A 3D MODELING OF UNDERWATER SOUND PROPAGATION AND ITS APPLICATION IN TONKIN GULF

MÔ HÌNH HÓA 3D TRUYỀN ÂM DƯỚI NƯỚC VÀ ÁP DỤNG VÀO VỊNH BẮC BỘ

Hoàng Việt Trung

Viện Khoa học và Công nghệ Quân sự

Trần Cao Quyền

Bộ môn thông tin Vô tuyến, Khoa Điện tử-Viễn thông

Trường Đại học Công nghệ (ĐHQGHN)

Email: quyentc@vnu.edu.vn

ABSTRACT

Underwater sound propagation has many important applications of SONAR (Sound Navigation and Ranging) not only in military sector but also in civilization industry. The problem of 3D modeling of underwater sound propagation is coming from the need of calculation, estimation of sound pressure in order to support to SONAR applications. This paper propose a method of 3D modeling of underwater sound propagation using Normal mode (NM) theory and PDE (Partial Differential Equation) tool box of Matlab (There is currently no research paper using this approach). The parameters of Tonkin gulf were used in our simulations. With the water depth of 100 m, sea bed depth of 10 m and the transmission range of 2 km, the analysis and simulation results show the possibility of 3D visualization of the proposed method in Tonkin gulf as expected. The number of real mode are the same at all time but the mode fashions are different from time to time.

Keywords: *Sound propagation, Normal mode, PDE, 3D visualization*

TÓM TẮT

Truyền âm dưới nước có nhiều ứng dụng SONAR quan trọng không chỉ trong phần quân sự mà cả công nghiệp dân sự. Bài toán mô hình hóa 3D truyền âm dưới nước đến từ nhu cầu tính toán, dự đoán áp suất âm để hỗ trợ các ứng dụng SONAR. Bài báo này đề xuất một cách mô hình hóa 3D truyền âm dưới nước dùng lý thuyết Mode chuẩn và công cụ PDE của Matlab (Hiện nay chưa có bài báo nào áp dụng lối này). Các tham số của vịnh Bắc Bộ được dùng trong các mô phỏng của chúng tôi. Với độ sâu cột nước 100 m, độ sâu đáy 10 m và cự ly truyền 2 km, các phân tích và mô phỏng chỉ ra khả năng trực quan hóa 3D quá truyền âm dưới nước trong vịnh Bắc Bộ như kỳ vọng ban đầu. Số mode thực hình thành trong toàn thời gian mô phỏng nhưng hình dạng các mode thì thay đổi theo thời gian.

Từ khóa: *Truyền âm, Mode chuẩn, PDE, trực quan hóa 3D*

1. INTRODUCTION

Underwater sound propagation has many important applications of SONAR not only in military sector but also in civilization industry. The nature of sound propagation problem is to find the way of propagation of sound pressure. Generally, sound pressure has to satisfy the sound wave equation (a second order Partial Differential Equation) and initial conditions of sound sources and boundary conditions of medium [1].

According to Normal mode (NM) theory [1-3], the sound pressure at the receiver is a summation of all propagation modes in space. If we assume that underwater sound propagation in a ocean waveguide which is limited by sea surface and sea bed then modes propagation are carried out in 2 dimensions: in vertical and in horizontal (depth and range) [4-5]. In this case, to be simplify but not loss the general, 2 layers of sound propagation are considered, i.e, water column and sea bed. The water column is limited by sea surface and sea bed with sound velocity of 1500 m/s whereas sea bed is made of sand with sound velocity of 1700 m/s (In reality, sea bed can be sand or mud or both of them).

Recently, to visualize the way of sound propagation one build a 3D modeling of underwater sound propagation [6]. The model has to be visualized, in 3 dimensions, as well as to be satisfied the requirement of the symmetry property of sound propagation, i.e, cylindrical spreading or spherical spreading. For underwater sound propagation, it is most reasonable permission of the former.

The problem of 3D modeling of underwater sound propagation is coming from the need of calculation, estimation of sound pressure in order to support to SONAR applications both of military sector and civilization industry [6]. This paper deal with the 3D modeling of underwater sound propagation using Normal mode theory and PDE tool box of Matlab (There is currently no research paper using this approach).

The analysis and simulation of 3D modeling of underwater sound propagation using PDE tool box combining 2 layers of sound propagation (water column and sea bed) are performed successfully. The parameters of Tonkin gulf were used in our simulation [7]. With the water depth of 100 m, sea bed depth of 10 m and the range investigated is 2 km, the analysis and simulation results show the possibility of 3D visualization of underwater sound propagation in Tonkin gulf as expected. The number of real mode are the same at all time but the mode fashions are different from time to time.

The rest of the paper are organized as follows. The Normal mode theory is introduced in Part 2. The 3D modeling of underwater sound propagation is presented in Part 3. Part 4 shows the simulation results. Finally, a conclusion is given.

2. THE NORMAL MODE

Staring from Helmholtz equation in two dimensions with sound speed c and density ρ depending only on depth z [1]

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\boldsymbol{\psi}}{\partial r})+\rho(z)\frac{\partial}{\partial z}(\frac{1}{\rho(z)}\frac{\partial\boldsymbol{\psi}}{\partial z})+\frac{\omega^2}{c(z)^2}\boldsymbol{\psi}=-\frac{\delta(r)\delta(z-z_s)}{2\pi r} \quad (1)$$

where $\boldsymbol{\Psi}$ is sound pressure, z_s is source depth, z and r are variables of depth and distance respectively.

Using separation of variables $\boldsymbol{\psi}(\mathrm{r,z})=\boldsymbol{\Phi}(\mathrm{r}).\mathbf{V}(\mathrm{z})$, we obtain the modal equation

$$\rho(z)\frac{d}{dz}[\frac{1}{\rho(z)}\frac{d\mathbf{V}_m(z)}{dz}]+[\frac{\omega^2}{c(z)^2}-k_{rm}{}^2]\mathbf{V}_m(z)=0 \quad (2)$$

with the boundary conditions such as

$$V(0)=0,\frac{dV}{dz}|_{z=D}=0 \quad (3)$$

where D is the depth of water column.

The former condition implies a pressure release surface and the latter condition is from a perfect rigid bottom. The modal equation that is the center of the NM, has an infinite number of modes. Each mode represents by a mode amplitude $V_m(z)$ and a horizontal propagation constant k_{rm}. $V_m(z)$ and k_{rm} are also called *eigenfunction* and *eigenvalue* respectively

Noting that the modes are orthonormal, i.e.,

$$\begin{aligned}&\int_0^D \frac{V_m(z)V_n(z)}{\rho(z)}dz = 0, \quad m \neq n \\ &\int_0^D \frac{V_m(z)^2}{\rho(z)}dz = 1, \quad m = n\end{aligned} \tag{4}$$

Since the modes forms a complete set, the pressure can represents as a sum of the normal modes

$$\psi(r,z) = \sum_{m=1}^{\infty} \Phi_m(r) V_m(z) \tag{5}$$

After some manipulations, we obtain

$$\psi(r,z) = \frac{i}{4\rho(z_s)} V_m(z_s) H_0^1(k_{rm} r) \tag{6}$$

where H_0^1 is the Hankel function of the first kind.

Substitute (6) back to (5) we have

$$\psi(r,z) = \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} V_m(z_s) V_m(z) H_0^1(k_{rm} r) \tag{7}$$

Finally, using the asymptotic approximation of the Hankel function, the pressure can be written as

$$\psi(r,z) \approx \frac{i}{\rho(z_s)\sqrt{8\pi r}} e^{-i\pi/4} \sum_{m=1}^{\infty} V_m(z_s) V_m(z) \frac{e^{ik_{rm} r}}{\sqrt{k_{rm}}} \tag{8}$$

3. THE 3D MODELING OF UNDERWATER SOUND PROPAGATION

3.1. Sound wave equations

According to [4], in water column layer, the sound wave is satisfied the wave equation as follows

$$\nabla^2 \mathbf{\Psi} + k_1 \mathbf{\Psi} = 0, k_1 = \frac{\omega}{c_1} \tag{9}$$

In sea bed layer, the sound wave is satisfied the wave equation as follows

$$\nabla^2 \mathbf{\Psi} + k_2 \mathbf{\Psi} = 0, k_2 = \frac{\omega}{c_1} \tag{10}$$

The boundary conditions are as follows

$$V(0) = V(\infty) = 0$$
$$\frac{dV}{dz}\Big|_{z=D} = 0 \qquad (11)$$

The condition of formation of real modes is

$$k_2 < k(z) < k_1 \qquad (12)$$

3.2. The medium of Tonkin gulf

As far as ocean waveguide is concerned,two layers are considered, i.e, water column and sea bed. The water column is bounded by sea surface and sea bed whereas the sea bed has different structures. It can be made of sand, mud or a composite of both of them.

In this paper, the parameters of Tonkin gulf are investigated.The parameters of Tonkin gulf [7] are given in the table 1 as follows

Table 1. The parameters of Tonkin gulf

Parameter	Value
Depth of water column	100 m
Depth of sea bed (made of sand)	10 m
Sound velocity in water column	1500 + 0.3z (m/s)
Sound velocity in sea bed	1700 (m/s)

Therefore, the depth of water column of Tonkin gulf is less than 100 m with the sound velocity is approximated by $c_1 = 1500\, m/s$; the sea bed is made of sand with its depth is less than 10 m and the sound velocity is approximated by $c_2 = 1700\, m/s$.

3.3. Using PDE tool box of Matlab

In PDE too box, we construct two layers as follows

a) Water column with its depth of 100 m
b) Sand sea bed with its depth of 10 m.

The sound wave equations for those layers are equations (9) and (10). Sound source is located at the depth of 50 m with the frequency of 250 Hz. Therefore, we have the $k_1 = \frac{\omega}{c_1} = \frac{\pi}{3}$ and the $k_2 = \frac{\omega}{c_2} = \frac{5\pi}{17}$.

The initial conditions for the point source located at the depth of and with frequency of 250 Hz is established.The boundary conditions [8] are setting up as follows:

a) Upper surface, Lower water column, Upper sea bed, Lower sea bed: Neumann condition

b) Left, Right: Dirichlet condition

4. SIMULATION RESULTS

The underwater sound propagation in Tonkin gulf is simulated using PDE tool box of Matlab. They are depicted in Figure 1, Figure 2 and Figure 3 as follows. From all three figures, we can see clearly the 3D visualization of the underwater sound propagation in Tonkin gulf.

It can be seen that the depth of water column is 100 m, the depth of sea bed is 10 m, the transmission range is 2 km and the sound pressures are oscillating according to mode theory. In addition, the number of real mode are the same at all time, i.e, in all figures. However, the mode fashions are different from time to time. It means that they are different from Figure 1 to Figure 3.

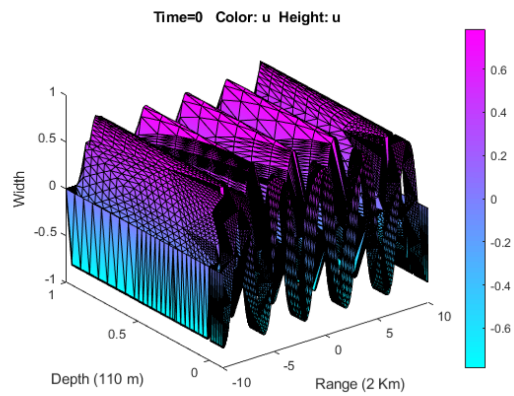

Figure 1. 3D visualization of underwater sound propagation at time 0

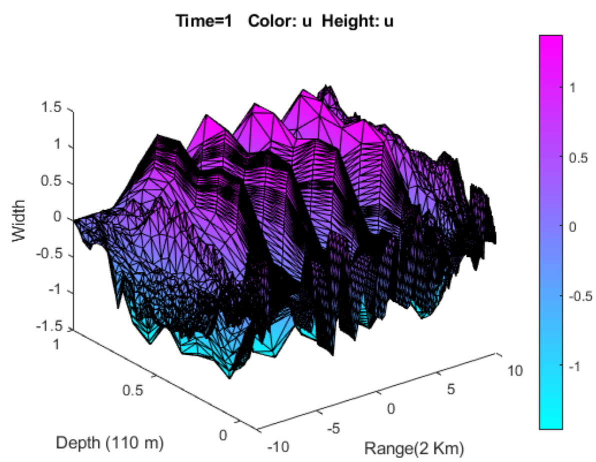

Figure 2. 3D visualization of underwater sound propagation at time 1

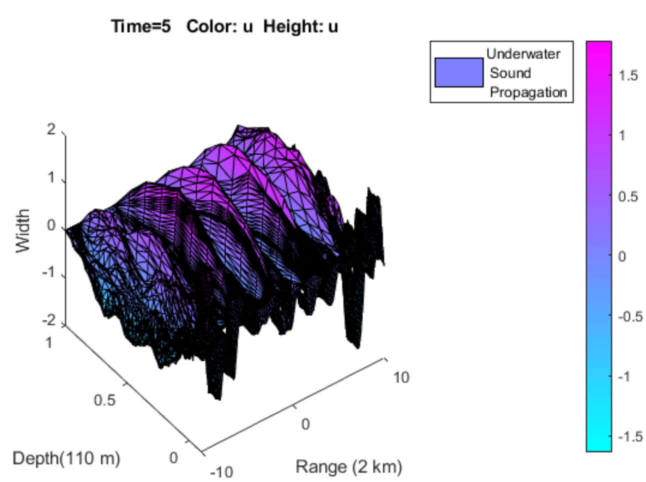

Figure 3. 3D visualization of underwater sound propagation at time 5

5. CONCLUSION

In this paper, we successfully investigate a method of 3D modeling of underwater sound propagation in Tonkin using Normal mode theory and PDE tool box of Matlab. With the water depth of 100 m, sea bed depth of 10 m and the transmission of 2 km, the analysis and simulation results show the possibility of 3D visualization of the proposed method in Tonkin gulf as expected. The number of real mode are the same at all time but the mode fashions are different from time to time. This obtained results are very useful for SONAR applications.

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