{\lambda}_d - \sum_{e\in E(l)} \sum_{s\in S(l)} \bar{\pi}_{es} \leq 0, d \in D, l \in L^1(d).$$

The formula $\bar{\lambda}_d - \sum_{e\in E(l)} \sum_{s\in S(l)} \bar{\pi}_{es}$ in the left hand side is called *reduced cost*. It is obvious that if the above condition holds for all $l \in L(d), d \in D$ (i.e., $(\bar{\lambda}, \bar{\mu}, \bar{\pi})$ is feasible solution for the dual program of (MP)), then $(\bar{\lambda}, \bar{\mu}, \bar{\pi})$ is also an optimal solution of the dual program of Master Problem. Otherwise, we try to seek for a light-path $l \in L(d)\backslash L^1(d)$, for a demand $d \in D$ such that

$$\bar{\lambda}_d - \sum_{e\in E(l)} \sum_{s\in S(l)} \bar{\pi}_{es} > 0. \quad (20)$$

This is called the *sub-problem*. This sub-problem (also called, the pricing problem) is a problem of finding, for each demand $d \in D$, a new light-path l which gives positive (and possibly, the largest) reduced cost. When a such light-path is found, new variable x_{dl} corresponding to it will be added to the Restricted Master Problem. In our column generation implementation, at each iteration and for each demand, we look for and include into set L^1 a light-path which provides the largest positive reduced cost. If no such light-path exists for all demands, the algorithm stops (i.e., the Master Problem is solved optimally). In such case, we will solve the integer linear programming formulation of the final restricted master problem (RMP) to obtain an approximate solution for the RSA problem.

The column generation-based algorithm for the RSA problem is summarized as follows.

Column generation-based algorithm

Step 1. Find initial sets $L^1(d)$ of light-paths for each demand $d \in D$, and set: $L^1 = \bigcup_{d\in D} L^1(d)$.
Step 2. Solve the linear programming problem MP(L^1) to obtain an optimal solution as well as an optimal dual solution $(\bar{\lambda}, \bar{\mu}, \bar{\pi})$.
Step 3. For each demand $d \in D$, solving the sub-problem optimally to find a light-path $l \in L(d)\backslash L^1(d)$, and update $L^1(d) := L^1(d) \cup \{l\}$.
Step 4. Iterate steps 2-3 until no light-path satisfying the condition (20) can be found.
Step 5. Solve the integer linear programming (ILP) formulation of the final restricted master problem (RMP) to have an approximate solution.

The sub-problem

Solving the sub-problem leads to solving shortest path problems on a weighted graph. In the current iteration, consider a fixed demand d with $\bar{\lambda}_d > 0$, we solve this problem as follows: for each frequency channel satisfying this demand, we compute the weight on edges $e \in E$ of the graph G based on the information of $\bar{\pi}_{es}$ that are non-negative values. Then, the classical Dijkstra algorithm is used to find the shortest path on this weighted graph. The obtained shortest path combining with that frequency channel would be a candidate light-path for satisfying d. Among the light-paths generated in this way, only one will be added to the MP(L^1) if that light-path makes the reduced cost positive and largest.

III. Numerical Experiment

In this section, we evaluate the performance of our column generation algorithm on three realistic networks given in the Fig. 1, namely, (a) COST239 with 11 nodes and 52 links, (b) NSFNET with 14 nodes and 42 links and (c) ITALY network with 14 nodes and 58 links. In the first network, we have 20 sample tests, and each has $D = 110$ demands; the size of frequency slices is set to 100. In the NSFNET and ITALY topologies, we have 20 sample tests, and each has $D = 182$ demands and the size of frequency slices is set to 200, which is large enough to accommodate all demands. The algorithms are written in MATLAB, and are tested on a computer armed with Intel Core i5 1.6 GHz, RAM 8G. We used the solver CPLEX 12.8 for the linear program MP(L^1), and the ILP formulation of the last RMP. We also used the function `graphshortestpath` in Matlab for finding shortest paths in the sub-problem. The time for solving the MILP model of the last RMP is limited to 2 hours.

We will compare the results provided by our column generation approach with a typical heuristic method, namely First-Fit algorithm [10]. For each test instance, the heuristic method

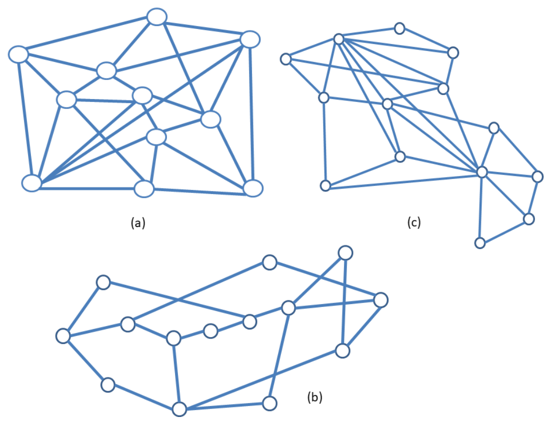

Fig. 1. Network topologies for evaluation

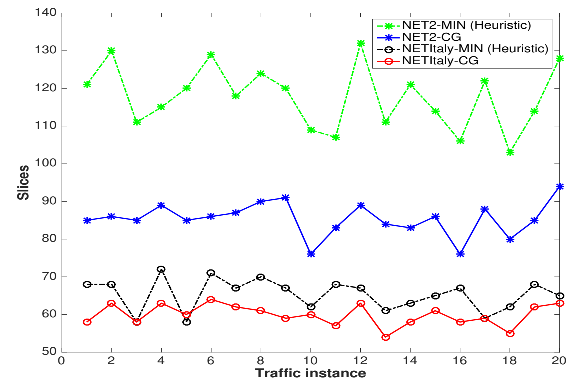

Fig. 2. Comparative results between NET2 (NSF topology) and NET3 (ITALY topology)

is run with the parameter $k = 2, 3, ..., 20$. In Table I, II and III we resume the mean, standard deviation, minimum value and maximal value of number slices found by the heuristic method for all parameter k on three networks. To start column generation, we use the worst solution (the initial light-paths) provided by this heuristic method corresponding to the case of $k = 2$. This solution also provides an upper bound for the number of used spectrum slices. In these tables: *Dual Bound* column denotes the optimal value of Master Problem (MP), *Light-Paths* column represents the number of light-paths generated during the column generation algorithm, CPU time is reported in second. In this case, we define

$$\textbf{Gap}(\%) = \frac{\text{Min(Heuristic Method) - Slices(Column Generation)}}{\text{Min(Heuristic Method)}}.$$

From Tables I, II, and III we can see that although the heuristic method can give feasible solution very quickly, about 0.69s, 2.62s, and 2.01s on the COST239, NSFNET and ITALY topology respectively, but the solution quality is not fine enough. It can be observed that our column generation method produces far better solutions than the heuristic does in the majority of cases. On the network COST239, the difference between the best solution provided by the heuristic method and the solution of column generation varies from 9.38% to 23% (16.01% in average). On the larger network NSFNET, this varies from 22.33% to 33.85% (27.34% in average). On the network ITALY, the column generation still gives better solutions on 17/20 traffic instances except the ones 3, 5, and 17. In this case the solution gap variation could be up to 16.18% in the most favorable conditions and 8.04% in average.

Next, comparing the optimal objective for NSFNET and ITALY networks whose difference is on the number of links, from Tables II, III and Figure 2 we can see the negative relationship between number of links in the network and number of used slices in the solutions provided by the heuristic method as well as the column generation. Interestingly, we observe that the large number of links can reduces the solution gap provided by the two methods.

IV. Conclusion

In this paper, we have proposed an efficient column generation approach for solving the problem of routing and spectrum allocation (RSA) in flexgrid elastic optical networks. Some numerical results have demonstrated the efficiency of our approach in comparison with a widely used First-Fit heuristic. In future works, we will investigate some branching techniques to get a Branch-and-Brice scheme for global solution, as well as compare with the other methods on larger scale networks.

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References

[1] D. T. Hai, "An optimal design framework for 1+1 routing and network coding assignment problem in wdm optical networks," *IEEE Access*, vol. 5, pp. 22 291–22 298, 2017.

[2] ——, "A bi-objective integer linear programming model for the routing and network coding assignment problem in wdm optical networks with dedicated protection," *Computer Communications*, vol. 133, pp. 51 – 58, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0140366418300148

[3] ——, "On routing, spectrum and network coding assignment problem for transparent flex-grid optical networks with dedicated protection," *Computer Communications*, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0140366418306546

[4] ——, "Leveraging the survivable all-optical wdm network design with network coding assignment," *IEEE Communications Letters*, vol. 21, no. 10, pp. 2190–2193, Oct 2017.

[5] ——, "On solving the 1 + 1 routing, wavelength and network coding assignment problem with a bi-objective integer linear programming model," *Telecommunication Systems*, vol. 71, no. 2, pp. 155–165, Jun 2019. [Online]. Available: https://doi.org/10.1007/s11235-018-0474-9

[6] T. H. Dao, "On optimal designs of transparent wdm networks with 1+1 protection leveraged by all-optical xor network coding schemes," *Optical Fiber Technology*, vol. 40, pp. 93 – 100, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1068520017303012

[7] D. T. Hai, "Re-designing dedicated protection in transparent wdm optical networks with xor network coding," in *2018 Advances in Wireless and Optical Communications (RTUWO)*, Nov 2018, pp. 118–123.

[8] D. T. Hai, L. H. Chau, and N. T. Hung, "A priority-based multiobjective design for routing, spectrum, and network coding assignment problem in network-coding-enabled elastic optical networks," *IEEE Systems Journal*, pp. 1–12, 2019.

TABLE I
COMPARATIVE RESULTS ON NET1 (COST239 TOPOLOGY).

Traffic instance	Heuristic Method					Column Generation				Gap
	Slices				Time (s)	Dual Bound	Slices	Light-Paths	Time (s)	
	Mean	STD	Min	Max						
1	32.74	2.05	30	39	0.70	21.38	26	2136	3624.66	13.33%
2	34.16	2.17	32	42	0.69	22.38	27	2028	1411.20	15.63%
3	34.32	2.03	31	40	0.69	22.13	26	2211	3621.80	16.13%
4	34.21	1.81	32	39	0.69	21.63	27	2144	2374.42	15.63%
5	33.95	2.20	32	40	0.69	22.00	29	1946	217.73	9.38%
6	28.84	1.42	27	33	0.68	19.29	23	1685	886.01	14.81%
7	28.26	1.37	27	32	0.69	19.43	23	1702	2150.50	14.81%
8	35.42	1.54	33	40	0.69	23.63	28	1899	3617.83	15.15%
9	34.58	1.89	33	42	0.69	21.63	26	2372	3625.39	21.21%
10	37.26	2.90	35	48	0.68	23.50	27	2838	3636.51	22.86%
11	32.95	2.55	32	43	0.69	21.75	26	2069	2611.49	18.75%
12	33.32	2.26	31	40	0.69	21.63	26	1773	410.98	16.13%
13	33.95	2.37	32	42	0.69	22.31	28	2249	223.00	12.50%
14	33.05	1.87	30	38	0.69	22.44	27	2143	3623.38	10.00%
15	33.05	1.99	31	39	0.69	22.86	27	1866	1726.90	12.90%
16	34.58	1.39	32	39	0.70	21.38	26	2039	3621.65	18.75%
17	35.16	1.21	34	38	0.69	22.86	26	2118	2149.66	23.53%
18	31.84	2.43	29	38	0.68	20.88	25	2038	2049.61	13.79%
19	34.32	3.40	32	45	0.69	23.00	27	2197	2500.90	15.63%
20	33.16	0.83	31	34	0.69	21.13	25	1810	1160.78	19.35%
Average	**33.46**	**1.98**	**31.30**	**39.55**	**0.69**	**21.88**	**26.25**	**2063.15**	**2262.22**	**16.01%**

TABLE II
COMPARATIVE RESULTS ON NET2 (NSF TOPOLOGY).

Traffic instance	Heuristic Method					Column Generation				Gap
	Slices				Time (s)	Dual Bound	Slices	Light-Paths	Time (s)	
	Mean	STD	Min	Max						
1	129.37	4.67	121	136	2.89	70.25	85	3626	7292.03	29.75%
2	136.89	5.29	130	148	2.94	69.75	86	4661	7309.78	33.85%
3	116.16	3.18	111	122	2.90	68.50	85	3324	7259.82	23.42%
4	118.84	3.91	115	128	3.00	76.00	89	3714	7270.50	22.61%
5	125.05	3.94	120	136	2.96	76.25	85	3675	7264.62	29.17%
6	137.68	5.50	129	145	2.89	69.25	86	4307	7294.00	33.33%
7	133.26	7.59	118	151	2.60	72.75	87	4145	7401.77	26.27%
8	137.53	7.06	124	144	2.49	74.50	90	3945	7481.52	27.42%
9	126.26	4.31	120	138	2.52	75.25	91	3587	7277.39	24.17%
10	116.47	3.24	109	126	2.48	60.75	76	4952	7611.80	30.28%
11	116.00	6.50	107	124	2.51	69.00	83	3866	7273.92	22.43%
12	142.53	4.55	132	150	2.46	71.75	89	3649	7264.89	32.58%
13	132.79	7.84	111	139	2.46	66.25	84	3313	7246.94	24.32%
14	130.74	5.92	121	146	2.47	72.75	83	4148	7303.93	31.40%
15	122.37	4.75	114	134	2.46	71.25	86	3797	7274.40	24.56%
16	109.21	3.34	106	118	2.46	63.00	76	3741	7263.48	28.30%
17	127.84	3.02	122	135	2.50	74.75	88	3444	7279.65	27.87%
18	116.84	5.76	103	122	2.46	68.75	80	2909	7254.97	22.33%
19	126.37	5.23	114	131	2.46	70.75	85	3568	7265.39	25.44%
20	134.89	3.54	128	143	2.47	80.5	94	4131	7576.35	26.56%
Average	**126.86**	**4.96**	**117.75**	**135.80**	**2.62**	**71.14**	**85.40**	**3825.10**	**7323.36**	**27.34%**

TABLE III
COMPARATIVE RESULTS ON NET3 (ITALY TOPOLOGY).

Traffic instance	**Heuristic Method**					**Column Generation**				**Gap**
	Slices				**Time (s)**	**Dual Bound**	**Slices**	**Light-Paths**	**Time (s)**	
	Mean	**STD**	**Min**	**Max**						
1	73.26	6.73	68	99	2.66	48.75	58	3361	7266.67	14.71%
2	72.05	6.60	68	94	2.28	55.33	63	2923	7252.09	7.35%
3	62.95	7.87	58	88	2.38	47.00	58	2677	7242.25	0.00%
4	79.79	10.28	72	120	2.28	51.25	63	3812	7303.98	12.50%
5	64.47	9.14	58	96	1.84	49.33	60	2660	7248.77	-3.45%
6	77.63	7.17	71	105	1.85	55.75	64	3119	7264.55	9.86%
7	74.42	9.95	67	108	2.82	50.50	62	3553	7271.22	7.46%
8	76.32	10.49	70	116	1.98	50.33	61	3046	7255.62	12.86%
9	71.68	5.94	67	93	1.85	49.00	59	3128	7253.19	11.94%
10	67.58	7.34	62	91	1.84	50.33	60	2468	7233.45	3.23%
11	74.11	5.91	68	92	1.85	45.60	57	3405	7265.52	16.18%
12	72.47	7.97	67	101	1.98	54.75	63	2914	7257.11	5.97%
13	66.26	6.38	61	86	1.90	44.40	54	2995	7254.73	11.48%
14	70.26	10.47	63	106	1.82	48.50	58	3581	7272.71	7.94%
15	73.00	11.40	65	115	1.81	51.00	61	3652	7297.92	6.15%
16	71.32	6.50	67	94	1.82	46.20	58	3124	7258.07	13.43%
17	66.74	8.85	59	91	1.81	47.40	59	3592	7277.71	0.00%
18	68.37	8.11	62	95	1.89	48.00	55	2620	7240.57	11.29%
19	73.58	6.99	68	100	1.82	49.75	62	2868	7277.10	8.82%
20	70.42	5.59	65	87	1.81	50.33	63	2269	5112.03	3.08%
Average	**71.33**	**7.98**	**65**	**99**	**2.01**	**49.72**	**60**	**3088**	**7155.26**	**8.04%**

[9] H. Dao Thanh, M. Morvan, and P. Gravey, "On the usage of flexible transponder in survivable transparent flex-grid optical network," in *2014 9th International Symposium on Communication Systems, Networks Digital Sign (CSNDSP)*, July 2014, pp. 1123–1127.

[10] D. T. Hai, M. Morvan, and P. Gravey, "Combining heuristic and exact approaches for solving the routing and spectrum assignment problem," *IET Optoelectronics*, vol. 12, no. 2, pp. 65–72, 2018.

[11] H. D. Thanh, M. Morvan, P. Gravey, F. Cugini, and I. Cerutti, "On the spectrum-efficiency of transparent optical transport network design with variable-rate forward error correction codes," in *16th International Conference on Advanced Communication Technology*, Feb 2014, pp. 1173–1177.

[12] P. J. Winzer, "Would scaling to extreme ultraviolet or soft x-ray communications resolve the capacity crunch?" *Journal of Lightwave Technology*, vol. 36, no. 24, pp. 5786–5793, Dec 2018.

[13] H. Dao, M. Morvan, and P. Gravey, "An efficient network-side path protection scheme in ofdm-based elastic optical networks," *International Journal of Communication Systems*, vol. 31, no. 1, p. e3410, e3410 dac.3410. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/dac.3410

[14] D. T. Hai, "Multi-objective genetic algorithm for solving routing and spectrum assignment problem," in *2017 Seventh International Conference on Information Science and Technology (ICIST)*, April 2017, pp. 177–180.

[15] D. T. Hai and K. M. Hoang, "An efficient genetic algorithm approach for solving routing and spectrum assignment problem," in *2017 International Conference on Recent Advances in Signal Processing, Telecommunications Computing (SigTelCom)*, Jan 2017, pp. 187–192.

[16] Y. Ujjwal and J. Thangaraj, "Review and analysis of elastic optical network and sliceable bandwidth variable transponder architecture," *Optical Engineering*, vol. 57, no. 11, pp. 1 – 18, 2018. [Online]. Available: https://doi.org/10.1117/1.OE.57.11.110802

[17] H.-C. Le, T.-L. Mai, and N. T. Dang, "Spectrum utilization of coarse granular routing elastic optical networks," in *Proceedings of the Eighth International Symposium on Information and Communication Technology*, ser. SoICT 2017. New York, NY, USA: ACM, 2017, pp. 197–203. [Online]. Available: http://doi.acm.org/10.1145/3155133.3155180

[18] D. T. Hai, "On the spectrum-efficiency of qos-aware protection in elastic optical networks," *Optik*, p. 163563, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0030402619314615

[19] ——, "Network coding for improving throughput in wdm optical networks with dedicated protection," *Optical and Quantum Electronics*, vol. 51, no. 12, p. 387, Nov 2019. [Online]. Available: https://doi.org/10.1007/s11082-019-2104-5

[20] D. Hai, M. Morvan, and P. Gravey, "On the routing and spectrum assignment with multiple objectives," in *Advanced Photonics for Communications*. Optical Society of America, 2014, p. JT3A.12. [Online]. Available: http://www.osapublishing.org/abstract.cfm?URI=PS-2014-JT3A.12

[21] P. Gravey, D. Hai, and M. Morvan, "On the advantages of co-ofdm transponder in network-side protection," in *Advanced Photonics for Communications*. Optical Society of America, 2014, p. PW1B.3. [Online]. Available: http://www.osapublishing.org/abstract.cfm?URI=PS-2014-PW1B.3

[22] B. Jaumard, C. Meyer, and B. Thiongane, "On column generation formulations for the rwa problem," *Discrete Applied Mathematics*, vol. 157, no. 6, pp. 1291 – 1308, 2009, reformulation Techniques and Mathematical Programming. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0166218X08004277

[23] M. Klinkowski, M. otkiewicz, K. Walkowiak, M. Piro, M. Ruiz, and L. Velasco, "Solving large instances of the rsa problem in flexgrid elastic optical networks," *IEEE/OSA Journal of Optical Communications and Networking*, vol. 8, no. 5, pp. 320–330, May 2016.

[24] J. Enoch and B. Jaumard, "Towards optimal and scalable solution for routing and spectrum allocation," *Electronic Notes in Discrete Mathematics*, vol. 64, pp. 335 – 344, 2018, 8th International Network Optimization Conference - INOC 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1571065318300350

[25] L. Velasco, A. Castro, M. Ruiz, and G. Junyent, "Solving routing and spectrum allocation related optimization problems: From off-line to in-operation flexgrid network planning," *Journal of Lightwave Technology*, vol. 32, no. 16, pp. 2780–2795, Aug 2014.